Experimental realization of a microscopic Carnot engine

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I. INTRODUCTION

Carnot engine played a crucial role in the development of thermodynamics, setting a fundamental upper limit to the efficiency of a motor operating between two thermal baths. Nowadays, micromanipulation techniques make it possible to explore the thermodynamics of small systems at scales where fluctuations cannot be neglected. In this contribution, I will present an experimental realization of a Carnot engine with a single optically trapped Brownian particle as working substance. We have fully characterized the thermodynamics of the engine when operating both in and out of equilibrium, observing that our device reaches Carnot efficiency for slow driving. I will also discuss the fluctuations of the finite-time stochastic efficiency, showing that Carnot efficiency can be surpassed in individual or ensembles of a few number of non-equilibrium realizations of the engine. Finally I will briefly comment on the stochastic efficiency large deviation behaviour which could provide information about the fundamental characteristics of the engine. A number of open questions arise due to the stochastic nature of the processes realized with a single Brownian particle which will be addressed in the final section.

II. EXPERIMENTAL SETUP

Fig. (1) depicts our experimental setup[1,2] which consists in a single polystyrene sphere of radius $R = 500$nm immersed in water and trapped with an optical tweezer. A pair of aluminium electrodes located in the chamber are used to apply a voltage of controllable amplitude. When a random electric field is applied to the electrodes, the colloidal particle experiences a random force that mimics a higher temperature thermal reservoir[2]. The effective temperature of the particle is related to the intensity of the random force and can be obtained from the enhanced position fluctuations:

$$T_{\mathrm{kin}} = \frac{\kappa \langle x^2 \rangle}{k}. \tag{1}$$

Using our setup, we can realize any thermodynamic process in which the stiffness of the trap and the kinetic temperature of the particle can change with time arbitrarily following a protocol $\{\kappa(t), T_{\mathrm{kin}}(t)\}$.

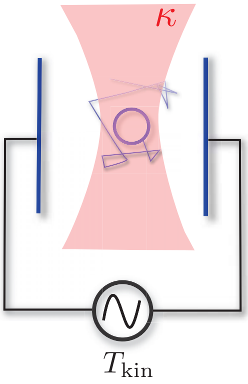

FIG. 1. Schematic of the experimental setup

III. MEASURING KINETIC ENERGY

For the case of microscopic dielectric beads immersed in water, the momentum relaxation time is of the order of nanoseconds. Therefore, in order to accurately measure the instantaneous velocity of a Brownian particle, it would be necessary to sample the position of the particle with sub-nanometer precision and at a sampling rate above MHz.

In our experiment, we do not have direct access to the instantaneous velocity due to our limited sampling frequency which is in the kHz range. However, we have developed a technique that allows to extrapolate the instantaneous velocity from the time averaged velocity (TAV) $\overline{v}_f$ over a time $\Delta t = 1/f$. With this technique, the kinetic energy changes can be measured giving access to the full thermodynamics of the particle. Work, heat, potential and kinetic energy and entropy changes can be measured in this setup[3].

IV. MICROADIABATICITY

Until now, the design of microscopic heat engines has been restricted to those cycles formed by isothermal processes or instantaneous temperature changes[4] . Among all the non-isothermal processes, adiabatic processes are of major importance in thermodynamics since they are the building blocks of the Carnot engine.

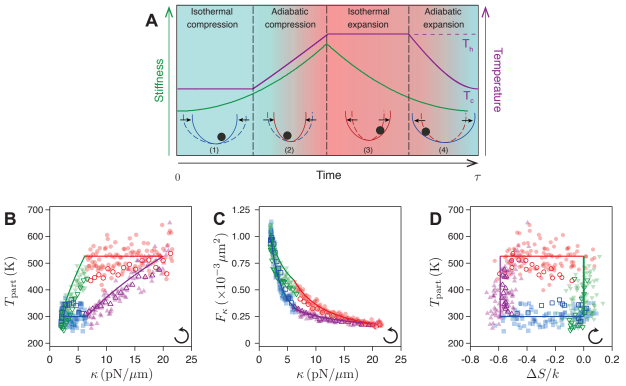

FIG. 2. The Brownian Carnot engine. (A) Time evolution of the experimental protocol. (B-D) Thermodynamic diagrams of the engine: Isoth. compression (blue); Adiab. compression (magenta); Isoth. expansion (red); Adiab. expansion (green). Solid lines are the values in the quasistatic limit. Filled symbols are obtained from ensemble averages over slow cycles while open symbols are obtained fast cycles. (B) $T-\kappa$ diagram. (C) Clapeyron diagram (D) $T-S$ diagram.

Microadiabaticity, i.e. adiabaticity at the microscopic scale, cannot be realized for single-trajectories due to the unavoidable heat flows between microscopic systems and their surroundings. However, a process where no net heat transfer is obtained when averaged over many trajectories could in principle be realized.

We have experimentally realized adiabatic processes[5] following a theoretical proposal aimed at keeping constant the phase space volume enclosed by the energy surface[6,7]. In the adiabatic protocol both T and the stiffness κ are modified keeping T^2/κ constant. To achieve this, we have taken advantage of the aforementioned technique to measure kinetic energy changes, as a full underdamped description is mandatory to take into account entropy changes in the velocity degree of freedom.

V. BROWNIAN CARNOT CYCLE

The experimental realization of a Carnot cycle with a single Brownian particle had previously remained elusive due to the difficulties of implementing an adiabatic process. In our setup, the Carnot cycle is implemented by modifying the stiffness κ and the temperature of the particle in a sequence of two isothermal steps joined by two adiabatic steps as in Fig. (2A)[8].

Taken all together, the thermodynamic diagrams under quasistatic driving (Figs. (2B-D)) are equivalent to those for a single particle ideal gas in a Carnot cycle. We have analyzed both the average power extracted and the efficiency of heat to work conversion. The efficiency is given by the ratio between the extracted work and the input of heat,which is usually considered as the heat flowing from the hot thermal bath to the system. In our experiment, however, there is a non-zero fluctuating heat in the adiabatic steps, which must be taken into account in the definition of the stochastic efficiency of the engine during a finite number of cycles. In the quasistatic limit our engine attains approximately Carnot efficiency. We have also tested a number of theoretical predictions of the stochastic thermodynamics for a Carnot engine in our setup regarding the distribution of the fluctuating efficiency[9,10].

VI. OPEN QUESTIONS

A number of questions remain open and are the subject of current and future work. In the contribution I will try to discuss some of them:

- Can adiabatic processes, either at realization or average level, be realized in other microscopic systems in contact with a thermal bath?
- In which cases must the efficiency include the fluctuating heat in the adiabatic steps?
- Are there reversible trajectories with finite power?
- Can we improve the efficiency of the finite time Carnot engine using measurements?

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