Effects of non-Gaussian α-stable noise sources on the transient dynamics of long Josephson junctions

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I. INTRODUCTION

The transient dynamics of noisy *Josephson junctions* (JJ) is studied by a theoretical analysis. A JJ is a mesoscopic system in which macroscopic quantities, as voltage and current, are directly dependent on the *order parameter* φ through the well-known Josephson relations. The parameter φ is the phase difference between the macroscopic wave functions describing the superconducting condensate in the two electrodes forming the device.

The behavior of Josephson junctions is strongly influenced by environmental perturbations, and specifically by the presence of noise source responsible for decoherence phenomena. The role played by random fluctuations in the dynamics of these devices has recently solicited a large amount of work and investigation on the effects both of thermal and non-thermal noise sources on their transient dynamics[1–4]. In the last decade, theoretical progress allowed one to calculate the entire probability distribution of the noise signal and its cumulants, performing a *full counting statistics* of the current fluctuations[2]. Moreover, the presence of non-Gaussian noise signals has been found experimentally in many systems[1,3]. Non-equilibrated heat reservoir is an example of non-Gaussian noise source. Within this context, experimental investigations have been performed, studying the effect of sources of non-Gaussian fluctuations on the average escape time from the superconducting metastable state of a current biased JJ[1,3].

Recently, the characterization of JJs as detectors, based on the statistics of the escape times, has been proposed[5–10]. Specifically, the statistical analysis of the switching from the metastable superconducting state to the resistive running state of the JJ has been exploited to detect weak periodic signals embedded in a noisy environment[6,7]. Moreover, the escape rate from one of the metastable wells of the tilted washboard potential of a JJ has been used to encode information on the non-Gaussian noise present in the input signal[5,8–10]. After the seminal paper of Tobiska and Nazarov[4], JJs used as threshold detectors allow to study non-Gaussian features of the current noise[8,9]. Specifically, when a JJ leaves the metastable zero voltage state, it switches to a running resistive state and a voltage appears across the junction. Therefore, it is possible to measure directly in experiments the escape times or switching times and obtain their probability distribution[11–14].

Motivated by these studies, in view of a deeper understanding of the transient dynamics of a JJ interacting with a noisy environment, we carry out an analysis on the role of non-Gaussian noise sources in the switching times of a long JJ. The dynamics of this system is ruled by a nonlinear PDE, the perturbed *sine-Gordon* (SG) equation, which includes in this work a stochastic term. The noise signal is modeled by using different α-stable (or *Lévy*) distributions $S_\alpha(\sigma, \beta, \mu)$, where $\alpha \in]0, 2]$ is the *stability index* (or characteristic exponent) which determines how the distribution tails go to zero, β ($|\beta| \leq 1$) is an *asymmetry parameter*, and $\sigma > 0$ and μ are two real numbers which define the profile of the distribution and are called, for this reason, *shape parameters*. Specifically $\beta = 0$ ($\beta \neq 0$) gives a symmetric (asymmetric) distribution. These statistics allows to describe real situations, in which some variables of the studied system show abrupt jumps and very rapid variations called *Lévy flights*[15]. The behavior of short and long JJs in the presence of non-Gaussian noise sources was previously studied[16,17] considering only few Lévy distributions, namely those with a probability density function for which an explicit form is known, that is Gaussian ($\alpha = 2$, $\beta = 0$), Cauchy-Lorentz ($\alpha = 1$, $\beta = 0$) and Lévy-Smirnov ($\alpha = 0.5$, $\beta = \pm 1$) distributions. In particular, the mean switching times (MST) and the mechanisms ruling the escape events from the metastable superconducting state were explored, considering also the effect of an external sinusoidal driving. Furthermore, the noise induced generation of nonlinear SG excitations along the junction, i.e. solitons and breathers, was taken into account. Fig. 1 shows the phase dynamics of junctions of different lenghts ($L = 2$ and $L = 15$)[17]. Considering lengths larger than a certain threshold value, the junction switches to the resistive state by soliton creation (left panel of Fig. 1), that is by formation of 2π steps in φ. Otherwise, the junctions elements move between the metastable states as a whole (right panel of Fig. 1). Solitons are the pillars of the transient dynamics in long JJ, so the noise induced effects in the MST behavior has to strongly depend on the solitons originated by random fluctuations. The soliton solutions are strongly stable, appearing clearly in the I-V characteristic of the junction, and maintaining their shape after collisions or reflections at the boundaries. Instead a breather is a travelling SG solution formed by a soliton-antisoliton bounded couple, oscillating with a proper internal frequency. Many practical difficulties to

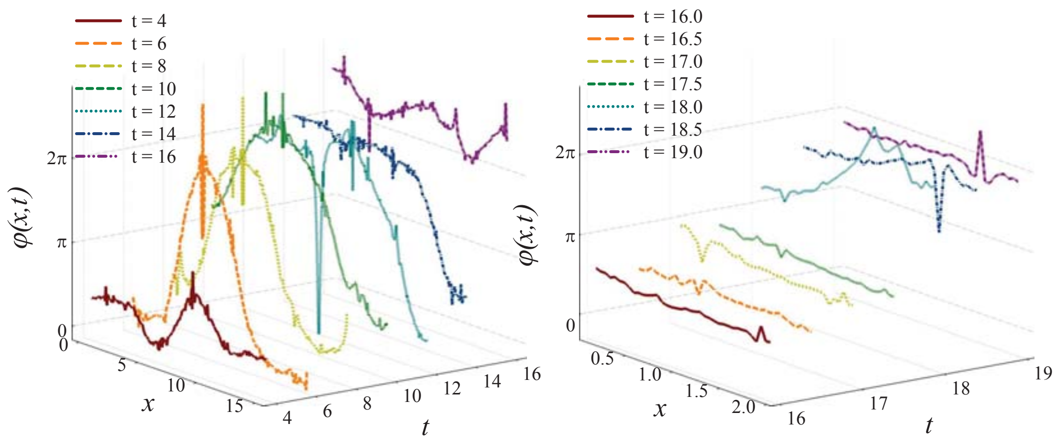

FIG. 1. LJJ phase dynamics during the switching to the resistive state using a Cauchy-Lorentz noise source. The length of junction is $L = 2$ (right panel) and $L = 15$ (left panel). The curves show the characteristic Lévy flights of the CL statistics inducing nonlinear excitations along the junction. Figure adapted from 17.

experimentally evidence breathers in long JJ exist. Indeed, breathers are unstable and rapidly decay in time, moreover the magnetic flux associated with a breather is zero, as well as the voltage difference across the junction. Therefore, an actual challenge in this field, both from experimental and theoretical point of view, is the capacity to generate, observe and sustain breathers in long JJ.

The mechanisms ruling the nonlinear modes excited in a SG system in the presence of non-Gaussian noise sources is a research field not completely explored. The focus of this work is therefore a deeper investigation of the transient dynamics of the phase difference φ in a long JJ, subject to noise sources with different α-stable distributions $S_\alpha(1,\beta,0)$. In particular, the MST as a function of the noise intensity, the frequency of the driving force and the junction length is studied, exploring the complete spectrum of α and β values within the intervals $]0,2]$ and $[-1,+1]$, respectively. Interesting nonmonotonic behaviors are expected to characterize the MST data, namely *noise enhanced stability* (NES) and *resonant activation* (RA) effects. From a theoretical point of view, the appearance of breathers, induced by Lévy flights, was highlighted by Valenti *et al*[17], modeling the noise source *only* by the CL statistics (see the narrow peaks in the curves of Fig. 1). When α-stable statistics different from Gaussian, Cauchy-Lorentz and Lévy-Smirnov are used, a lack of knowledge exists about the characteristics (rate of appearance, oscillation frequency, intensity) of nonlinear SG excitations induced by the noise. In particular, in view of a deeper comprehension of the role of the noise in the transient dynamics of long JJs, it has to be further studied how the features of different Lévy-distributed noises (fat tales, mode and limited space displacement around it) affect the SG nonlinear modes. Finally, another interesting open problem is the role of the breathers in the out of equilibrium dynamics of long JJs.

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