Stochastic resonance and diversity-induced resonance in complex systems

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Complex systems theory is a conceptual framework for understanding many phenomena from the nano-scaled condensed matter to the macroscopic social scale, characterized by emergent properties produced by the non-linear interactions between the constituent units, in general surprisingly different from those of the units themselves. Among the many examples known, such as synchronization, phase transitions, and pattern formation, here we explore some instances of emergent phenomena induced by disorder. Disorder can mean both *noise*, i.e. time-dependent disorder in the form of e.g. a random force acting on a particle, a noisy background, or random fluctuations of some parameter, and *heterogeneity*, i.e. quenched disorder affecting some features of the units composing the system, related to e.g. the different ion-channels in a cell, neurons with heterogeneous parameters in a neuronal network, different individuals in a social group, heterogeneous economic agents in a market economy, or species with different fitness in an ecological system.

The subject of this study is the effect resulting by the interplay between noise and diversity. For this reason the focus is on systems that present both stochastic resonance, that takes place at intermediate amplitudes of noise[1], and diversity-induced resonance, that appears for a suitable (i.e. neither too high nor too low) level of quenched disorder in the form of heterogeneity of the constituent units[2–4]. Previous investigations have concentrated on the study of noise and diversity treated as independent phenomena. For instance, some of the authors showed that a multi-neuronal model of the wake-sleep cycle presents stochastic resonance as well as diversity-induced resonance (with a clear evidence of diversity-induced resonance and a milder level of stochastic resonance)[5], see the example in Fig. 1. On the other hand, here we consider the actual interplay between noise and diversity.

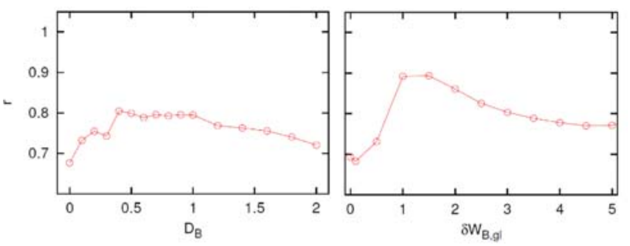

FIG. 1. The coefficient r providing a quantitative estimate of the quality of a sleep-wake cycle[5] presents both a stochastic resonance when studied in *versus* the noise amplitude (left) and diversity-induced resonance *versus* the diversity level of the neuronal activation thresholds (right), see Ref. [5] for further details.

A first question that we consider and try to answer is related to the way noise and diversity should interact with each other in order to produce the optimal response of a system, e.g. which parameters it is best to diversify and which parts of the system should undergo random fluctuations. In turn this depends on the specific type of dynamical system under study. To this aim we review previous work on this topic and presents the results of numerical studies of multicomponent systems with bistable, excitable, and other types of constituent units.

A second question is whether there exists a general connection between stochastic resonance and diversity-induced resonance, enabling one to predict the possible appearance of diversity-induced resonance in a system when the (corresponding noisy single-particle version of the) system is known to present stochastic resonance.

At a general level, this research is motivated by the well known fact that an appreciable level of both noise and quenched disorder is naturally present in all biological and social systems. It is a natural question to ask whether there is a reason for the evolution of different types of system toward a similar state with a combined level of noise and diversity, that are both known to be able to play a relevant role in e.g. improving the response, resilience, or performance of a system subject to external perturbations and interactions[4].

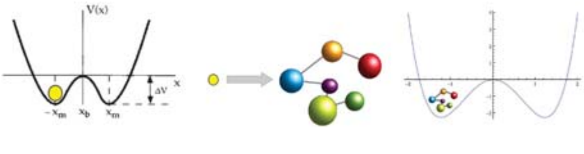

FIG. 2. The prototypical diversity-induced resonant system[2] (right) can be considered as a generalization of the prototypical stochastic resonant system[1] (left) obtained by replacing the Brownian particle by a heterogeneous polymer.

Finally, this work is related to and extends the implications of the links between stochastic resonance and diversity-induced resonance that have been pointed out or implicitly demonstrated by different authors. The original introduction of the concept of diversity-induced resonance is in fact in the perspective of an analogy with a stochastic resonant system[2]. The analogy can be visualized by constructing the diversity-induced resonant system from the famous example of a stochastic resonant system of a Brownian particle in a quartic bistable potential through the replacement of the Brownian particle by a heterogeneous polymer, see Fig. 2. As a different example, the studies of the phenomenon of supra-threshold stochastic resonance[6] employed some particular model of multi-component system that can be re-interpreted as a heterogeneous systems that in principle presents diversity-induced resonance, see Fig. 3.

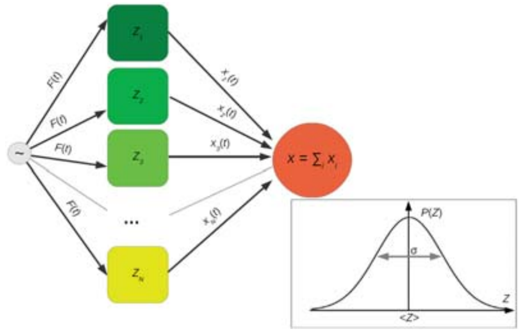

FIG. 3. This supra-threshold stochastic resonant system[6] with input signal $F(t)$ and output signal $x = \sum_i \Theta(F(t)-Z_i)$, where $\Theta(.)$ is the step function, can represent a stochastic resonant or a diversity-induced resonant system depending whether the thresholds Z_i are assumed to be diversified by independent noises $\xi_i(t)$, $Z_i = \langle Z \rangle + \xi_i(t)$), or extracted from a suitable distribution $P(Z)$.

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