Classical and quantum non-linear dynamics in optomechanical systems

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I. INTRODUCTION

Light-matter interaction is one of the most complex phenomena in natural sciences. It has been studied for 400 years as classical optics and for 50 years as quantum optics, and it still remains an active research field. The simplest device which demonstrates light-matter interaction is an optomechanical cavity, regarded as a single mode of electromagnetic radiation confined between two mirrors. One of the mirrors is static, and another one is attached to a spring and can mechanically move. Light and mechanical motion are coupled by the radiation pressure[1]. Massive effort in cavity optomechanics resulted in the recent years in a number of ground-breaking experiments, for example, demonstration of cooling of mechanical resonators to their quantum mechanical ground state, optomechanical squeezing of light, and observation of radiation pressure shot noise.

In optomechanical experiments, both cavities and mechanical resonators are perfectly linear.

An important development of the last few years was circuit (microwave) optomechanics — a technology in which mechanical elements are embedded into (superconducting) microwave cavities. Whereas microwaves and light are both electronagnetic waves, and the physical principles of cavity optomechanics and microwave optomechanics are similar, the setups and the operation regimes are very different. An important aspect we address here is that microwave cavities can be made intrinsically non-linear by adding Josephson junctions and can be integrated with non-linear mechanical elements, for example based on multi-layer graphene. This opens a whole new field of non-linear optomechanical systems.

The optomechanical system is described by a compact Hamiltonian, written in the second-quantized form as follows,

$$\hat{H} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar\omega_m \hat{b}^\dagger \hat{b} + \hbar g_0 \hat{a}^\dagger \hat{a} \left(\hat{b}^\dagger + \hat{b}\right) \ . \qquad (1)$$

Here, $\hat{a}$ and $\hat{b}$ are creation operators for photons and mechanical vibrations, respectively, the first term describes the optical or the microwave cavity with the frequency ω_c, the second one — the mechanical resonator with the frequency ω_m, and the last term corresponds to the radiation pressure coupling, with g_0 being the (single-photon) coupling constant. In optomechanical cavities, ω_c is the frequency of visible light (THz range), and therefore the mechanical frequency ω_m, which can range between kHz and GHz, is always several orders of magnitude less than ω_c. In microwave cavities, the cavity frequency is in the hundreds of GHz range, still much greater than ω_m.

A very common step in treating the Hamiltonian (1) is to assume that the number of photons in the cavity is large. Then the coupling term can be linearized, and the Hamiltonian becomes

$$\hat{H} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar\omega_m \hat{b}^\dagger \hat{b} + \hbar g \left(\hat{a}^\dagger + \hat{a}\right) \left(\hat{b}^\dagger + \hat{b}\right) \ , \qquad (2)$$

where g is the so-called multi-photon coupling. It is related to the single-photon coupling by $g = g_0\sqrt{N}$, where N is the average number of photons in the cavity. In the experiments, this number can be made very big, and therefore g is much stronger than g_0, sometimes by several orders of magnitude. In contrast to the non-linear Hamiltonian (1), (2) is linear and thus represents a significant simplification.

II. CLASICAL SQUID GEOMETRY

A simple example of a non-linear microwave cavity of a superconducting quantum interference device (dc SQUID), a superconducting loop with two junctions. dc SQUID is a very sensitive detector of magnetic field. If one of the arms of the SQUID is suspended and vibrates, the sensitivity is sufficient to detect these mechanical vibrations via the variation of the flux. This mechanism of detection of mechanical oscillations was proposed theoretically[2,3] and demonstrated experimentally[4].

Subsequent experiments demonstrated strong backaction at low voltages in a partially suspended SQUID (the mechanical frequency and the quality factor of the mechanical resonator were affected by the biasing conditions of the SQUId, such as the current bias and the magnetic flux)[5] and even self-sustained oscillations in a fully suspended (torsional) SQUID[6].

This is a purely classical problem, which has been solved theoretically by considering classical equations of motion. The two Josephson junctions were modeled in the framework of the resistively and capacitively shunted junction model, when a current-biased junction is represented as a combination of three circuit elements connected in parallel — a Josephson junction proper, a resistor, and a capacitor. The coupling between superconducting phases and the mechanical motion was provided by the position dependence of the magnetic flux and by the Lorentz force acting on the mechanical resonator. Rather than giving here the details of the calculation[5,6], we present here an argument which facilitates the understanding of the origin of the self-sustained oscillations. The equation of motion for the resonator reads

$$m\ddot{x} + m\omega_m Q^{-1}\dot{x} + m\omega_m^2 = F\cos\omega t + \alpha\tilde{I} \ , \qquad (3)$$

where m, Q, and x are the mass, the quality factor, and the coordinate of the (single-mode) mechanical oscillator, the first term on the right hand side stands for the driving force, and the second term represents the Lorentz force and is proportional to the current through the suspended arm of the SQUID.

In practice, the dynamics of the mechanical resonator is several orders of magnitude slower than the dynamics of the phases of Josephson junctions. Furthermore, in the experiments the junctions were overdamped. In this situation, the current $\tilde{I}$ can be approximated by the time-averaged (with respect to the plasma frequency of the SQUID) value, $\tilde{I} \approx C\dot{V} + V/R$, where $V/R = (I^2 - I_c^2)$, I is the bias current, and I_c is the flux-dependent critical current through the SQUID. The flux dependence of I_c provides the dependence of $\tilde{I}$ on x and $\dot{x}$, thus renormalizing the frequency and the quality factor in the oscillator. In particular, the renormalization of the quality factor originates from the term $\dot{V}$. Its sign depends on the magnetic field, and its value diverges at $I \to I_c$, thus providing a possibility for the negative effective quality factor — a necessary condition for self-sustained oscillations, observed in the experiment.

III. OPTOMECHANICALLY INDUCED ABSORPTION

One of the signatures of the quantum nature of electromagnetic radiation in optomechanical systems is electromechanically induced transmission / absorption (OMIT/OMIA), which is a sharp peak/dip in the transmission/reflection spectrum of the cavity exactly at the cavity frequency ω_c if the cavity is driven at the red-shifted frequency $\omega_c - \omega_m$. For a linear cavity, the peak is harmonic. It has been observed in several experiments, including a recent measurements in the microwave cavity with graphene membrane serving as one of the mirrors[7].

The same device[7] showed clear signs of non-linear behavior as the power of the probe increased. This behavior was due to mechanical non-linearities of the graphene membrane. We performed systematic theoretical studies of non-linear response of the microwave cavity coupled to a Duffing oscillator — the simplest model of a non-linear oscillator[8]. The main conclusions are as follows,

- The shape of the peak does not necessarily correspond to the response profile of a driven Duffing oscillator. Instead, the response develops a sharp peak without an inflection point.

- The depth of the OMIA dip is almost independent on the probe power, however, the position depends on it.

- At low probe powers, there is hysteretic behavior, which disappears at higher probe powers.

These theoretical conclusions are in a good agreement with experimental results.

IV. UNSOLVED PROBLEMS

Despite a great success of microwave optomechanics, there are currently more questions than answers in the field. The most interesting issue is how one can reliable manipulate with the mechanical resonator in the quantum regime, and, in particular, whether one can create non-classical states (defined as states with negative Wigner function). A superposition of two lowest mechanical Fock states has been previously achieved but not in the cavity geometry.

For a linear cavity, one can only transfer a Gaussian state of the cavity into a Gaussian state of the mechanical resonator. Thus, to create non-classical states, one needs non-linearity. To access the quantum regime, one needs strong coupling, and thus strongly coupled non-linear cavities become of interest.

Lifetime of quantum states of a mechanical resonator is not really well understood, and the mechanisms of decoherence are not understood at all. The decoherence in quantum non-linear mechanical systems will certainly be one of the most important issues in the coming research.

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