Study on the origin of 1/f noise in bulk acoustic wave resonators

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I. INTRODUCTION

The Centre National d'Etudes Spatiales (CNES), Toulouse, France and the FEMTO-ST Institute, Besancon, France, investigate the origins of 1/f noise in bulk acoustic wave resonators. Several European manufacturers of high quality resonators and oscillators are involved in this partnership. The goals are first to improve the yield of excellent resonators and second to assess the intrinsic lower limit for the resonator noise.

In this contribution, we first give some relevant information about the realization of the 5 MHz SC-cut resonators used in this study. Then, we report the resulting short term stability of the resonators as a function of the position of the resonators inside the crystal block. Concerning the second goal, a theoretical approach, based on the fluctuation-dissipation theorem, is used in order to put numerical constraints on a model of 1/f noise caused by an internal (or structural) dissipation proportional to the amplitude and not to the speed. The order of magnitude of the noise is then discussed using a candidate physical process. Finally, we conclude on the work that could be done to solve the remaining open problems.

II. RESONATOR REALIZATION

For this investigation, quartz crystal resonators have been cut from a quartz crystal block supplied specifically for this study on 1/f noise (cf. Fig. 1). This crystal block is obtained from a seed cut in a previous synthetic crystal which was grown using a natural seed.

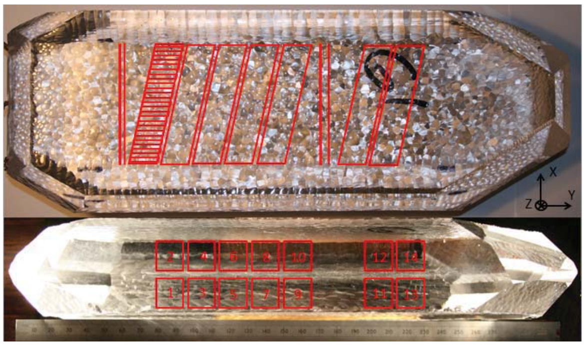

Figure 1 : Quartz crystal resonators according to their positions in the mother block.

Its dimensions were approximately 220 mm along the Y-axis, 36 mm along the Z-axis and 110 mm along the X-axis. Two Y-cut slices have been cut before and after an oriented block used to achieve ten quartz bars. The Y-cut slices are used to obtain dislocations evaluation by X-ray topographies. The red marks show how the crystal is cut in order to get SC-cut blanks from 14 initial bars. First, Fourteen quartz bars pre-oriented on the first rotation angle have been achieved. The length of the bars was about 70 mm. Taking into account the width of the cutting saw, about 24 blanks could be obtained in each bar. They were distributed to the various manufacturers part of the project, so that they could make 5 MHz SC-cut resonators out of them and give them back for analysis to FEMTO-ST and CNES.

III. EXPERIMENTAL RESULTS

The passive technique using carrier suppression is used to characterize the inherent phase stability of the ultra-stable resonators.

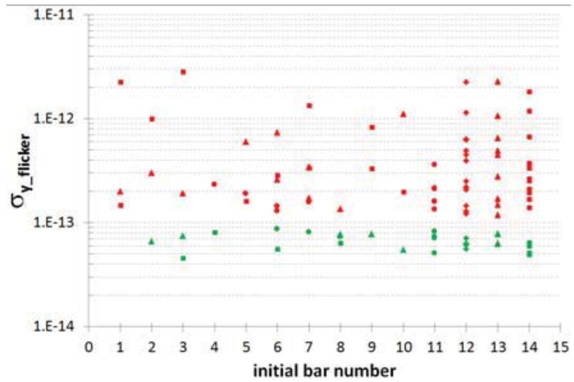

Figure 2 : Short-term stability of quartz crystal resonators according to the position in the mother block.

The short-term stabilities of a large portion of these resonators have been measured to be lower than $8{\cdot}10^{-14}$. However, rest of them shows much worse results around few 10^{-12}, though they have been fabricated with exactly the same process alike the best ones. Although the positions of the blanks are known, no clear correlation between the noise results and the blanks positions (e.g. center or edges) can be found for theses bars. We are consequently carrying investigations on the origin of these differences.

IV. THEORETICAL APROACH

The fluctuation-dissipation theorem[1,2] (FDT) is used to estimate the power spectral density of thermal noise coming from fluctuations in the thickness (2h) of quartz resonators (cf. Fig. 3).

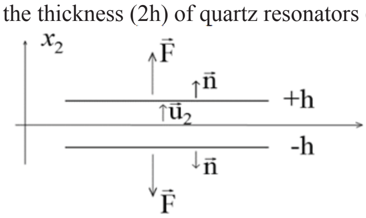

Figure 3 : Resonator design.

An internal friction term, φ, is added in the formulation, in order to obtain a 1/f spectrum at low frequencies. Indeed, for this mode (characterized by the mechanical displacement inside the resonator $u_2(x_2,t)$), the fundamental principle of dynamics for continuum media can be written as:

$$\rho\frac{\partial^2 u_2}{\partial t^2} = c_{22}(1+j\varphi)\frac{\partial^2 u_2}{\partial x_2^2} + \eta_{22}\frac{\partial^3 u_2}{\partial x_2^2 \partial t} \quad (1)$$

with c_{22} the elastic constant and η_{22} the viscoelastic damping constant of quartz crystal. φ is an internal friction coefficient[1,2], ρ the quartz mass per unit volume. Searching for solutions of the type:

$$u_2(x_2,t) = [a\sin(kx_2) + b\cos(kx_2)]e^{j\omega t} \quad (2)$$

with limit condition given by:

$$T_2(\pm h,t) = F\cdot e^{j\omega t}/S \quad (3)$$

with F the modulus of the harmonic mechanical force applied to the surface S of the electrodes (perpendicular to x_2). The complex mechanical admittance of the system is defined by:

$$\bar{Y}(\omega) \equiv \frac{\frac{\partial u_2(\pm h,t)}{\partial t}}{\pm F\cdot e^{j\omega t}} = \frac{j\omega a\sin(kh)}{F} \quad (4)$$

The FDT then states that the spectral power density of the thickness fluctuations in a bandwidth BW, can be computed by[1,2]:

$$\frac{u_2^2(\pm h,\omega)}{BW} = \frac{4k_BT}{\omega^2}Re\left(\bar{Y}(\omega)\right) \quad (5)$$

with T the absolute temperature (in K) and k_B the Boltzmann constant (in J/K).

The assumptions $\varphi \ll 1$ and $\omega \ll c_{22}/\eta_{22}$ lead to:

$$\frac{u_2^2(\pm h,\omega)}{BW} \approx \frac{4k_BTh}{S\omega c_{22}^2}(\eta_{22}\omega + c_{22}\varphi) \quad (6)$$

Moreover, we can consider that the circular frequency at resonance $\omega_r \sim 1/h$, thus:

$$S_y(\omega) \equiv \frac{(\delta\omega_r)^2}{\omega_r{}^2BW} = \frac{u_2^2(\pm h,\omega)}{(2h)^2BW} \approx \frac{1}{\omega}\times\frac{2k_BT}{Vc_{22}}\left(\frac{\eta_{22}}{c_{22}}\omega + \varphi\right) \quad (7)$$

where V is the volume of the resonator. One can then see from the previous expression that for circular frequencies lower than $\varphi\, c_{22}/\eta_{22}$, the internal friction becomes dominant and gives a $1/f$ spectrum, with an Allan standard deviation given by:

$$\sigma_{y\_flicker} = \sqrt{2\ln(2)\frac{2k_BT}{Vc_{22}}\varphi} \quad (8)$$

We note that φ could depend upon the temperature and that no assumption where made about this possible dependence. We consider here numerical values typical for a 5 MHz oscillator equipped with an SC-cut quartz crystal resonator. Due to the rotation of the axis, the 2 axis is not the usual one, so that the constants must be evaluated in the rotated basis: $c_{22} = 115$ GPa, $\eta_{22} = 1.36\cdot10^{-3}$ Pa·s, $T = 350$ K and $V = 0.104$ cm^3. This gives:

$$\sigma_{y\_flicker} \approx 1.06\cdot10^{-12}\sqrt{\varphi} \quad (9)$$

In order to recover measured values of $\sigma_{y\_flicker}$ with this expression, we would need to have φ between 10^{-2} and 10^{-4}, which would mean that even at resonance the internal damping would be dominant over viscoelastic damping. We therefore conclude that internal damping of thickness fluctuations by any force proportional to strain and independent of frequency, may not be the dominant noise mechanism for the best SC-cut quartz resonators. However, other modes may be noisier…

Nonetheless, we try to evaluate this coefficient by the modified Granato-Lücke theory[3] of the energy loss due to some kinds of dislocation motion in the low frequency range. They supposed first that the pinning force F of the impurity atom which arises from elastic interactions depends on the orientation of the dislocation line. Second, they supposed that, once a dislocation has broken away from its pinning points, its motion is not necessarily limited by its line tension, but that the distance it moves may be determined by the stress field of neighboring impurity atoms. With these assumptions, they found an expression of the decrement for the impurity spacing controlled dislocation motion that, in the small stress amplitude limit, is given by:

$$\Delta = \frac{\beta NbL_N}{\pi c^{1/3}\epsilon} \quad (10)$$

Where β is a parameter having approximate value of 1.5. N is the total length of dislocation line in a unit volume of material $\approx 2\times$ surface dislocation density. This value is of the order of 6 cm/cm^3 judging from an X-ray image of the surface of one of the resonator. b is the mean length of a Burger's vector $\approx 3\times10^{-8}$ cm. L_N is the network length $=\sqrt{(3/N)}$. c is the atom fraction of impurity which must be lower than 1 ppm to get Q values as high as a few 10^6. ϵ is the fractional difference between the radius of impurity and host atoms taken to be of the order of 20 %. Finally, we recall that, previously, we saw that $1/Q_{eff} = 1/Q_{viscous} + \varphi$, with $1/Q_{viscous} = \frac{\eta_{22}\omega}{c_{22}}$. Therefore at low frequencies $1/Q_{eff} \approx \varphi$. Hence, we attempt to identify Δ with $\pi\varphi$ at low frequencies, in a first approximation in spite of the fact that we are not in the dominantly viscous regime. This would give $Q_{eff} \approx 10^5$ and $\varphi \approx 10^5$ in the low frequency regime, which would be an interesting order of magnitude to attribute at least some non-negligible part of the $1/f$ noise to the fluctuations of thickness. However, this would also mean that at resonance $\frac{1}{Q_{eff}} = \frac{1}{Q_{viscous}} + \varphi \approx 4\cdot10^{-7} + 10^{-5} \approx 10^{-5} = \varphi$, hence that the viscous damping would not be dominant at resonant frequency which is contradictory to experimental facts.

V. CONCLUSION

We have seen that it is possible to find 1/f noise through the fluctuation-dissipation theorem, by adding a constant complex part to the elastic constant in the usual differential equation characteristic of a viscously damped harmonic oscillator. This corresponds to a frequency independent energy loss in the limit of small frequencies. The hysteretic motion of the dislocations described by a modified Koehler-Granato-Lücke model could a priori describe such a loss mechanism. Indeed, it could provide an explanation for the experimental observations that the logarithmic decrement generally decreased when the dislocation density decreased when quartz were not as good as now and that sometimes a slightly higher concentration of impurity could improve the quality factor. However, numerical estimations seem to provide values that are at least an order of magnitude too high. Hence the physical origin of 1/f noise in quartz crystal ultra-stable oscillators still remains an open question. We therefore plan to study another approach based on thermally activated nucleation and motion of kink-antikink pairs along dislocations, with possibly several different activation energies. This could lead to 1/f noise by the mechanism of Lorentzian summation.

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