Brownian motion (and more) in disordered media

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What are the possible sources of observed anomalous (sub)diffusion?

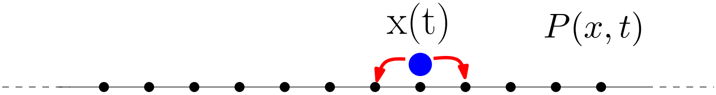
Need a more precise question. What does subdiffusive mean ?
$\langle x^2(t) \rangle \sim t^\alpha$, $0 < \alpha < 1$ (or logarithms etc.)

- Stochastic process: correlated increments. non-stationary increments.
- Fractional Brownian motion (homogeneous medium). Diffusion on scale-free disorder: percolation clusters, fractals.
- Continuous time random walk with anomalously long waiting times between steps. Traps.
- Aggregating particles.

What is diffusion on (scale-free) disordered media ?

- Disordered diffusivity.
- Disordered confinement (reflecting barriers or confining potentials)

Continuous time random walk



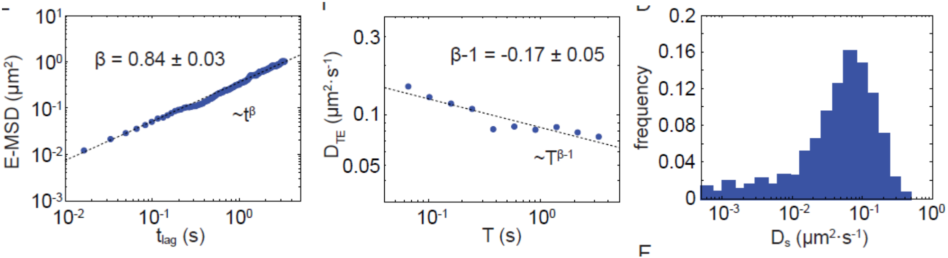
$$\psi(t, x) = \psi(t)\lambda(x) \qquad \Psi(t) = \int_t^{\infty} \psi(t')\,\mathrm{d}t'$$

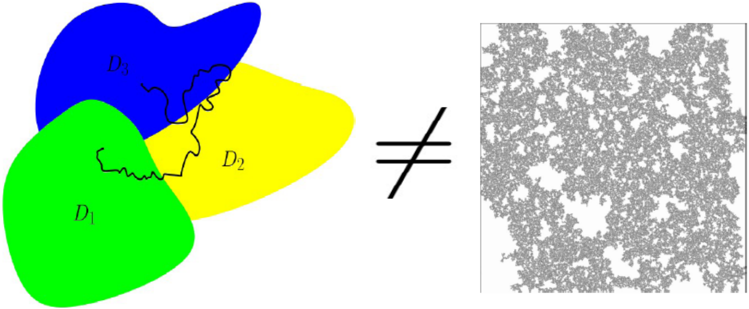
$$P(k, s) = \frac{\Psi(s)}{1 - \psi(s, k)}$$

$\langle t \rangle_{\text{step}} = \int_0^{\infty} t\psi(t)\,\mathrm{d}t$ finite ?	$\langle x^2 \rangle_{\text{step}} = \int_0^{\infty} x^2 \lambda(x)\,\mathrm{d}x$ finite ?

$$\langle x^2(t) \rangle \sim Dt$$

$$\langle x^2(t) \rangle \sim K_\alpha t^\alpha \quad 0 < \alpha < 1$$





Manzo, Torreno-Pina, Massignan, Lapeyre, Lewenstein, García-Parajo, PRX **5** 011021 (2015)

$$P_{D,r}(D,r) = P_D(D) P_r(r|D)$$

$$P_D(D) \sim D^{\sigma-1} \text{ with } \sigma > 0, \quad \text{for small } D \text{ (e.g. } \Gamma \text{ dist.)}$$

$$P_r(r|D) \text{ has mean } \mathrm{E}[r|D] = D^{(1-\gamma)/2}, -\infty < \gamma < \infty$$

$$\text{or } P_\tau(\tau|D) \text{ has mean } \mathrm{E}[\tau|D] = D^{-\gamma}$$

$$\lambda(x|\tau,t) = \frac{1}{\sqrt{2\pi D(\tau)t}} \exp\left(\frac{-x^2}{2D(\tau)t}\right), \quad \psi(\tau) \sim \tau^{-\sigma/\gamma-1}$$

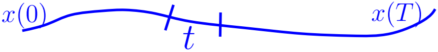
$\langle x^2(t)\rangle \sim t^{\alpha}$	(0) $\gamma < \sigma$	(I) $\sigma < \gamma < \sigma+1$	(II) $\sigma+1 < \gamma$
Annealed	1	σ/γ	1 - $1/\gamma$
Quenched 1d	1	$2\sigma/(\sigma+\gamma)$	?

Massignan, Manzo, Torreno-Pina, García-Parajo, Lewenstein, Lapeyre, PRL **112** 150603 (2014)

Continuous time Random Walk (CTRW)

Waiting time distribution $\psi(t) \sim t^{-\alpha-1}$ $0 < \alpha < 1$

Ensemble averaged MSD $\langle x^2(t) \rangle \sim t^{\alpha}$



Time-ensemble Avg. MSD

Time ensemble averaged MSD $\langle \overline{x^2(t)} \rangle_T \sim T^{\alpha-1} t = \left(\frac{t}{T}\right)^{1-\alpha} t^{\alpha}$

He, Burov,Metzler,Barkai PRL (2008) Lubelski, Sokolov, Klafter, PRL (2008)

$t \ll T$

Subordination. CTRW is "subordinator" of another process.

2nd process. MSD $\langle x^2(t) \rangle \sim t^{\beta}$ $\quad 0 < \beta < 1$

Combined processes. MSD $\langle x^2(t) \rangle \sim t^{\alpha\beta}$

Time-ensemble MSD $\langle \overline{x^2(t)} \rangle_T \sim T^{\alpha-1} t^{1-\alpha+\alpha\beta} = \left(\frac{t}{T}\right)^{1-\alpha} t^{\alpha\beta}$

Meroz, Sokolov, Klafter PRE (rc) (2010)

Weigel, Simon, Tamkun, Krapf, PNAS (2011)

$$\langle \overline{x^2(t)} \rangle_T = \left(\frac{t}{T}\right)^{1-\alpha} \langle x^2(t) \rangle \quad t \ll T$$

Disordered Confinement

$$P(r) \sim r^{-1-c}, \quad c > 0$$

Molecular motors robustly drive active gels to a critically connected state Nat.Phys. (2013)

José Alvarado[1], Michael Sheinman[2], Abhinav Sharma[2], Fred C. MacKintosh[2]★ and Gijsje H. Koenderink[1]★

Sheinman, Sharma, Alvarado, Koenderink, MacKintosh PRL (2015)

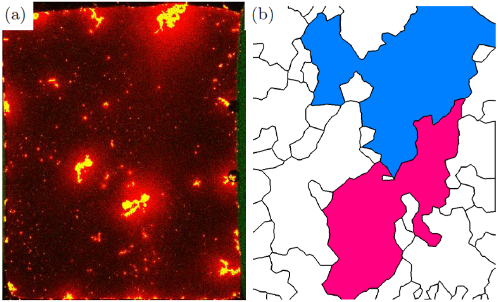
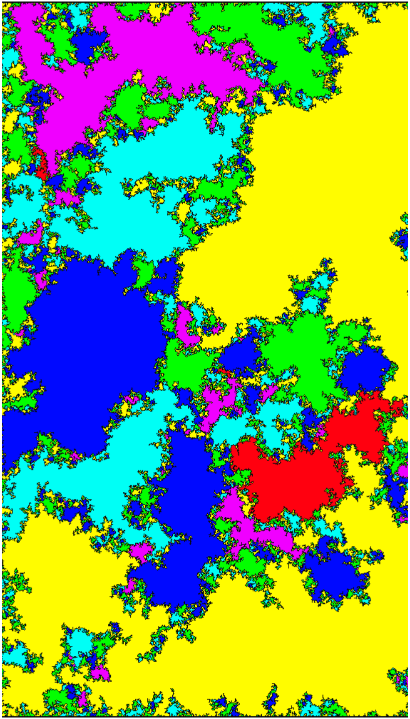


Figure 1: (Color online) (a) Experimental results of a fascin-crosslinked actin network, collapsed by myosin motors (see SI for details and movie of the collapse). (b) Initial configuration of the collapsed clusters in (a). Colours indicate the



$\tau \approx 1.82$
Fischer exponent

$$\Pr(|\mathcal{C}| = s) \sim s^{-0.82} \ (!)$$

$$P^*(r) \sim r^{-1-c}, \quad c = 1.64$$

Probability density of "radii" $P(r) \sim r_0^c \, r^{-c-1}, \quad 0 < c$

Average MSD over random radii $\langle \widetilde{x^2(t)} \rangle = \int_0^\infty P(r) \langle x^2(t) \rangle_r \, \mathrm{d}r$

$$\langle \widetilde{x^2(t)} \rangle \sim \int_{r^*}^\infty r_0^c \, r^{-c-1} D_a \, t^a \; f\left(\frac{D_a \, t^a}{r^2} \right) \mathrm{d}r$$

Change variable. Get dimensionless integral.... Convergence ?

$$\langle \widetilde{x^2(t)} \rangle \sim r_0^c D_a^{\frac{2-c}{2}} \, t^{\frac{a(2-c)}{2}} \int_0^{z^* \to \infty} z^{\frac{c-2}{2}} f(z) \, \mathrm{d}z \qquad f(z) \sim z^{-1}$$

Converges for $0 < c < 2$

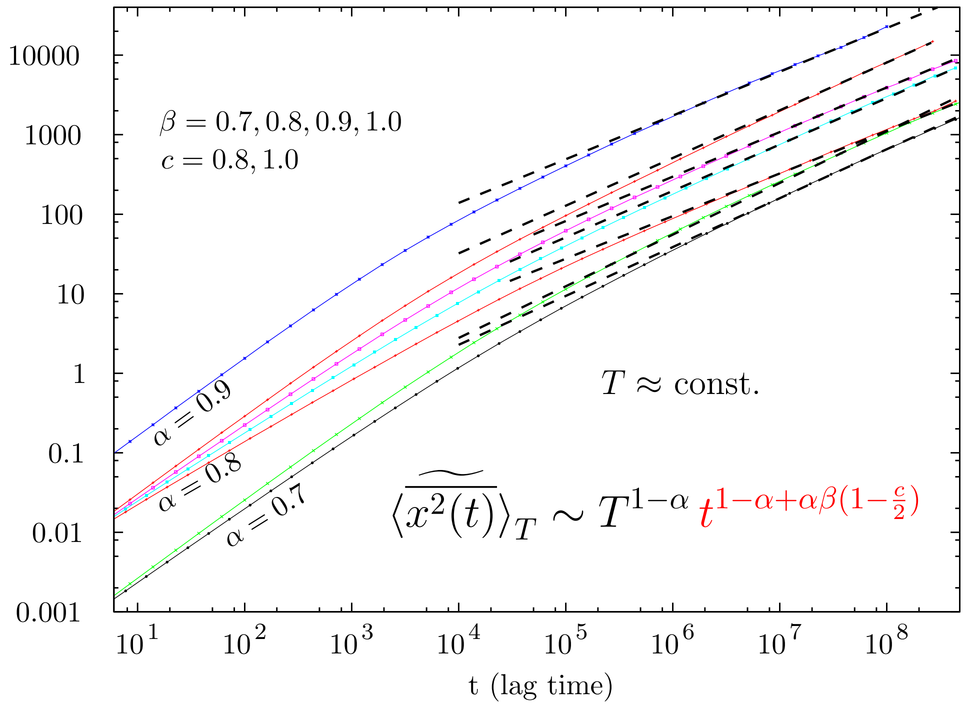
$$\langle \widetilde{x^2(t)} \rangle \sim r_0^c \, D_a^{1-\frac{c}{2}} \, t^{a(1-\frac{c}{2})}, \; 0 < c < 2$$

$$a \to a\left(1 - \tfrac{c}{2}\right)$$

$$\langle \widetilde{\overline{x^2(t)}} \rangle_T \sim r_0^c \, D_a^{1-\frac{c}{2}} \left(\frac{t}{T} \right)^{1-\alpha} t^{a(1-\frac{c}{2})}$$

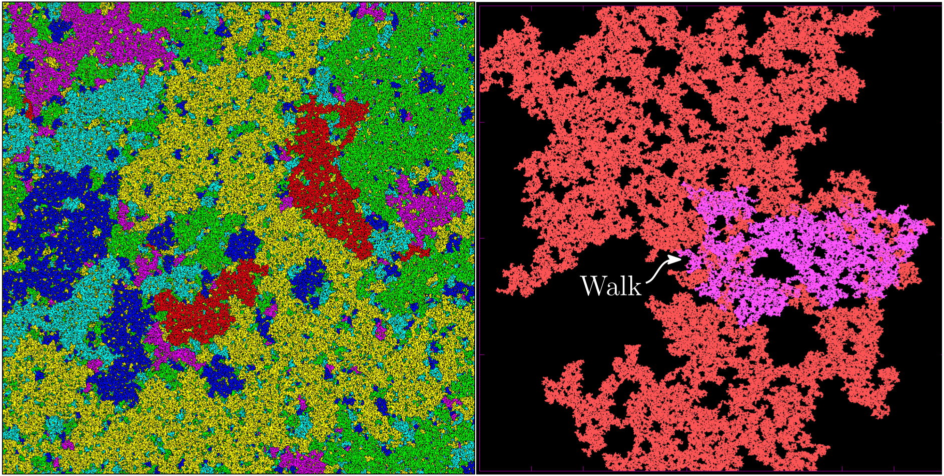
$\langle \overline{x^2(t)} \rangle_T = \left(\frac{t}{T}\right)^{1-\alpha} \langle x^2(t) \rangle$

Displacement of every particle bounded. Ensemble MSD unbounded


$\beta = 0.7, 0.8, 0.9, 1.0$
$c = 0.8, 1.0$
$\alpha = 0.9$
$\alpha = 0.8$
$\alpha = 0.7$
$T \approx$ const.
$\langle\widetilde{\overline{x^2(t)}}\rangle_T \sim T^{1-\alpha}\, t^{1-\alpha+\alpha\beta(1-\frac{c}{2})}$
t (lag time)
10000
1000
100
10
1
0.1
0.01
0.001
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8

Percolation: "Natural" scale-free confined disorder.

All finite clusters — Only infinite cluster



$$k = \frac{2\nu - \beta}{2\nu - \beta + \mu}.$$

$$\langle x^2(t) \rangle \sim t^{k}$$

$$k' = \frac{2\nu}{2\nu - \beta + \mu}$$

Gefen, Aharony, Alexander, PRL (1983)

Walk on percolation $p = p_c$, $\langle x^2(t) \rangle \sim t^k$

$$k' = \frac{2\nu}{2\nu - \beta + \mu}$$

Free diffusion on "incipient" infinite cluster
Subdiffusion due purely to walk on random fractal

$$k = \frac{2\nu - \beta}{2\nu - \beta + \mu}$$

Walk only on all finite clusters of occupied sites.
Subdiffusion has two sources:
1) Walk on random fractal
2) Scale free confinement.

$$\frac{k}{k'} = \frac{2\nu - \beta}{2\nu}$$

Ratio of exponents, with and without confinement
No conductivity exponent μ.

$$t^a \to t^{a(1-\frac{c}{2})}$$

Scale-free confinement → Ratio of exponents $= 1 - \frac{c}{2}$

$\Pr(|\mathcal{C}| = s) \sim s^{1-\tau}$
$r \sim s^{\sigma\nu}$
$2 - \tau = -\sigma\beta$

From known exponents, one easily finds $c = \frac{\beta}{\nu}$ for percolation.

Does it agree?

$$1 - \frac{c}{2} = 1 - \frac{\beta}{2\nu} = \frac{2\nu - \beta}{2\nu} = \frac{k}{k'}$$

✓

Thanks!

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