From cell membranes to ultracold gases: classical and quantum diffusion in inhomogeneous media

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I. INTRODUCTION

Brownian motion is one of the most fundamental phenomena of physics, and its discovery and study contributed to the birth of contemporary statistical physics and theory of stochastic processes. It finds wide applications in many branches of science, including physics, chemistry, biology and economics.

A long series of studies, however, indicates that transport in condensed matter and living systems is often far from random. As an example, many cellular components exhibit anomalous diffusion, i.e., a mean-squared displacement MSD$\sim t^{\beta}$ with $\beta \neq 1$, and recent works even evidenced clear signatures of nonergodic behavior. Presently open are many questions on what are the physical mechanisms generating non-ergodicity, what are the implications of anomalous diffusion for biological function, and more generally how complex environments affect Brownian motion.

II. CLASSICAL BROWNIAN MOTION

A celebrated model yielding anomalous, subdiffusive and nonergodic dynamics, widely used in biology and condensed matter is the so-called Continuous-Time Random Walk (CTRW), whose underlying assumption is that particles, while diffusing, wait at random positions for anomalously long times. However, transient trapping is not the only possible source of transport anomalies, as spatial and temporal disorder may have important consequences in this direction.

In a recent theoretical work, we introduced models which describe particles diffusing in a complex and inhomogeneous medium consisting of patches with random sizes and random diffusivities[1]. The particles are never trapped, but rather perform continuous Brownian motion with the local diffusion constant. Under simple assumptions on the distribution of diffusivities D in each patch, such as

$$P_D(D) = \frac{D^{\sigma-1} e^{-D/b}}{b^{\sigma}\Gamma(\sigma)}, \tag{1}$$

and of the traversal times τ of each patch, such as

$$P_\tau(\tau|D) = \frac{D^{\gamma}}{k} e^{-\tau D^{\gamma}/k}, \tag{2}$$

with b and k appropriate dimensionful constants, we find that the mean squared displacement displays subdiffusion due to non-ergodicity for both annealed and quenched disorder, see Fig. (1). Our model is formulated as a walk continuous in both time and space, similar to the Lévy walk.

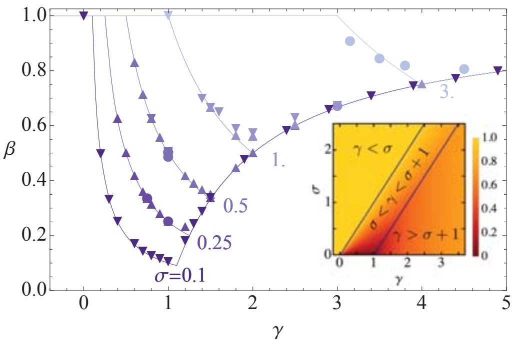

FIG. 1. **Subdiffusion exponent** β**.** Lines are the analytic predictions for different values of σ. Symbols are numerical simulations of various annealed models with spatial or temporal disorder. Lines and symbols vary from dark to light with increasing σ. The inset shows a density plot of β vs. both γ and σ.

In a complementary experimental work, we used single particle tracking on living cells to demonstrate that the motion of the transmembrane receptors DC-SIGN reveals nonergodic subdiffusion on living cell membranes, see Fig. (2). In contrast to previous studies, this behavior resulted incompatible with transient immobilization, and therefore it can not be interpreted according to continuous time random walk theory. We show instead that receptors undergo changes of diffusivity, consistent with the current view of the cell membrane as a highly dynamic and diverse environment. Simulations based on the above mentioned theoretical model of ordinary random walk in an inhomogeneous medium quantitatively reproduce all our observations, pointing towards diffusion heterogeneity as the cause of DC-SIGN anomalous behavior. By studying different receptor mutants, we further correlated receptor motion to its molecular structure, thus establishing a strong link between nonergodicity and biological function. Our results highlight the fundamental role of disorder in cell membranes, and its connection with function regulation.

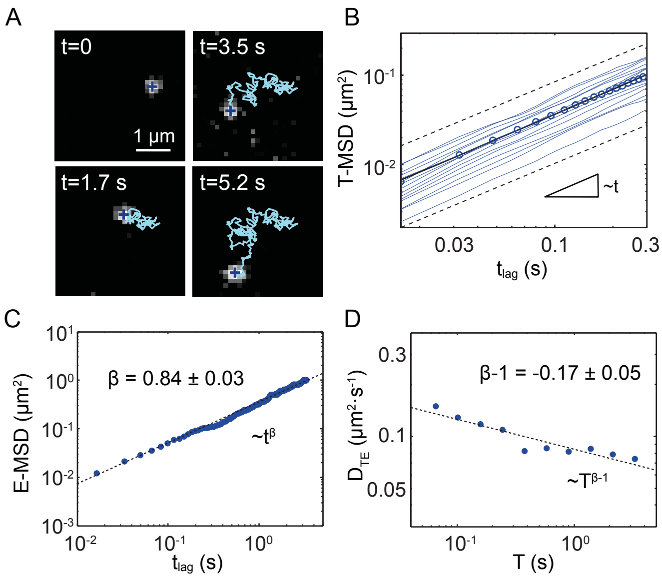

FIG. 2. **DC-SIGN diffusion shows weak ergodicity breaking and aging.** (A) A quantum-dot-labeled wtDC-SIGN molecule diffusing on the cell membrane. (B) The time-averaged MSD for individual trajectories scales linearly in time, compatibly with pure Brownian motion. (C) The ensemble-averaged MSD shows marked subdiffusion. (D) The time-ensemble-averaged diffusion coefficient shows non-stationarity (aging) of the process as a function of the total observation time T.

III. QUANTUM BROWNIAN MOTION

Quantum Brownian motion, although studied since half a century, has not yet found convincing experimental realizations and observations. Recent experiments on trapped ultracold atomic gases provide unprecedented precision and control that allow us to hope to observe effects of quantum Brownian motion in a very near future. But the presence of the external trapping potential introduces a novel complexity level into the well studied problem. In a recently published work, we revise the standard theory of quantum Brownian motion and consider in detail the case when a quantum Brownian particle is moving in a spatially inhomogeneous environment, such as the one provided by a trap[3]. This leads to spatially dependent diffusivity and, consequently, to spatially dependent decoherence and damping rates. As a result of these intrinsically nonlinear relations novel quantum effects occur: the interaction of the quantum Brownian particle with such environment might induce effective cooling of its state, and even squeezing of the fluctuations of its motion, see Fig. (3).

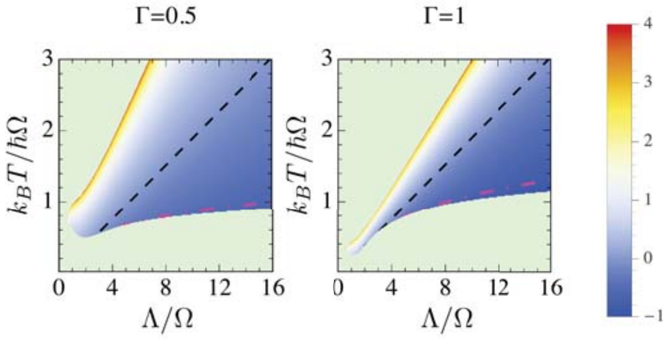

FIG. 3. **Shape of the stationary distributions.** Aspect ratio of the impurity wavefunction, $\ln(\delta_x^2/\delta_p^2)$, for the case of a quadratic coupling with the environment, as a function of spectral density cut-off Λ and temperature T; left (right): weak (strong) damping. The impurity shows "cooling" (i.e., $\delta_x^2 < \delta_p^2$) below the black dashed line, and "quantum squeezing" (i.e., $\delta_x^2 < 1$) below the magenta dotted-dashed line.

ACKNOWLEDGMENTS

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