

Frequency-dependent shot noise in single-electron devices interpreted by means of waiting time distributions

Vincent Talbo, Javier Mateos, Tomás González

Department of Applied Physics

University of Salamanca

Spain



VNIVERSIDAD
D SALAMANCA

Sylvie Retailleau, Philippe Dollfus

Institute of Fundamental Electronics

CNRS / University Paris-Sud

France



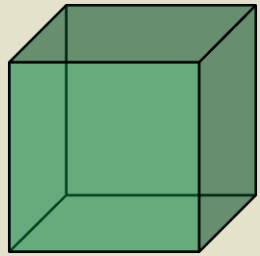
UNIVERSITÉ
PARIS
SUD

vtalbo@usal.es

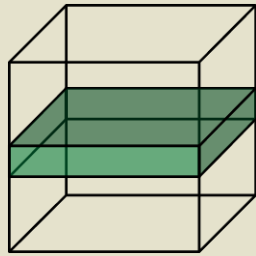
Introduction

COULOMB BLOCKADE AND APPLICATIONS

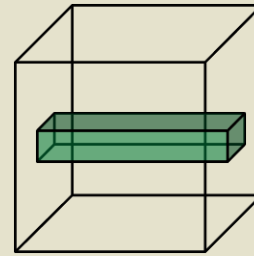
From micrometric to nanometric



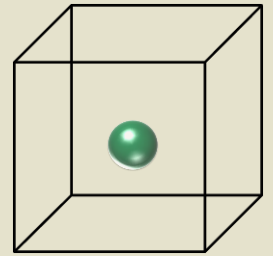
3D - bulk



2D - quantum well

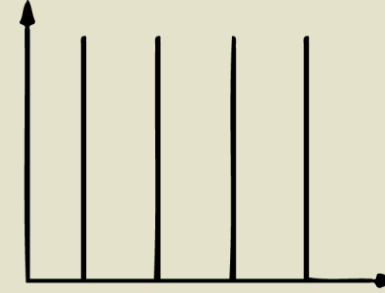
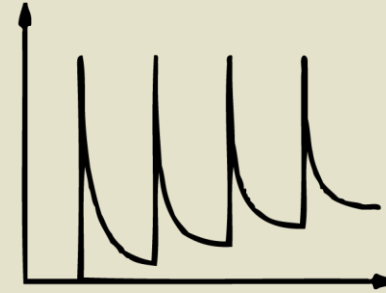
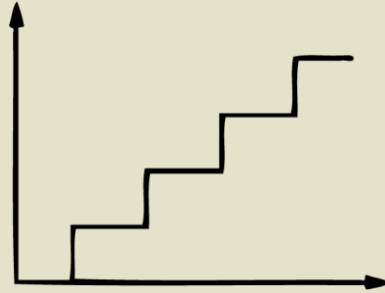
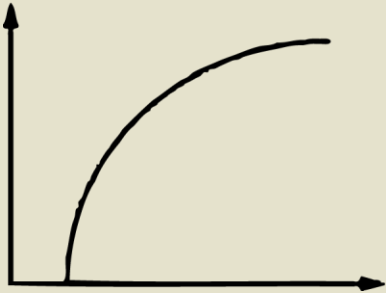


1D - nanowire



0D - quantum dot

Density of states



ENERGY

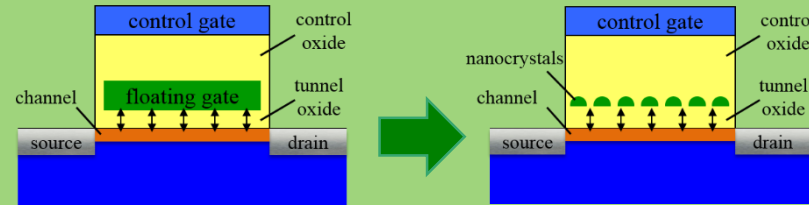
- discretization of energy levels
- gap broadening with reduction of size
 - blue shift



size of the dot

Applications in electronics

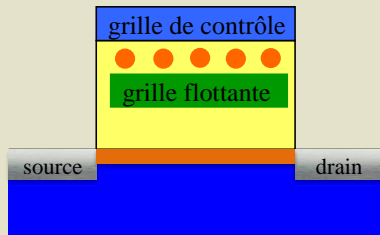
Granular floating gate FLASH memories



- Tiwari, IEDM, 1995
- Freescale

COULOMB BLOCKADE

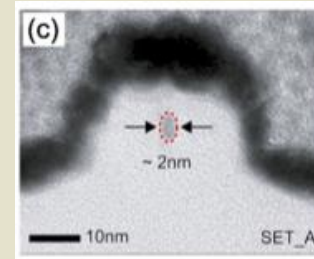
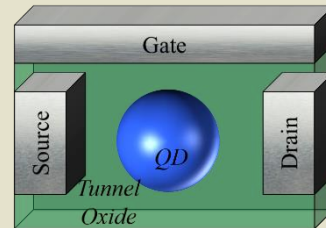
Multiple-tunnel junction FLASH memory



Deleryuelle, *Microelec Eng.*, 2004

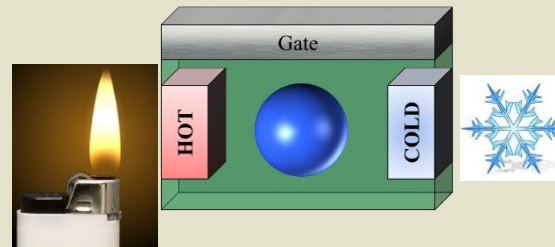
- Writing through nanocrystals
- Writing/Reading decoupled

Single-Electron Transistor (SET)

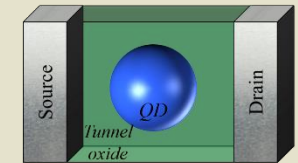


Shin, *APL*, 2010

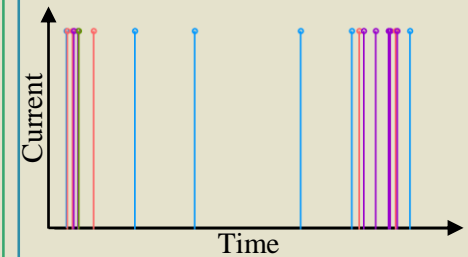
Thermoelectricity



Double-Tunnel junction (DTJ)

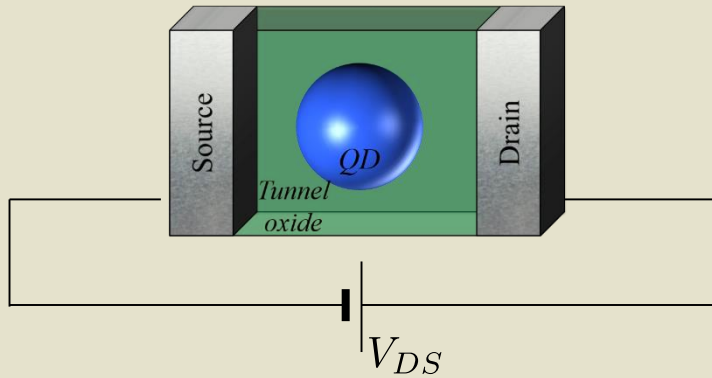


SHOT NOISE IN DTJ



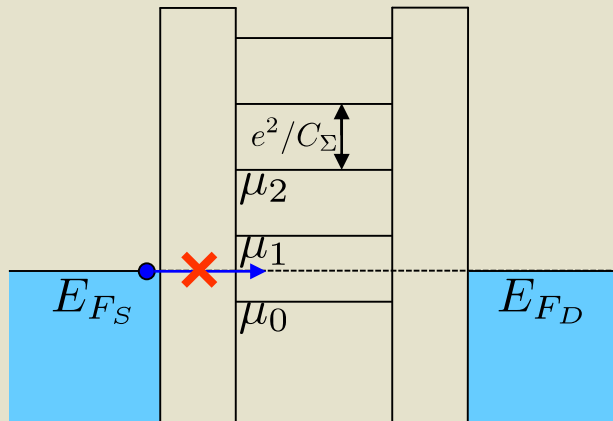
Coulomb blockade

double-tunnel junction: simple case of Coulomb blockade

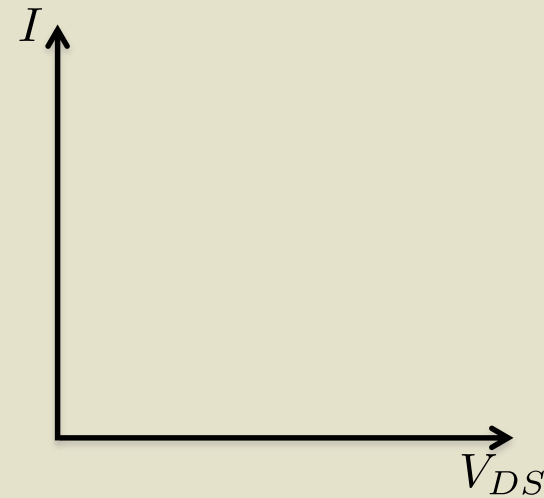


Charging energy: $E_{\text{charge}} = \frac{e^2}{C_{\Sigma}}$

(energy to bring to add an electron in the dot)

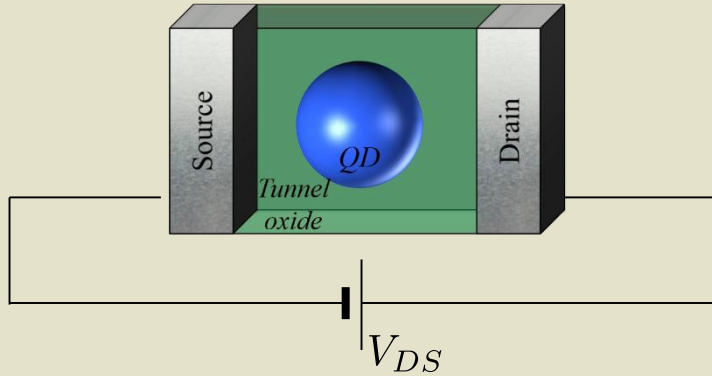


$V_{DS} = 0$



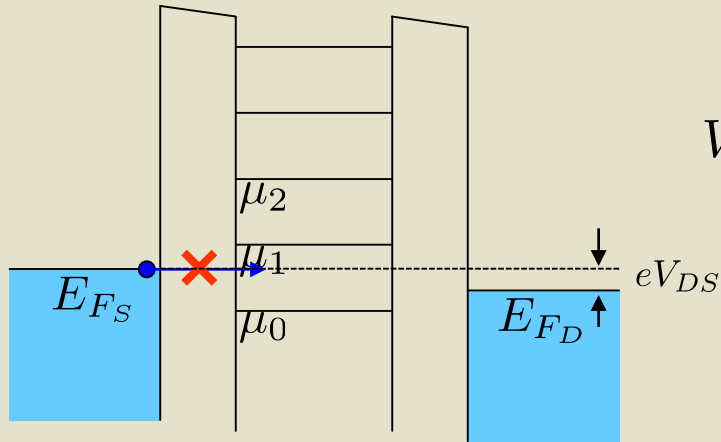
Coulomb blockade

double-tunnel junction: simple case of Coulomb blockade

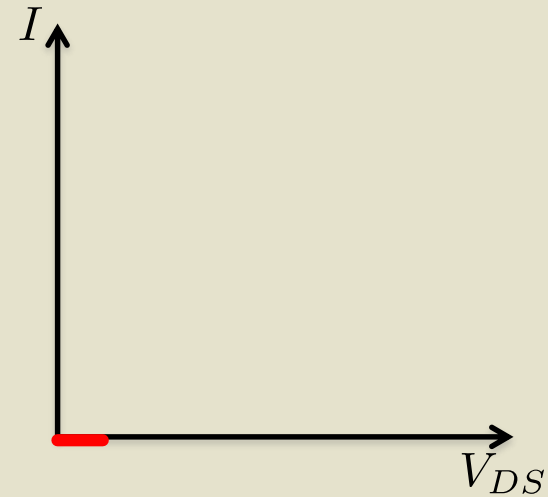


Charging energy: $E_{\text{charge}} = \frac{e^2}{C_{\Sigma}}$

(energy to bring to add an electron in the dot)



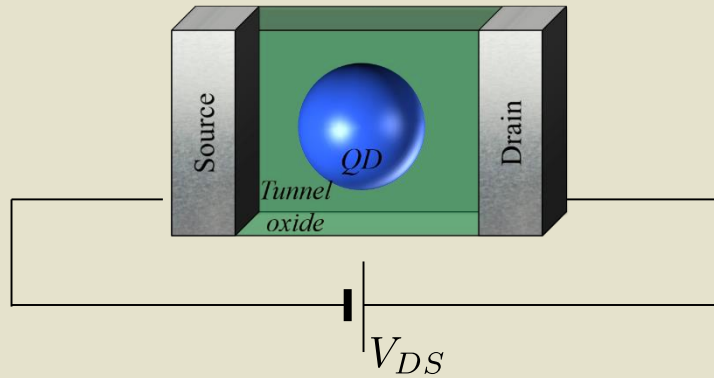
$$V_{DS} < \frac{e}{C_{\Sigma}}$$



COULOMB BLOCKADE

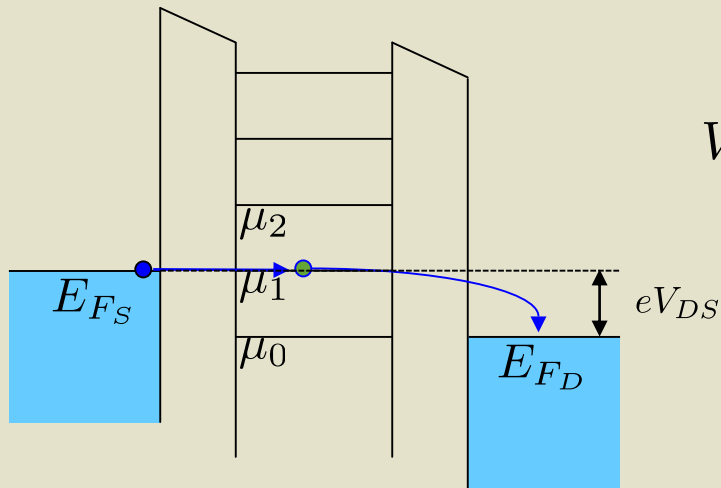
Coulomb blockade

double-tunnel junction: simple case of Coulomb blockade

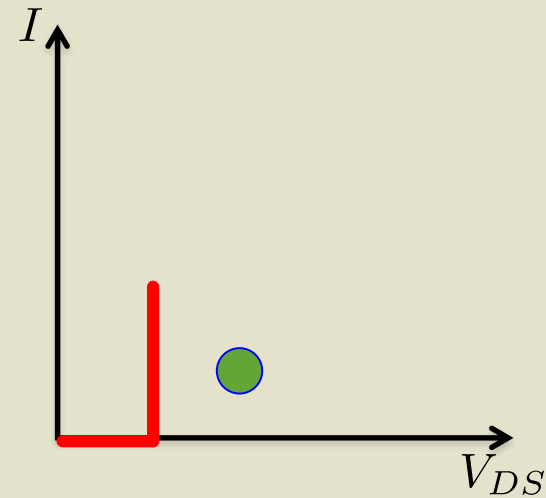


$$\text{Charging energy: } E_{\text{charge}} = \frac{e^2}{C_{\Sigma}}$$

(energy to bring to add an electron in the dot)

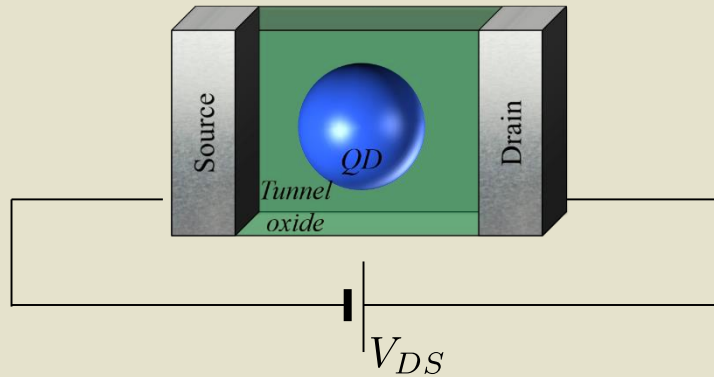


$$V_{DS} = \frac{e}{C_{\Sigma}}$$



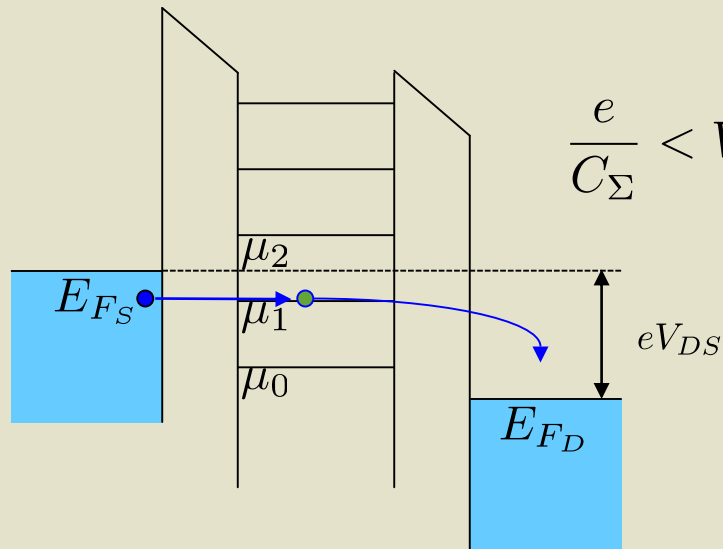
Coulomb blockade

double-tunnel junction: simple case of Coulomb blockade

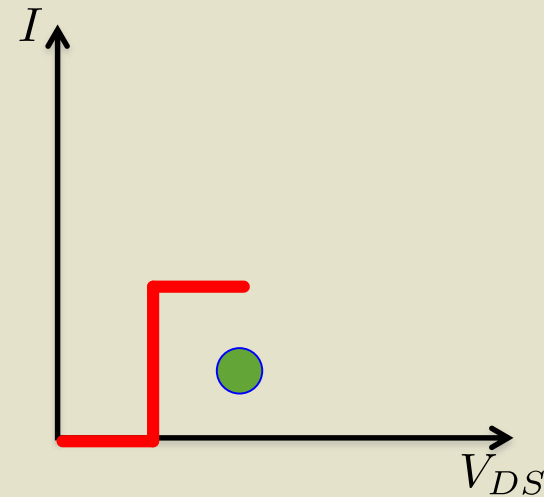


$$\text{Charging energy: } E_{\text{charge}} = \frac{e^2}{C_{\Sigma}}$$

(energy to bring to add an electron in the dot)

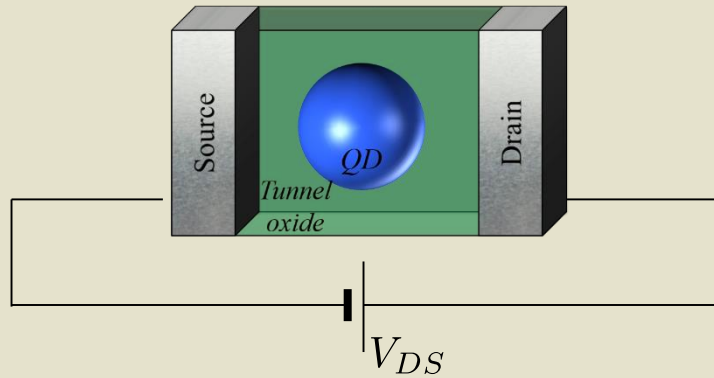


$$\frac{e}{C_{\Sigma}} < V_{DS} < \frac{3e}{C_{\Sigma}}$$



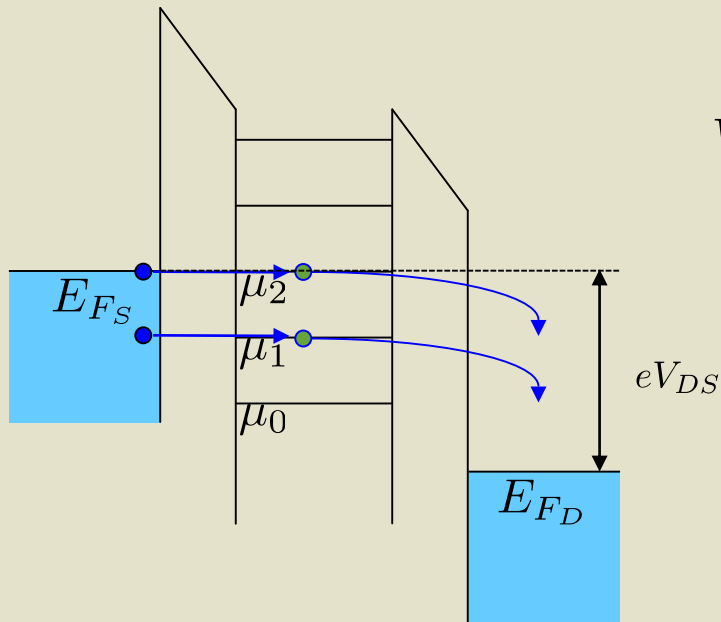
Coulomb blockade

double-tunnel junction: simple case of Coulomb blockade

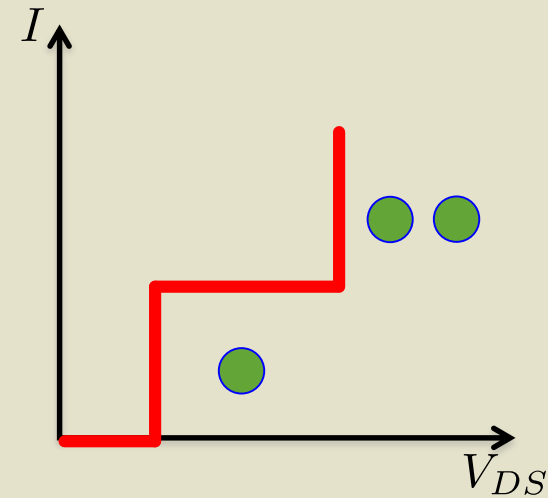


$$\text{Charging energy: } E_{\text{charge}} = \frac{e^2}{C_{\Sigma}}$$

(energy to bring to add an electron in the dot)

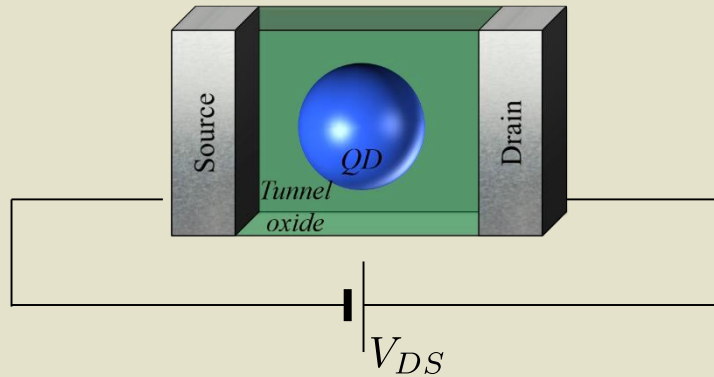


$$V_{DS} = \frac{3e}{C_{\Sigma}}$$



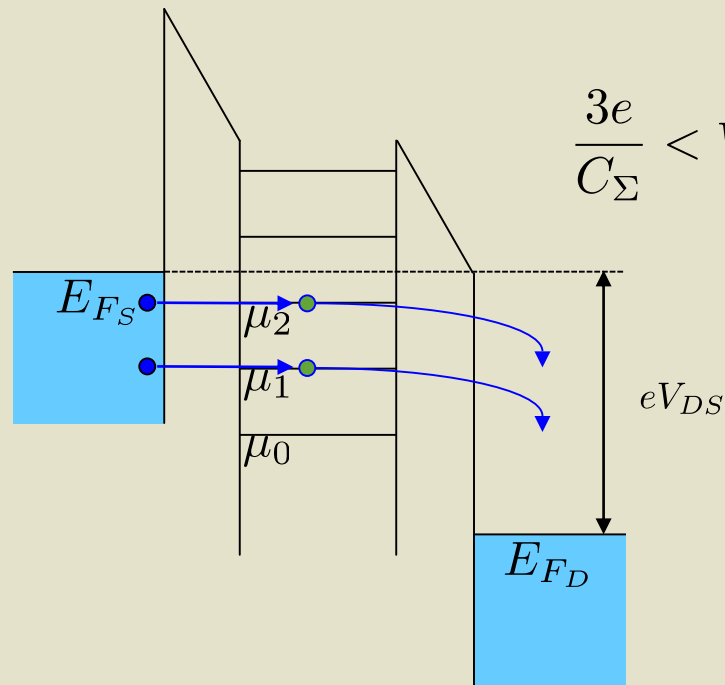
Coulomb blockade

double-tunnel junction: simple case of Coulomb blockade

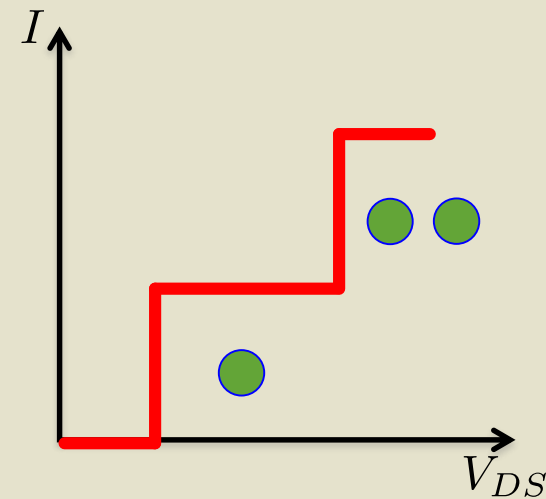


$$\text{Charging energy: } E_{\text{charge}} = \frac{e^2}{C_{\Sigma}}$$

(energy to bring to add an electron in the dot)



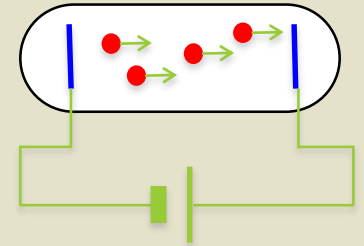
$$\frac{3e}{C_{\Sigma}} < V_{DS} < \frac{5e}{C_{\Sigma}}$$



COULOMB STAIRCASE

Shot Noise in double-tunnel junction

- Shot noise (SN): Consequence of charge granularity
→ **more information of the electronic transport**
Between two electrodes: **Poissonian transport**



- Comparison with Poissonian transport**

$$S(f)/2e\langle I \rangle$$

$S(f)$ current spectral density at frequency f

$2e\langle I \rangle$ current spectral density of a Poissonian process

- at $f = 0$, **Fano factor**
 - $F = S(0)/2e\langle I \rangle$
 - < 1 : sub-Poissonian noise**
 - = 1 : Poissonian noise**
 - > 1 : super-Poissonian noise**

➤ Behaviour at $f = 0$ already well understood

Sub-Poissonian noise : *Birk et al., Phys. Rev. Lett, 1995*

Super-Poissonian noise in multi-levels QDs: *W. Belzig, Phys. Rev. B, 2005*

Shot Noise in double-tunnel junction: *V. Talbo et al., Fluct. Noise Lett., 2012*

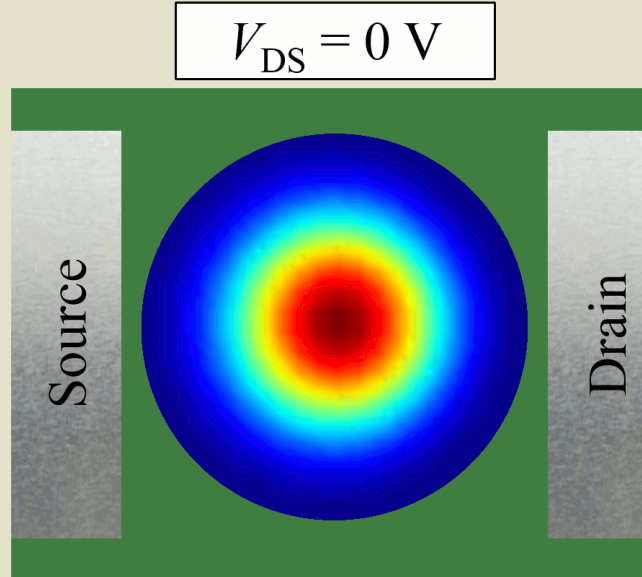
SENS Simulation

MODEL AND RESULTS

SENS model – Single-Electron Nanodevice Simulation

Electronic structure of Si QDs(wave-function ψ , energy E)

- 3D solver for Poisson and Schrödinger equations (geometry, bias, number of electrons)
- Hartree method, access to the electronic wave-function
- **delocalization of the wavefunction with bias**



double-tunnel junction: *J.Sée et al., IEEE TED, 2006*
double-dot structure: *A. Valentin et al., J. Appl. Phys., 2009*
single-electron transistor: *V. Talbo et al., IEEE TED, 2011*

SENS model – Single-Electron Nanodevice Simulation

Electronic structure of Si QDs(wave-function ψ , energy E)

- 3D solver for Poisson and Schrödinger equations (geometry, bias, number of electrons)
- Hartree method, access to the electronic wave-function

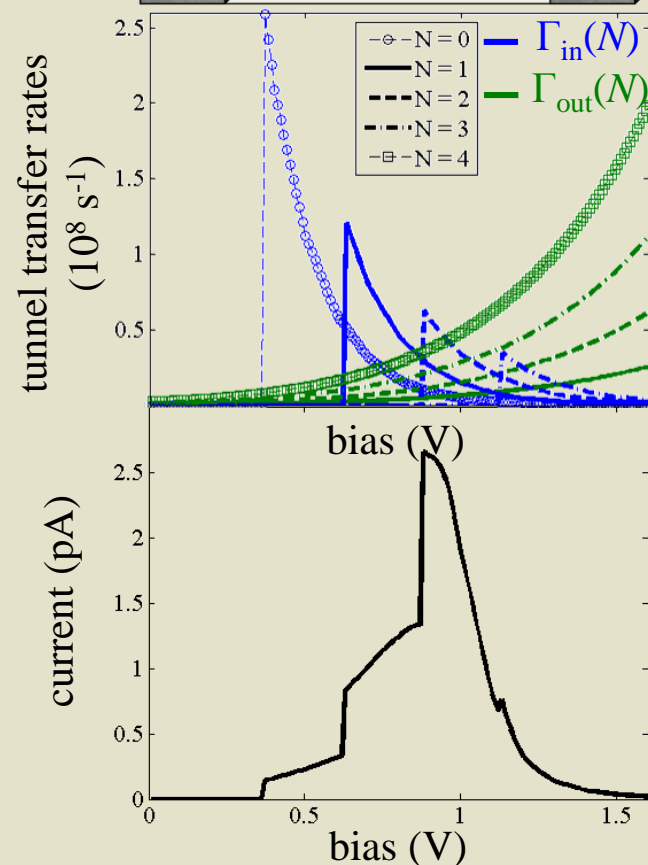
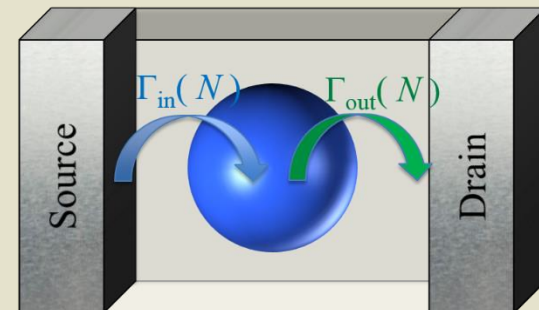
Tunnel transfer rates $\Gamma(\text{Bias}, \text{Temperature})$ from wave functions

- weak **coupling**
- Fermi golden rule and Bardeen formalism
- **decrease (increase) of Γ_{in} (Γ_{out}) with as consequence of delocalization of the wavefunction with bias**

Electronic characteristics

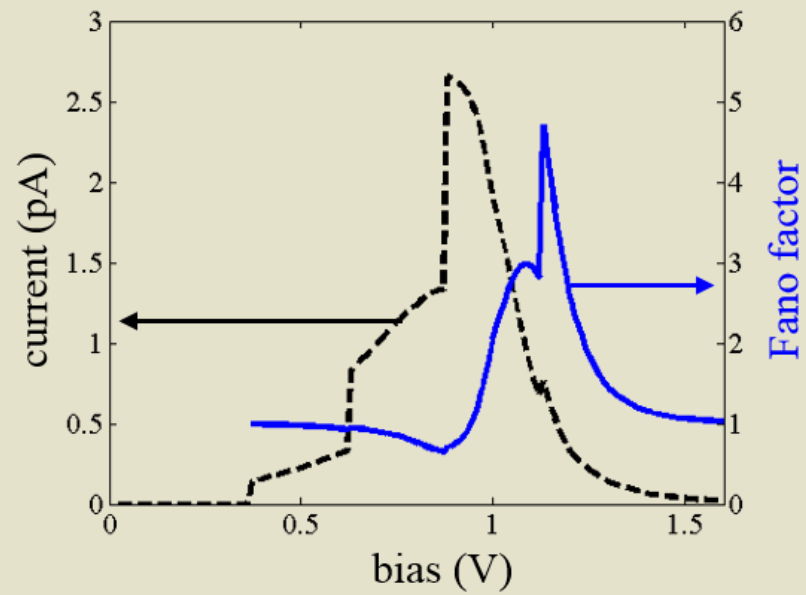
- **Monte-Carlo algorithm:** probability to find N electrons in the dot($P(N)$)
- and / or **master equation**, linked with **Korotkov** formalism for noise
- **negative differential conductance when $\Gamma_{\text{in}}(N) < \Gamma_{\text{out}}(N+1)$**

double-tunnel junction: *J.Sée et al., IEEE TED, 2006*
double-dot structure: *A. Valentin et al., J. Appl. Phys., 2009*
single-electron transistor: *V. Talbo et al., IEEE TED, 2011*

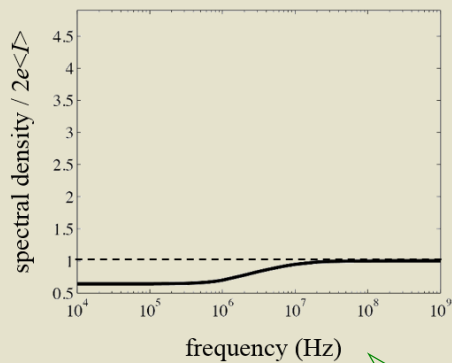
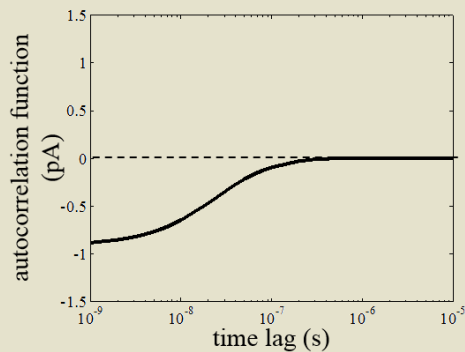
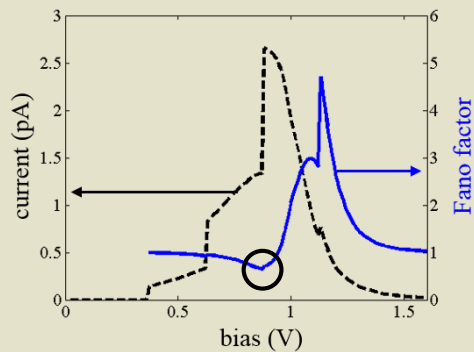


Results

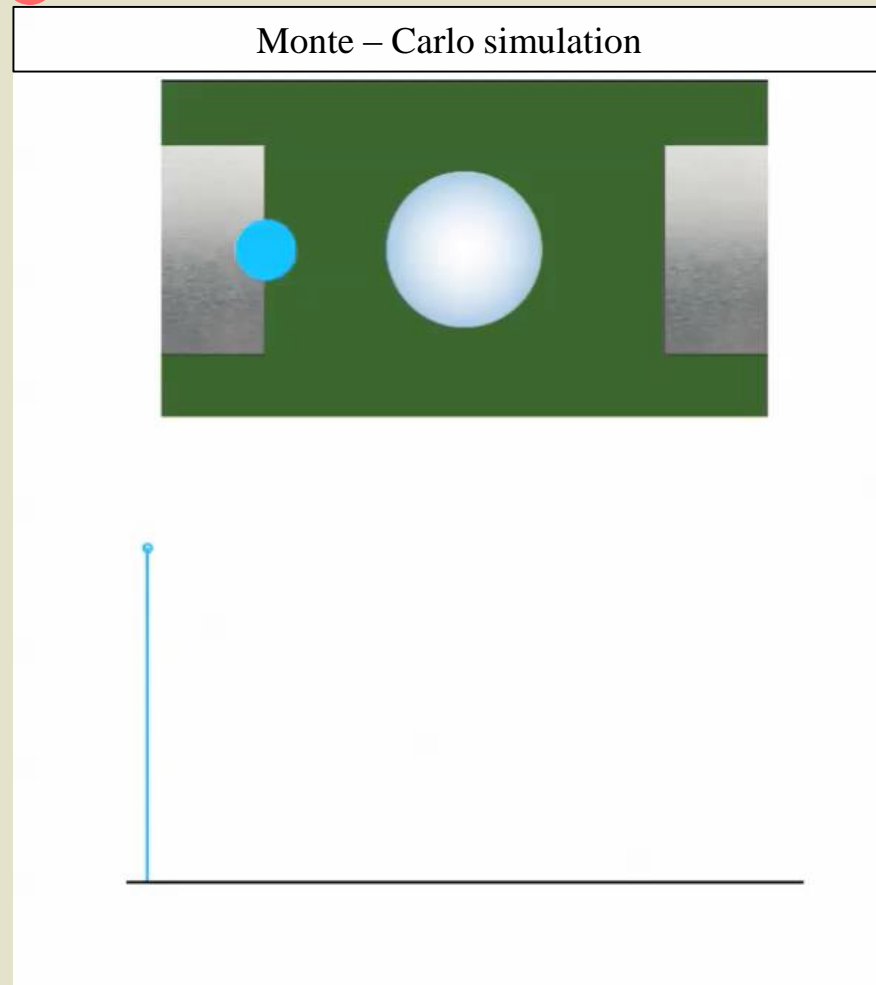
$\varnothing = 10 \text{ nm}$ $h_S = 1.2 \text{ nm}$ $h_D = 1.8 \text{ nm}$ $T = 0 \text{ K}$



Results – Sub-Poissonian case

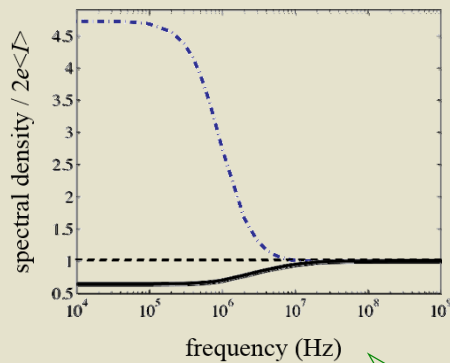
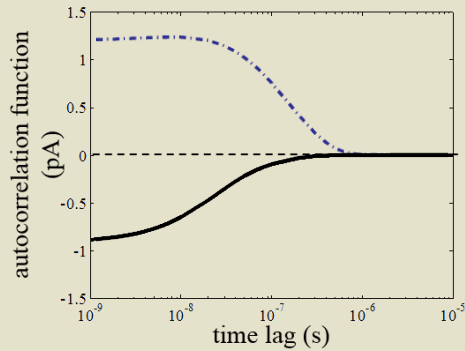
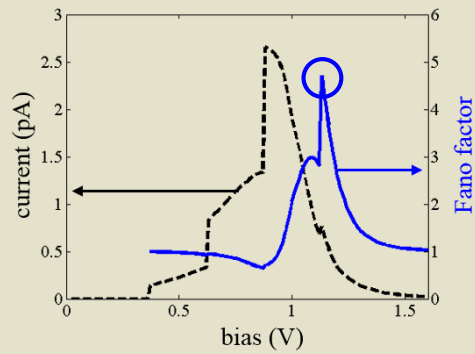


- 1st electron in excess
- 2nd electron in excess



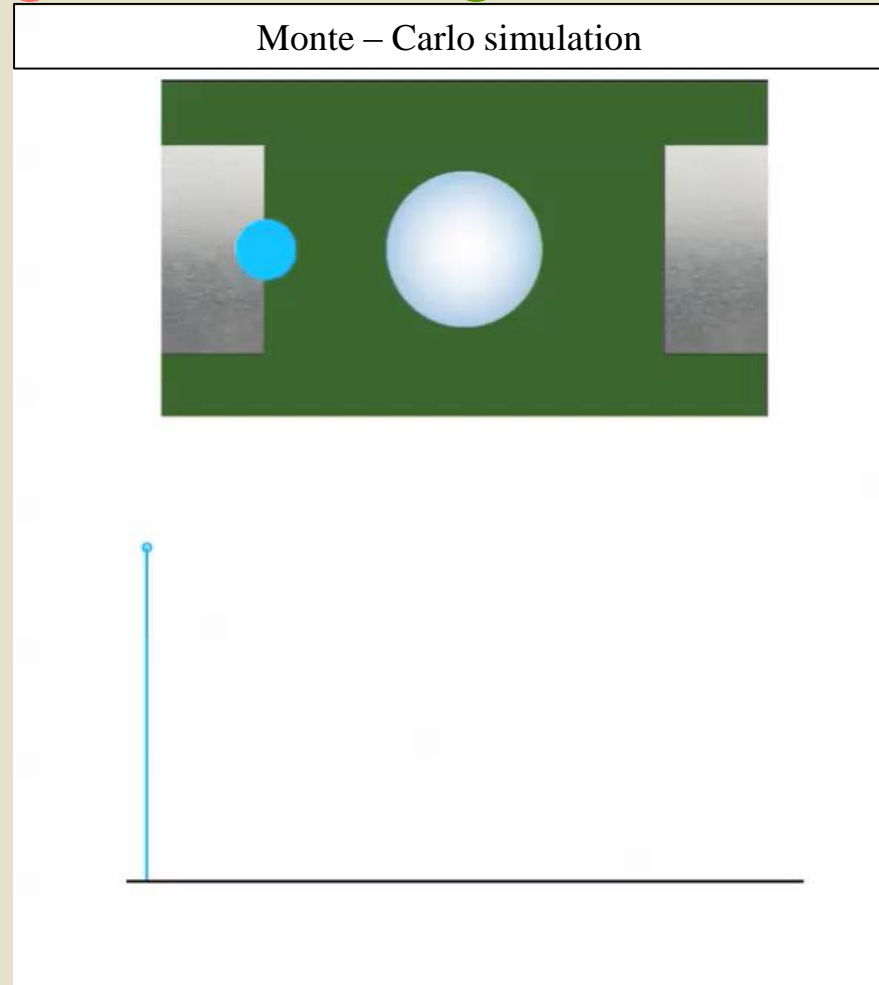
➤ remains sub-Poissonian with frequency

Results – Super-Poissonian “bunching”

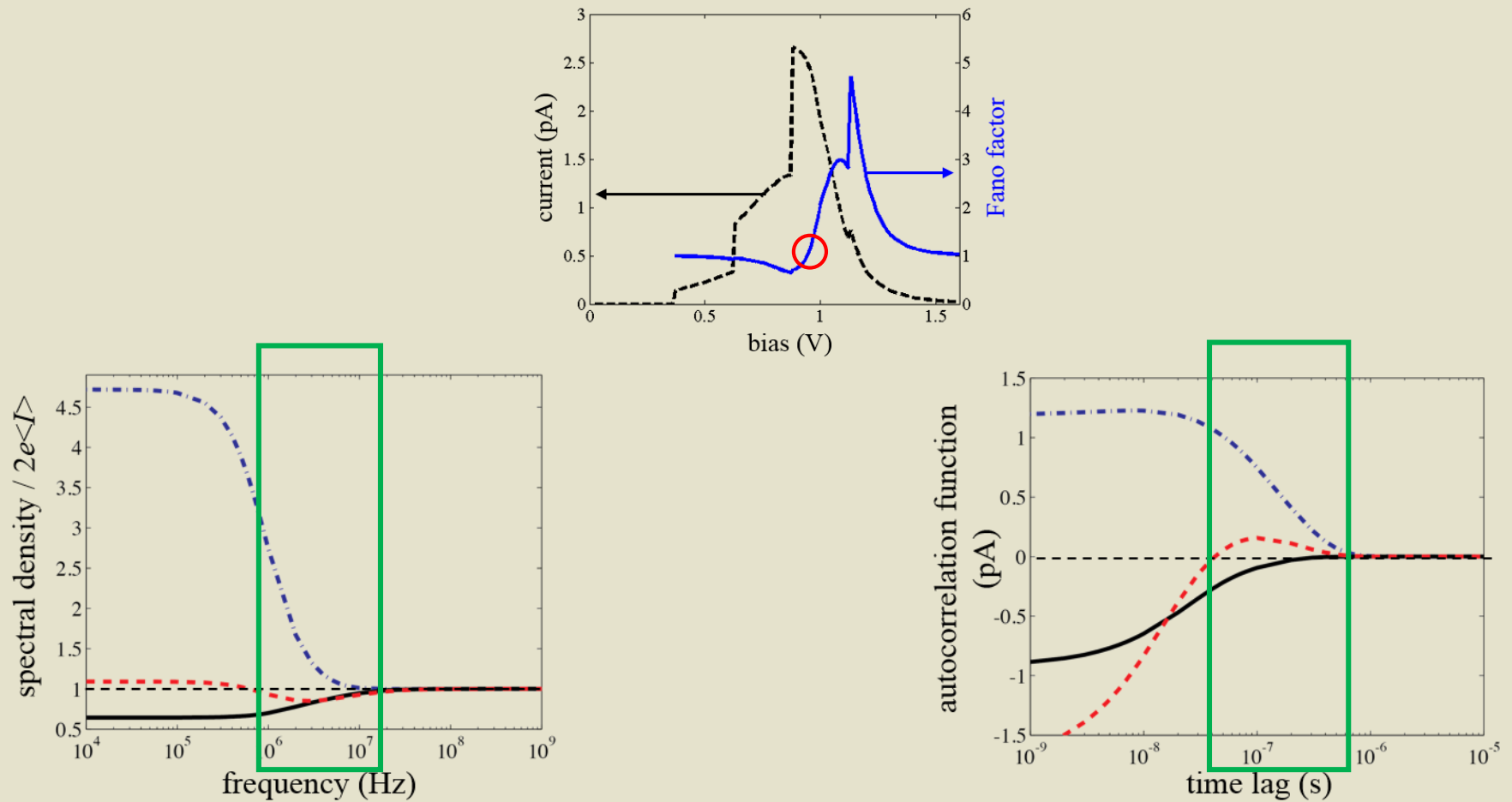


➤ remains super-Poissonian with frequency

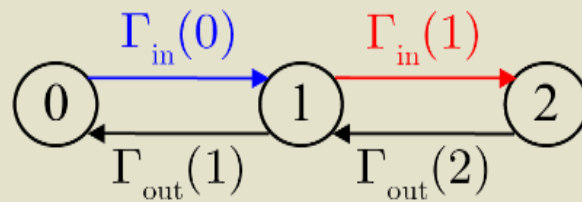
- 1st electron in excess
- 2nd electron in excess
- 3rd electron in excess
- 4th electron in excess



Results – Near-Poissonian ($F = 1.01$)



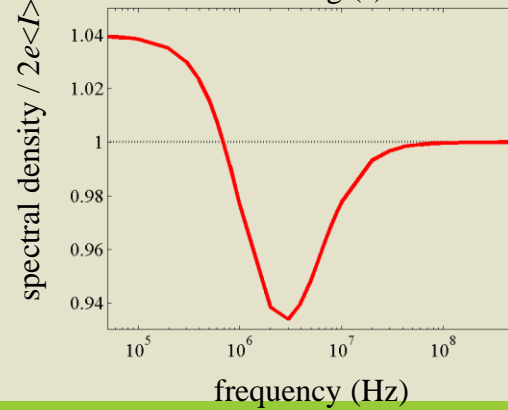
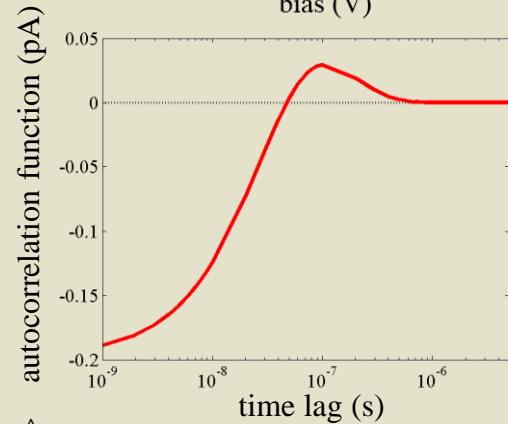
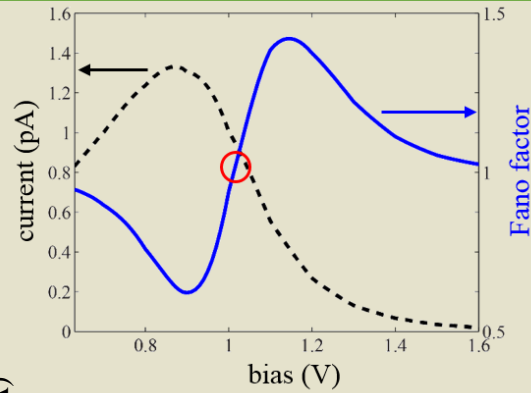
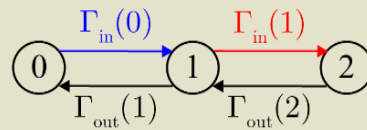
Where this specific dynamics is coming from ?



3-state case

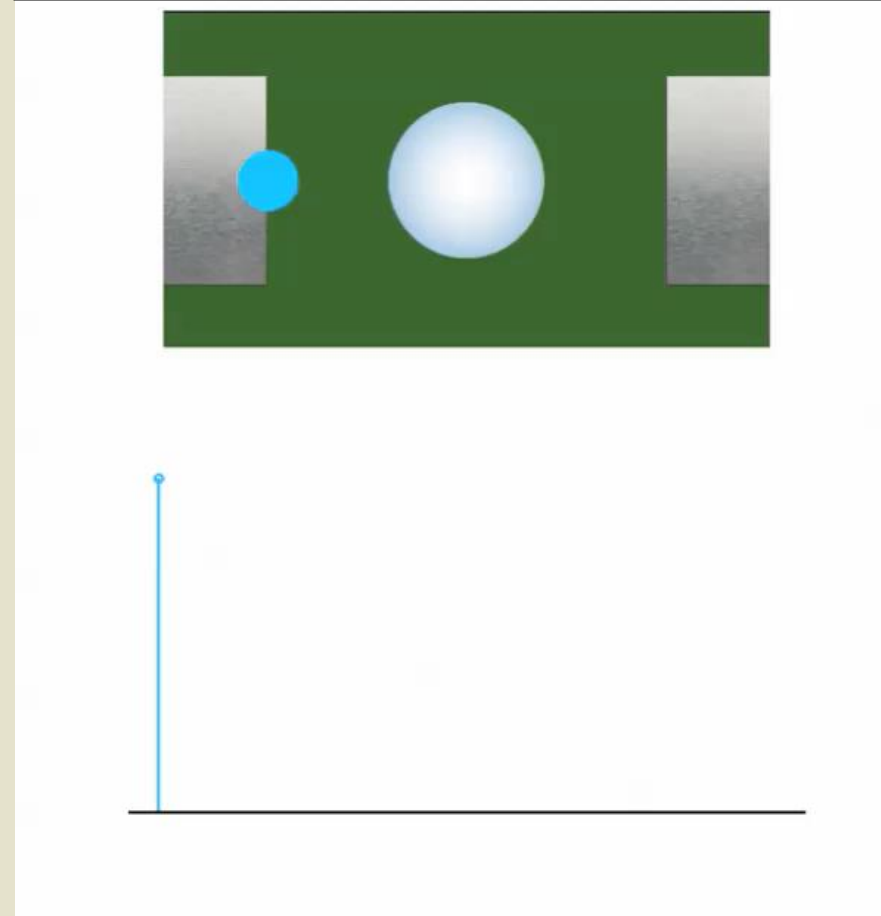
CORRELATIONS AND WAITING TIME
DISTRIBUTIONS

Results



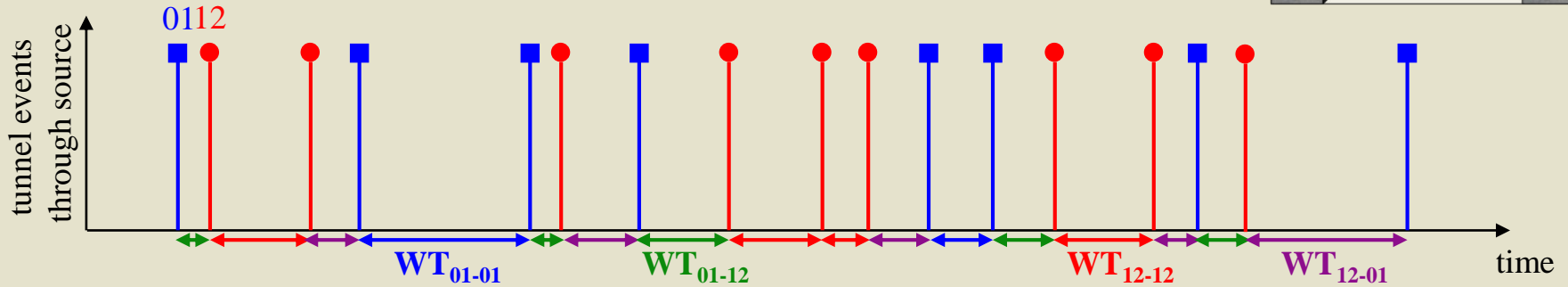
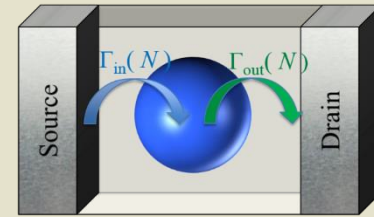
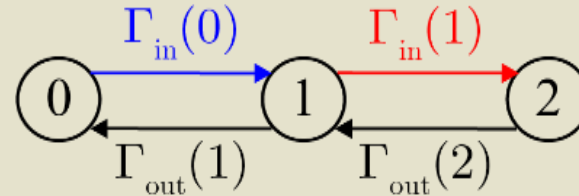
- 1st electron in excess
- 2nd electron in excess

Monte – Carlo simulation



How to understand the behaviour of auto-correlation functions ?

tunnel events through source:
01 and **12**



Probabilities of 01 and 12 peaks

partial AUTO-CORRELATIONS

$$C_{01-01} + C_{12-12}$$

+

partial CROSS-CORRELATIONS

$$C_{01-12} + C_{12-01}$$

=

total auto-correlation C_{II}

WAITING TIME DISTRIBUTIONS

$$\longleftrightarrow WT_{01-01}$$

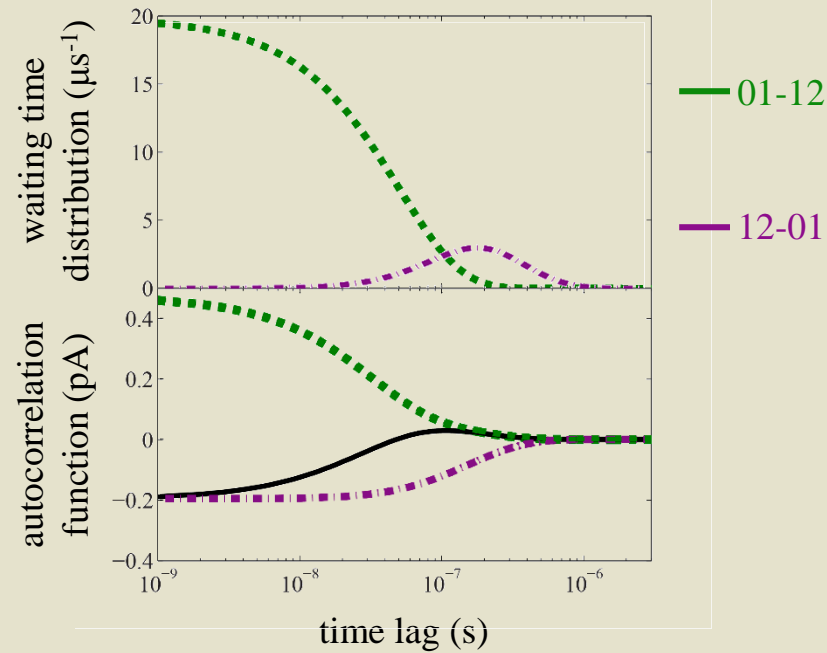
$$\longleftrightarrow WT_{12-12}$$

$$\longleftrightarrow WT_{01-12}$$

$$\longleftrightarrow WT_{12-01}$$

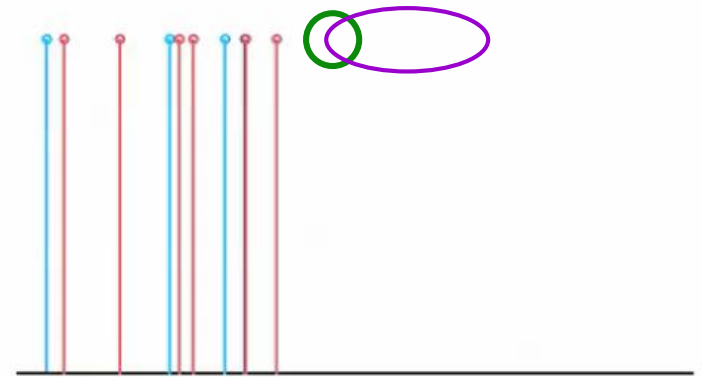
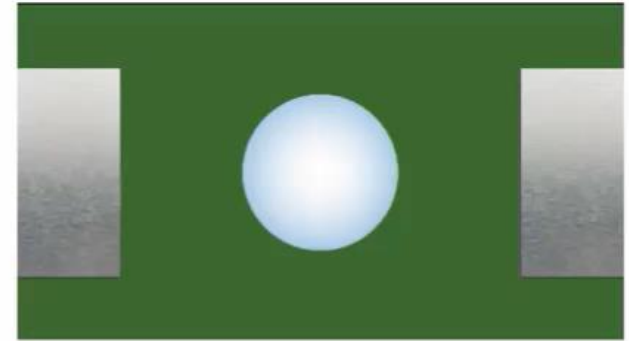
Electron Waiting Times in Mesoscopic Conductors
 M. Albert, G. Haack, C. Flindt, and M. Büttiker
Phys. Rev. Lett. 2012

Cross-correlations C_{01-12} and C_{12-01}



$$\text{Prob}_{01-12} = \text{Prob}_{12-01} = 23\%$$

Monte - Carlo simulation



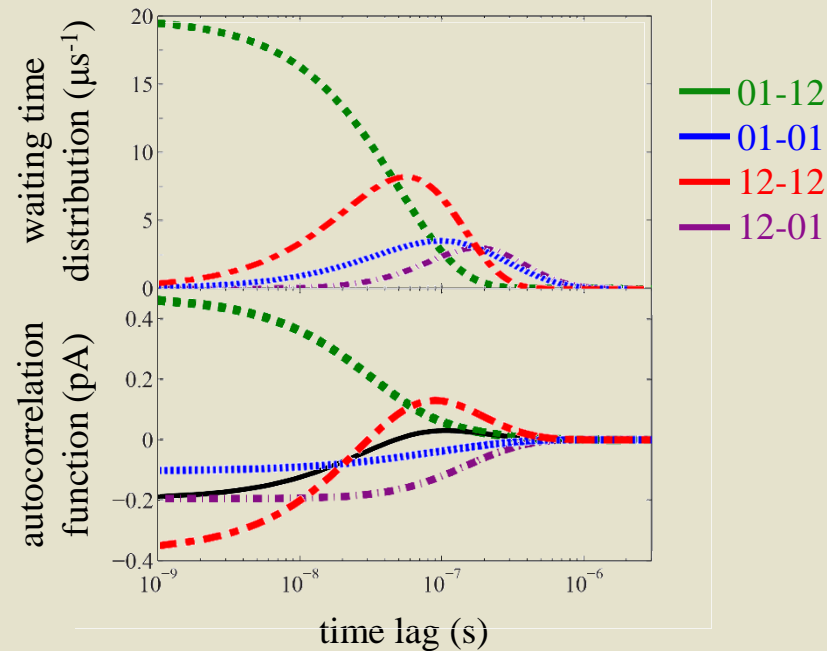
➤ No tunnel event between consecutive **01** and **12**

- Poissonian WTD (maximum at 0)
- High correlation C_{01-12} at low times

➤ 2 tunnel events (drain side) between **12** and **01**

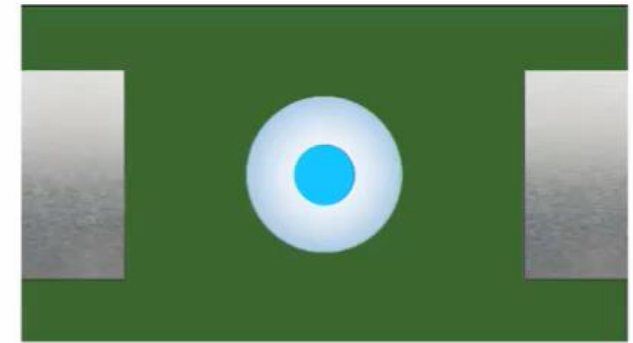
- WTD = 0 at short times
- Negative correlation C_{12-01}

Auto-correlations C_{12-12} and C_{01-01}



$\text{Prob}_{12-12} = 43\%$, $\text{Prob}_{01-01} = 12\%$

Monte - Carlo simulation

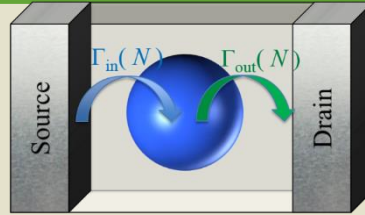


- one tunnel event between consecutive peaks
 - WTD = 0 at short times, increases after
 - negative correlation at low times, increasing later
- P_{12-12} is not high enough for C_{12-12} to remain negative when WTD increases



ORIGIN OF SPECIFIC DYNAMICS

Conclusion



The time-dependent physics of electronic transport in double-tunnel junction

- ✓ Obtained from SENS simulator: tunnel transfer rates
- ✓ Understood through the study of the link between:

auto- and cross-correlations, probabilities and waiting time distributions

High-majority of a given tunnel event: negative correlation

- Equality of tunnel events: positive correlation
- "in between": specific dynamics, going from negative to positive correlation

Thank you !