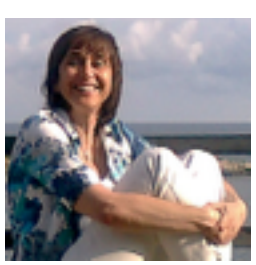
Fractional quantum Hall spectroscopy investigated by a resonant detector

Alessandro Braggio CNR-SPIN, Genoa

https://sites.google.com/site/alessandrobraggio/

M. Sassetti
Genoa



M.Carrega
Genoa

D.Ferraro
Geneve-Marseille

UNIVERSITÉ DE GENÈVE

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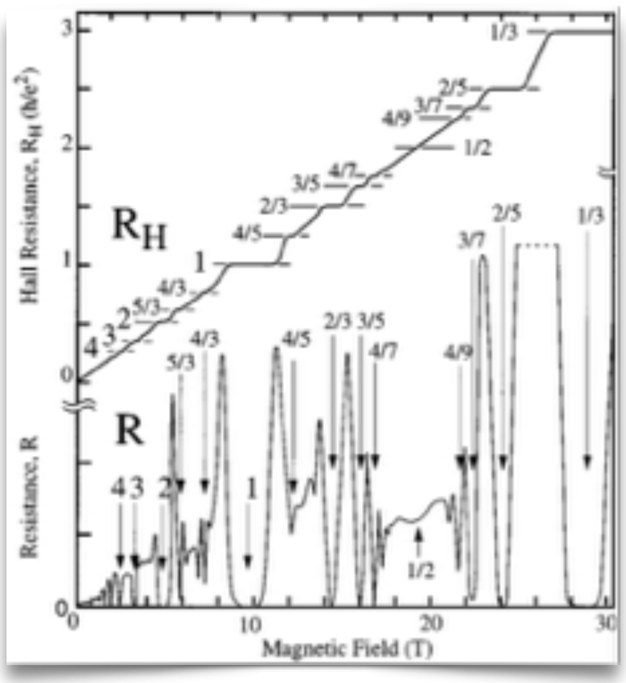
FUTURO IN RICERCA 2012

HORIZON 2020

FQHE: edge states & qps



- Topological protected edge states
- Fractional statistics & charges $\nu = \frac{N}{N_\Phi}$

  Laughlin PRL'83
- Chiral edge states with gapless modes

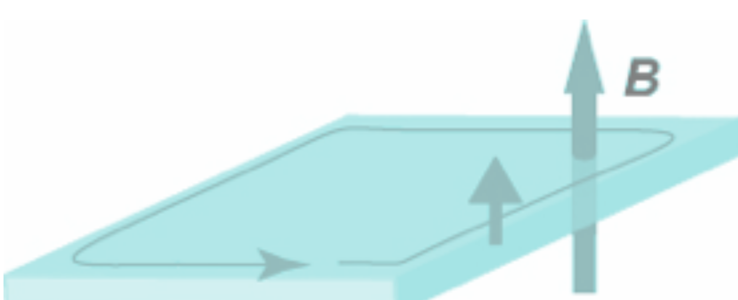
  Wen PRB90, *Halperin PRB 82, Buttiker PRB 88, Beenakker PRL 90*

$$\sigma_{xy} = \nu \frac{e^2}{h}$$

$$\sigma_{xx} = 0$$

- Laughlin sequence $\nu = \frac{1}{2np+1} = 1, \frac{1}{3}, \frac{1}{5}, \ldots$
- Jain sequence $\nu = \frac{p}{2np+1} = \frac{2}{5}, \frac{2}{3}, \cdots$

Jain PRL'89, Wen & Zee PRB'92, Kane & Fisher PRB'95



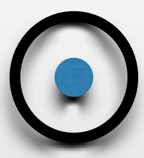
Hierarchical models

Multiple qp excitations

- Hierarchical theories

$$\Psi_l(x) \propto e^{l^T \cdot K \cdot \phi}$$

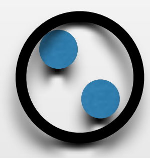
$m = 1$



Single-qp

$$e^* = \frac{1}{2np+1}$$

$m > 1$



m-agglomerate

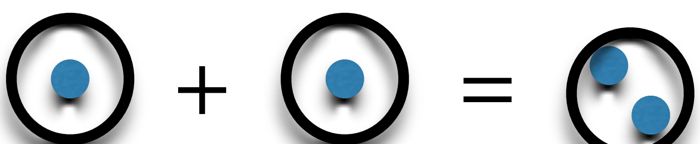
$$me^*$$

$$G^{(m)}(\tau) = \langle \Psi^{(m)}(\tau) \Psi^{(m)\dagger}(0) \rangle$$

$$G^{(m)}(\tau) \propto |\tau|^{-\Delta_m}$$

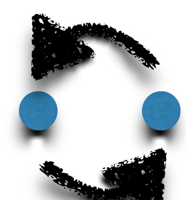
Δ_m Scaling dimension

Abelian



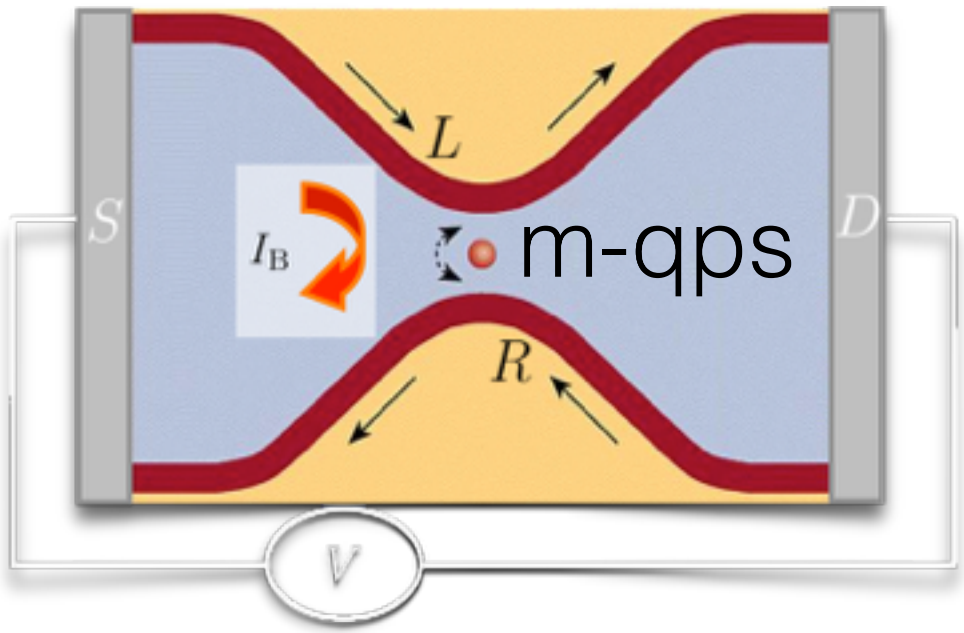
- Fractional statistics

$$\Psi^{(m)}(x)\Psi^{(m)}(y) = \Psi^{(m)}(y)\Psi^{(m)}(x)e^{-i\theta_m \mathrm{sgn}(x-y)}$$



Laughlin PRL 83, Arovas, Schrieffer & Wilczek PRL 84

QPC:Current & Noise



- Weak backscattering current

$$I = \nu \frac{e^2}{h} V - I_B \qquad I_B \ll I$$

- Power-law signatures in the scaling dimension Δ_m

$$G_B^{(m)} \propto T^{2\Delta_m - 2} \qquad I_B^{(m)} \propto V^{2\Delta_m - 1}$$

- Current noise signatures: charge measurement

$$S(\omega = 0) = \int_{-\infty}^{+\infty} \langle \{ \delta I_B(t), \delta I_B(0) \}_+ \rangle \qquad \delta I_B = I_B - \langle I_B \rangle$$

$$S^{(m)} = I_B^{(m)} \coth\left(\frac{m e^* V}{2 k_B T} \right)$$

$k_B T \gg m e^* V$: $\quad S^{(m)} \approx 2 k_B T G_B$

$k_B T \ll m e^* V$: $\quad S^{(m)} \approx m e^* I_B^{(m)}$

Multiple-qp evidences

- Fractional charges: single-qps evidences

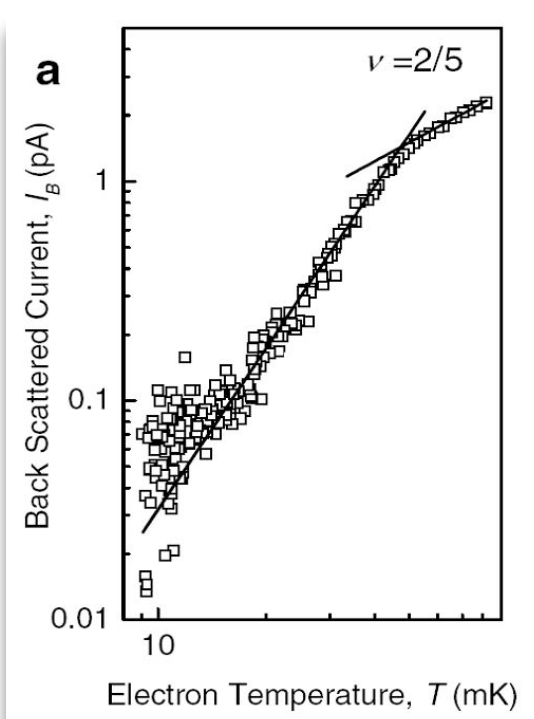
Theory:Kane & Fisher PRL 94, Fendley, Ludwig & Saleur PRL 95

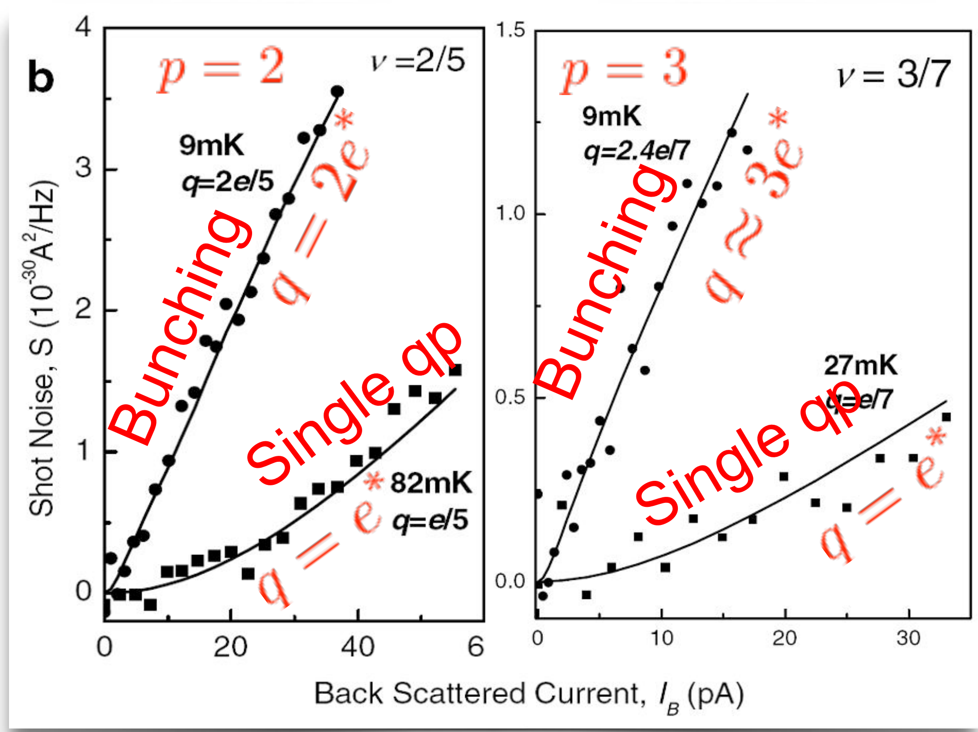
Exp:De-Picciotto… Nature 97,Saminadayar…. PRL'97,Reznikov… Nature'99

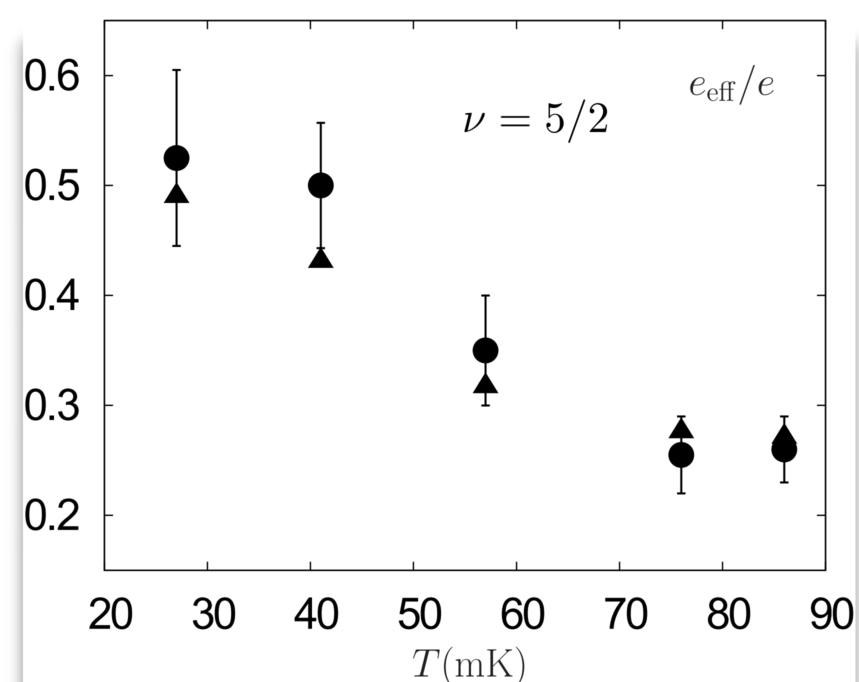
Robert B. Laughlin, Horst L. Störmer and Daniel C. Tsui
"for their discovery of a new form of quantum fluid with fractionally charged excitations"

- Multiple-qp. evidences

Chung…PRL03, Bid PRL03,Dolev….



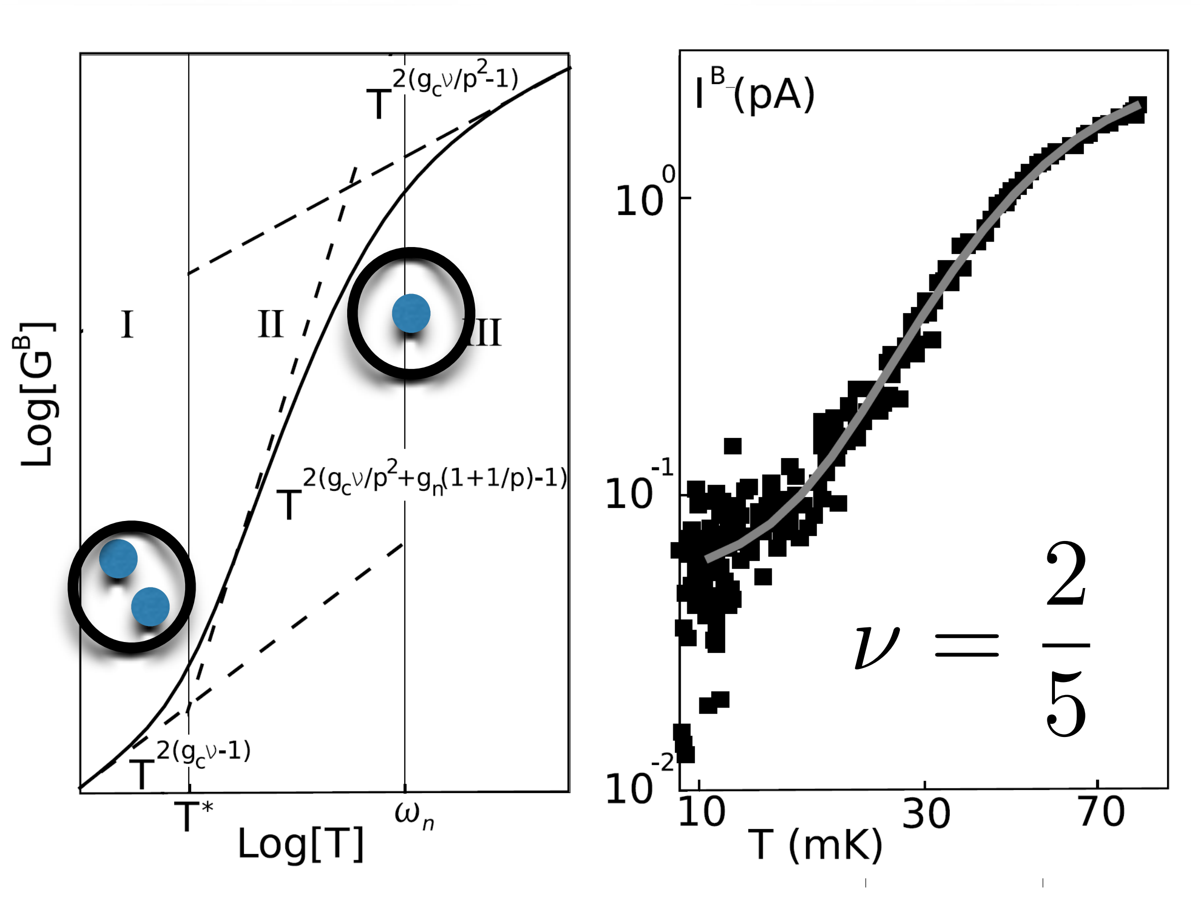


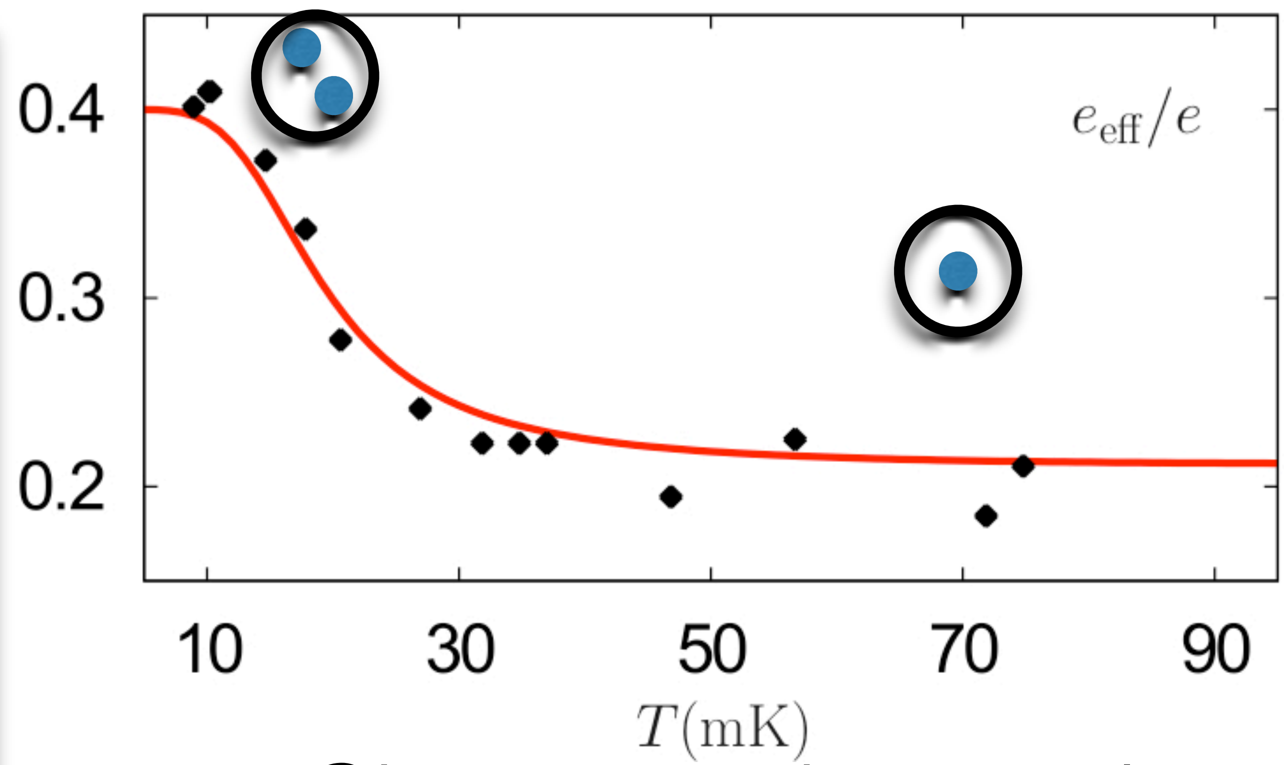


M. Heiblum (2/5,3/7,2/3,5/2,..),Willet(5/2),Yacoby (2/3),…

Theoretical explanations 1

- Single-qp and multiple-qp crossover





- Charge and neutral modes
- Mode velocity $v_n \ll v_c$
  $\omega_n \ll \omega_c$

D. Ferraro, A. B. , N. Magnoli, M. Sassetti PRL 08,PRB10,NJP10,PRL11

- Renormalization of scaling exponent $\Delta_m = g_c\Delta_m^c + g_n\Delta_m^n$

Coupling other degrees: Rosenow & Halperin PRL 02, Papa & MacDonald PRL 05

1/f noise + dissipation: A. B., D. Ferraro, M. Carrega, N. Magnoli, M. Sassetti NJP12

New questions

- Qp. charge measurements
  Kou et al. PRL12,D. T. McClure et al. PRL12, Safi & Sukhorukov EPL10
- Contropropagating neutral modes evidences
  Bid et al., Nature 10, Gross et al. PRL12, Gurman et al. Nature 12, Shtanko et al PRB14, Takei et al. PRB11, Dolev et al. PRL11
- Heat transport & neutral modes proliferation
  Altimiras et al PRL12, Aita et al PRB13, Inoue et al. Nature14
- Edge reconstruction & T dependent edge coupling
  J. Wang et al PRL13, Karzig et al NJP12, Zhang et al 1406.7296
- Imaging of the edge structure
  N. Paradiso et al. PRB11,PRL12, Pasher et al. PRX14, Kozikov et al NJP13
- Edge model identification Meier et al. 1406.4517

Why not at finite frequency ?

- Josephson resonances $\omega_m = me^* V/\hbar$

Blanter&Buettiker Phys.Rep.00, Rogovin&Scalapino Ann. Phys 74

- Rich theoretical tools & interesting non-equilibrium phys.

Chamon..PRB95; Chamon..PRB96; Dolcini..PRB05; Bena..PRB06; Bena..PRB07; Sukhorukov..PRB01; Sukhorukov..EPL10; Schoelkopf…03; Deblock…Science '03; Engel…'04;Hekking….06;……

- Interesting questions: how to measure it?

Lesovik..JETP97;Gavish U..PRB00;Gavish U.. arXiv:0211646; Bednorz& Belzig PRL13; Aguado..PRL00;

Symmetrized or non-symmetrized ?

- Symmetrized noise (Landau docet) $[I(t), I(t')] \neq 0$

$$S^{(m)}(\omega) = \int_{-\infty}^{+\infty} e^{i\omega t} \langle \{\delta I_B(t), \delta I_B(0)\}_+ \rangle = \sum_{i=\pm} S_i^{(m)}(\omega)$$

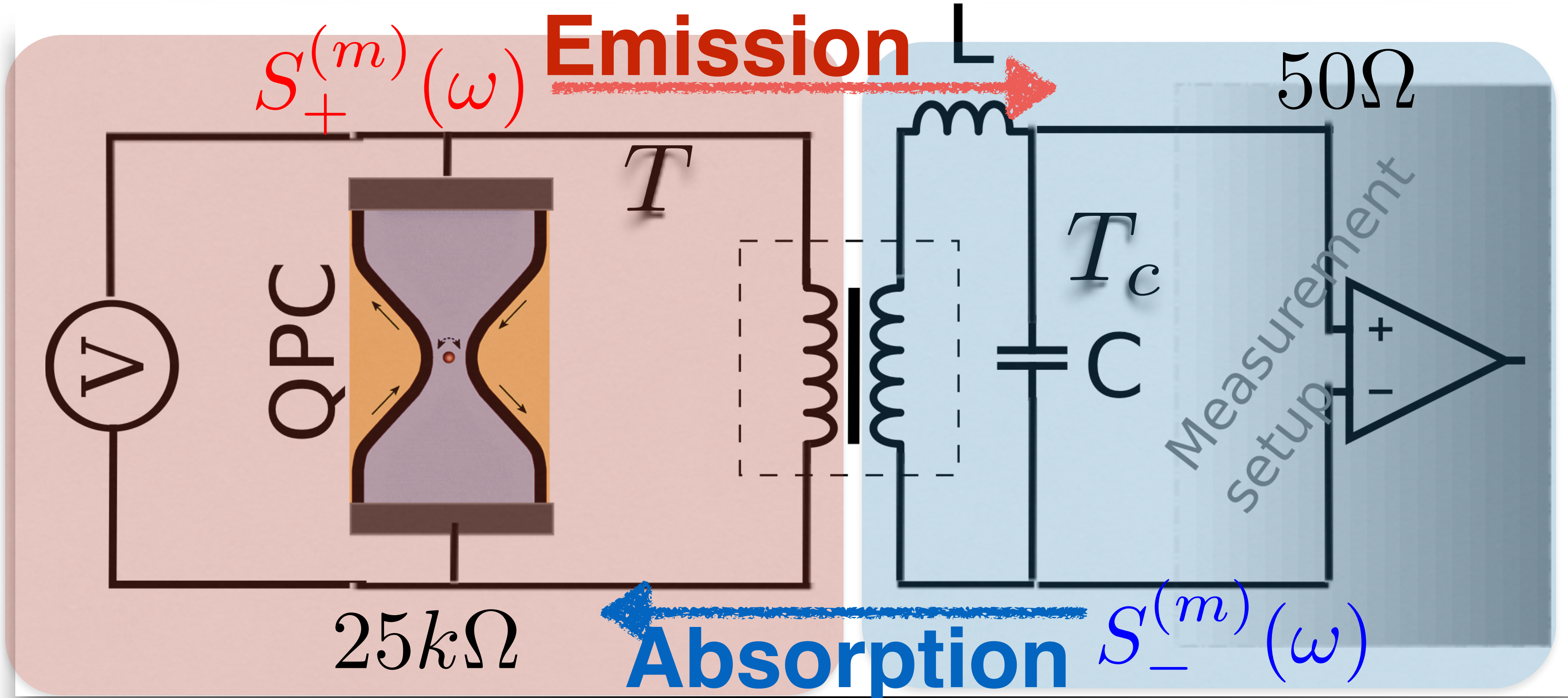
- Non-symmetrized (Emission/absorption from QPC)

Aguado PRL00, Blanter 05, Martin&Crepieux 04-05-06,…..

$$S_{+/-}^{(m)}(\omega) = \int_{-\infty}^{+\infty} e^{\pm i\omega t} \langle \delta I_B^{(m)}(t) \delta I_B^{(m)}(0) \rangle$$

Finite frequency detection

Lesovik G B and Loosen R JETP 65 295 (1997); Gavish U,….arXiv:0211646



Resonant

$\omega = \sqrt{1/LC}$

Cold detector

$T_c \ll T$

Hot detector

$T_c \gg T$

- Impedance matched resonant detection scheme

Altimiras et al. APL13, PRL14

$\omega \approx 5GHz \qquad T \approx 15mK$

- Output power proportional to variation of LC energy $\delta\langle x^2\rangle$

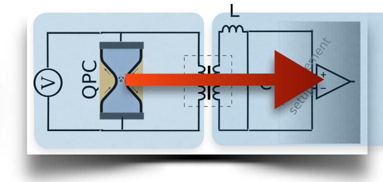
$$S_{meas}^{(m)}(\omega) = K\left\{S_+^{(m)}(\omega) + n_B(\omega)\left[S_+^{(m)}(\omega) - S_-^{(m)}(\omega)\right]\right\}$$

$$-\omega\,\Re e\left[G_{ac}^{(m)}(\omega)\right]$$

$$n_B(\omega) = \frac{1}{e^{\omega/T_C} - 1} \qquad K = \left(\frac{\alpha}{2L}\right)^2 \frac{1}{2\eta} \ll 1$$

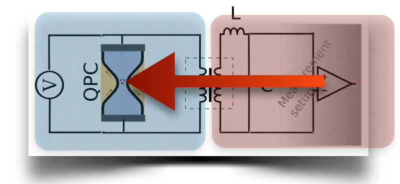
Noise properties in QPC-LC

- Detector quantum limit (Cold detector) $k_B T_c \ll \omega$



$$S^{(m)}_{meas}(\omega) \approx K S^{(m)}_{+}(\omega) + \mathcal{O}(e^{-\hbar\omega/k_B T_c})$$

- Absorbitive QPC limit (Hot detector) $k_B T_c \gg \omega$



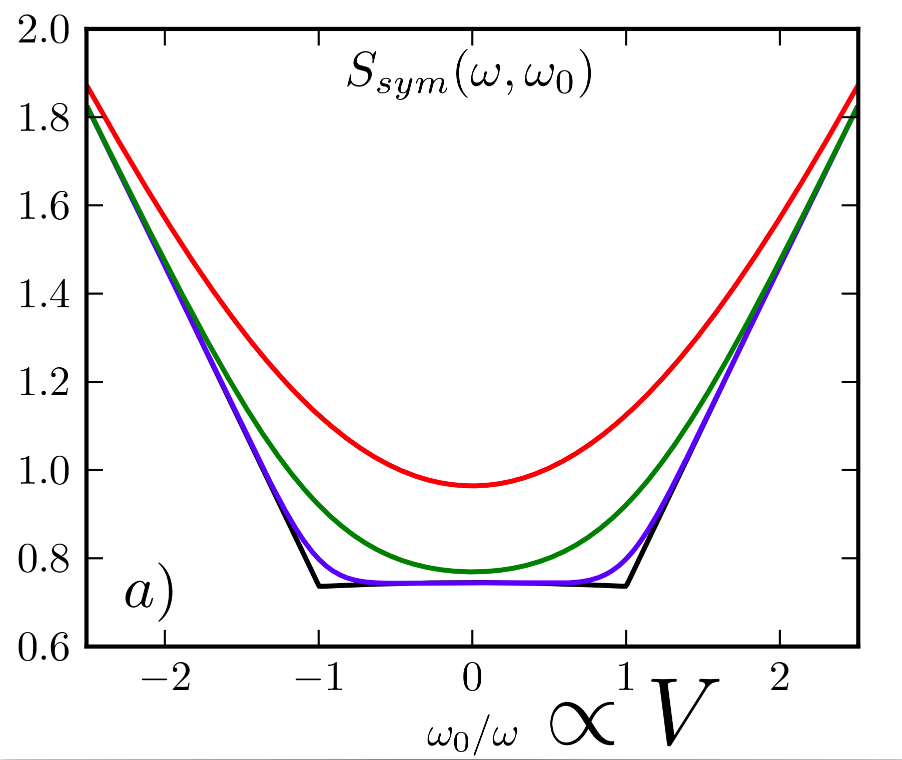
$$S^{(m)}_{meas}(\omega) \approx K \left\{ S^{(m)}_{+}(\omega) - k_B T_c \Re e \left[G^{(m)}_{ac}(\omega) \right] \right\}$$

- Is it measurable? $\omega_0 = e^* V/\hbar$
- $$S_{meas} \equiv S_{ex} \qquad T = T_c$$
- Lowest order in the tunnelling $|t_m|^2$(purely additive)

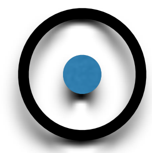
$$S_{sym}(\omega) = \sum_m S^{(m)}_{sym}(\omega) \qquad S_{meas}(\omega) = \sum_m S^{(m)}_{meas}(\omega)$$

- Keldysh formalism blow up in Fermi's rule: rate $\mathbf{\Gamma}^{(m)}(E)$

Non-interacting result



$\nu = 1$

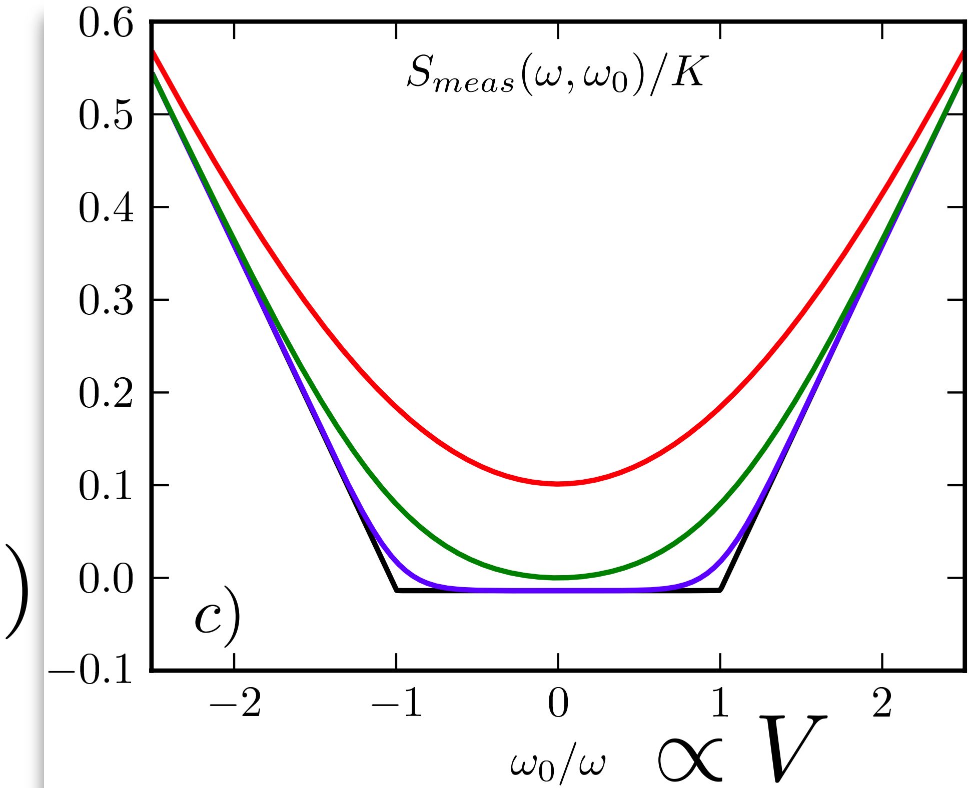


Electron

$T_c = 15\text{mK}$

$\omega = 7.9\text{GHz}(60\text{mK})$

$\omega_c = 660\text{GHz}(5\text{K})$



$T = 0.1, 5, 15, 30[\text{mK}]$

$$S_{sym}(\omega,\omega_0) = 2\frac{\tilde{S}_0}{\omega_c}\left[\theta(\omega_0-\omega)\omega_0 + \theta(\omega-\omega_0)\omega\right] \qquad \tilde{S}_0 = \frac{e^2}{2}\frac{|t_1|^2}{2\pi\alpha^2}\frac{1}{\omega_c}$$

$$S_{meas}(\omega,\omega_0) \approx KS_+(\omega,\omega_0) = \frac{K}{2}\left(S_{sym}(\omega,\omega_0) - 2\tilde{S}_0\frac{\omega}{\omega_c}\right) \qquad k_BT_c \ll \omega$$

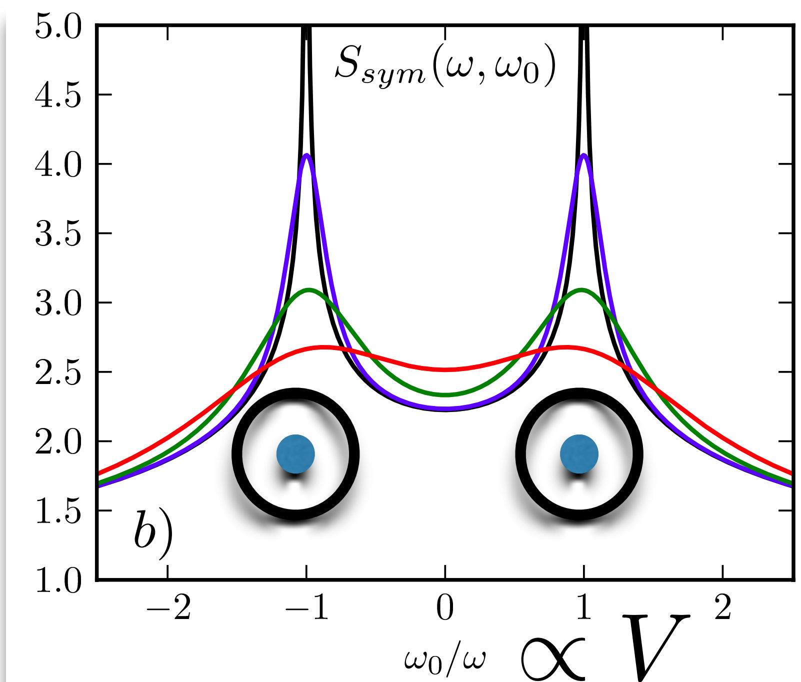
$$\mathbf{\Gamma}^{(1)}(E) \propto \theta(E)E$$

Lesovik G B and Loosen R JETP 65 295 (1997)

Interacting case: Laughlin

$$\nu = 1/3$$

$$e^* = \frac{e}{3}$$



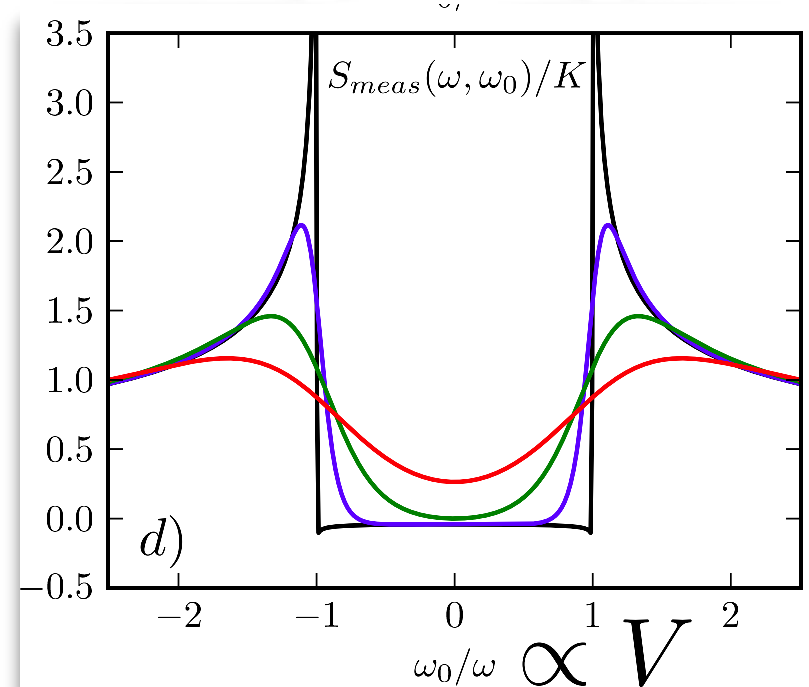
Single-qp

$$T_c = 15\text{mK}$$

$$\omega = 7.9\text{GHz}(60\text{mK})$$

$$\omega_c = 660\text{GHz}(5\text{K})$$

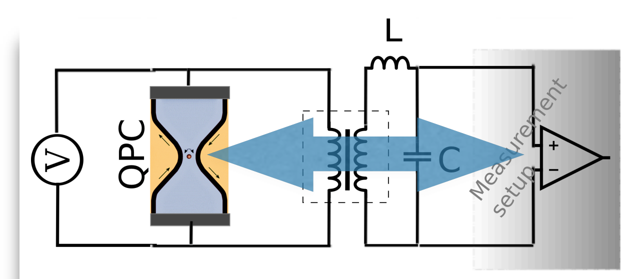
$$T = 0.1, 5, 15, 30[\text{mK}]$$



$$S_{sym}(\omega, \omega_0) \approx |\omega - \omega_0|^{4\Delta^{(1)}_{1/3} - 1}$$

Chamon, Freed & Wen PRB95,PRB96

- Detector quatum limit $\quad k_B T_c \ll \omega$
- QPC Shot noise $\quad k_B T \ll \omega_0$

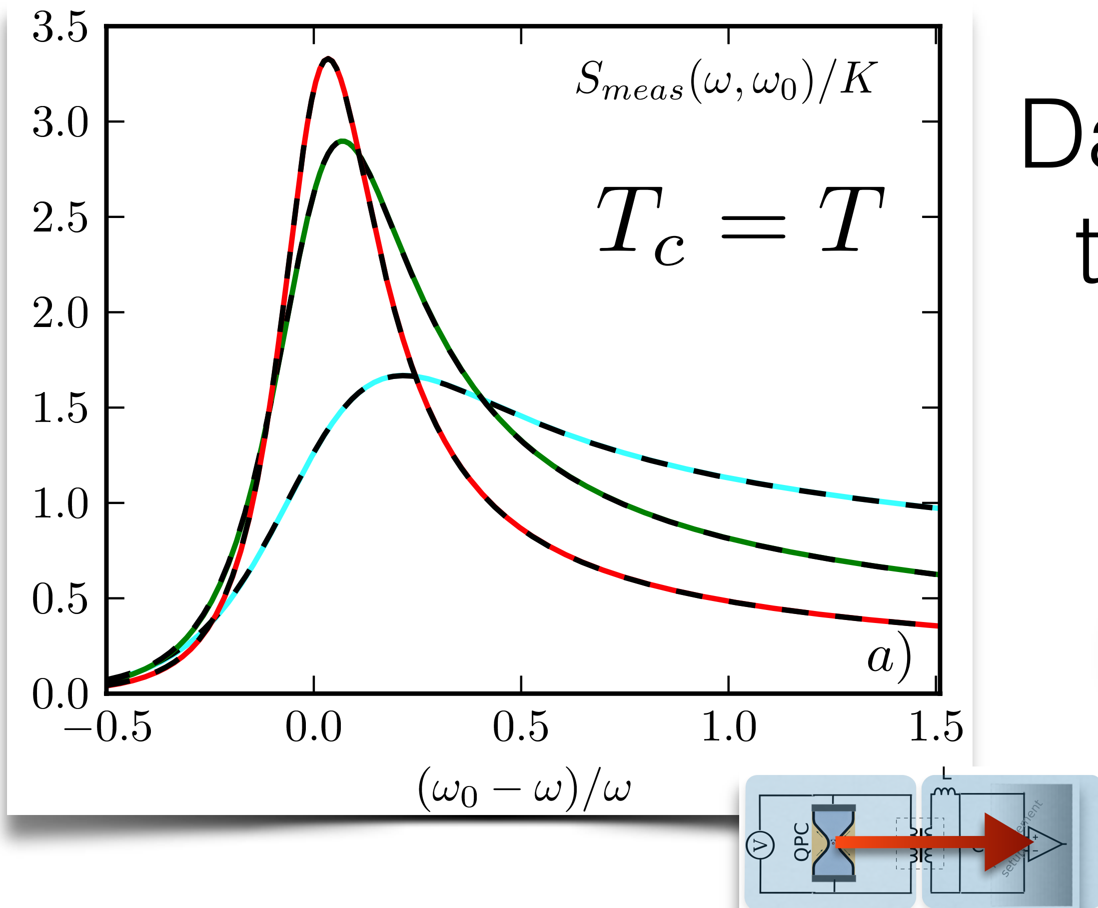


$$S^{(m)}_{meas}(\omega, \omega_0) \approx S^{(m)}_{+}(\omega) \approx K\frac{(me^*)^2}{2}\mathbf{\Gamma}^{(m)}(-\omega + m\omega_0) \qquad \omega \sim \omega_0 \qquad m = 1$$

$S^{(m)}_{meas}(\omega, \omega_0)$ returns directly the rates…….

Rate detection

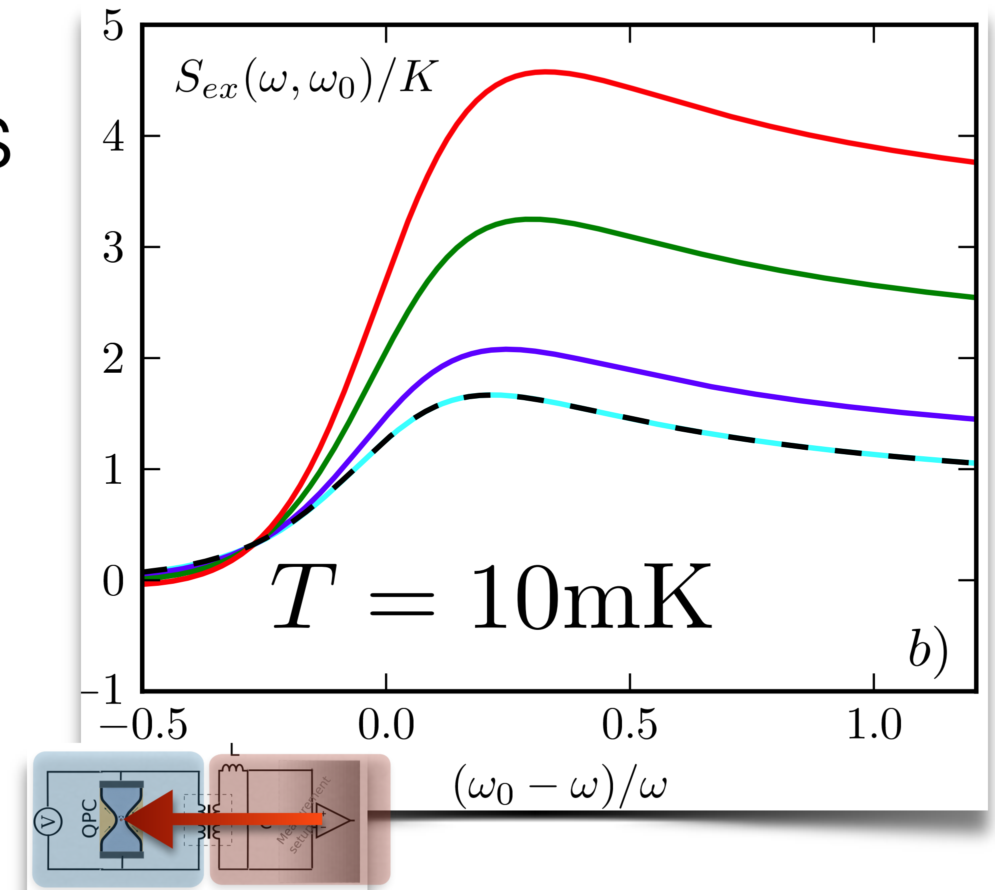
$\nu = 1/3, 1/5, 1/7$



Dashed lines theoretical rates

$$\Delta_\nu^{(1)} = \frac{\nu}{2}$$

$T_c = 10, 30, 60, 90$ mK



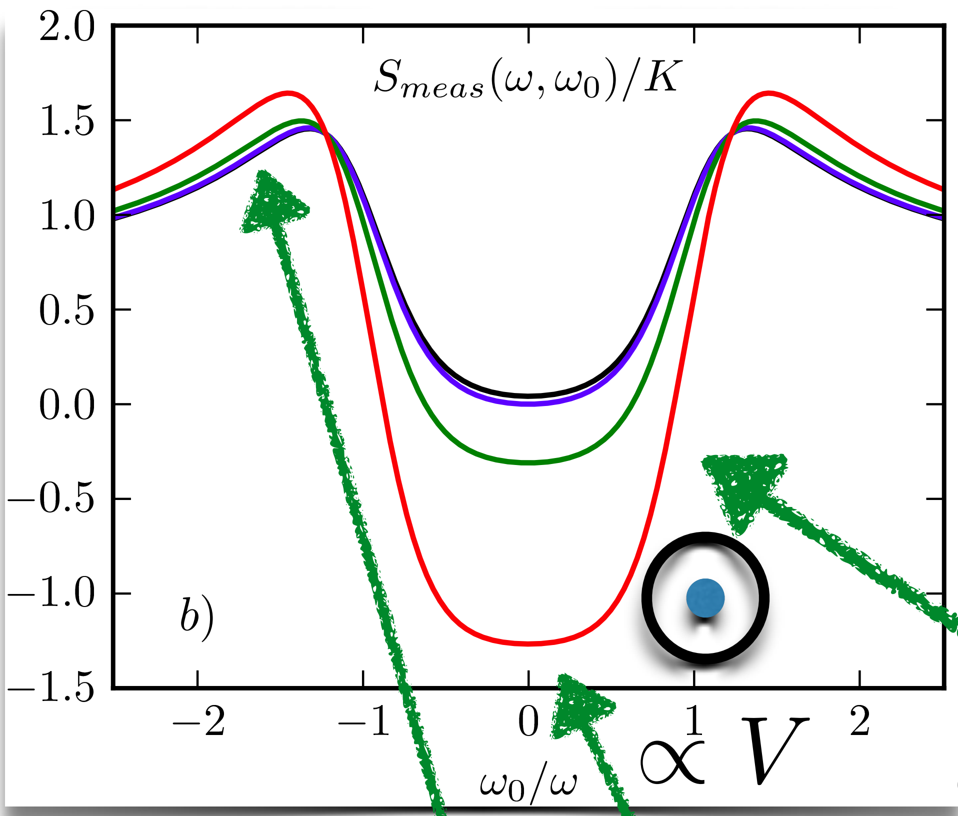
It is possible to extract the scaling dimensions without requiring an extended window in frequency and bias simplifying the experimental requirements

Note that $S_{meas} \equiv S_{ex}$ $\quad T = T_c$

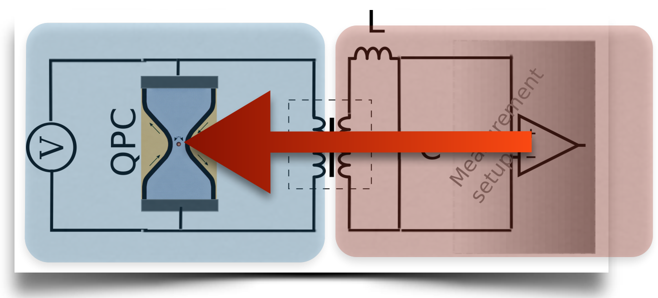
Hotter is better?

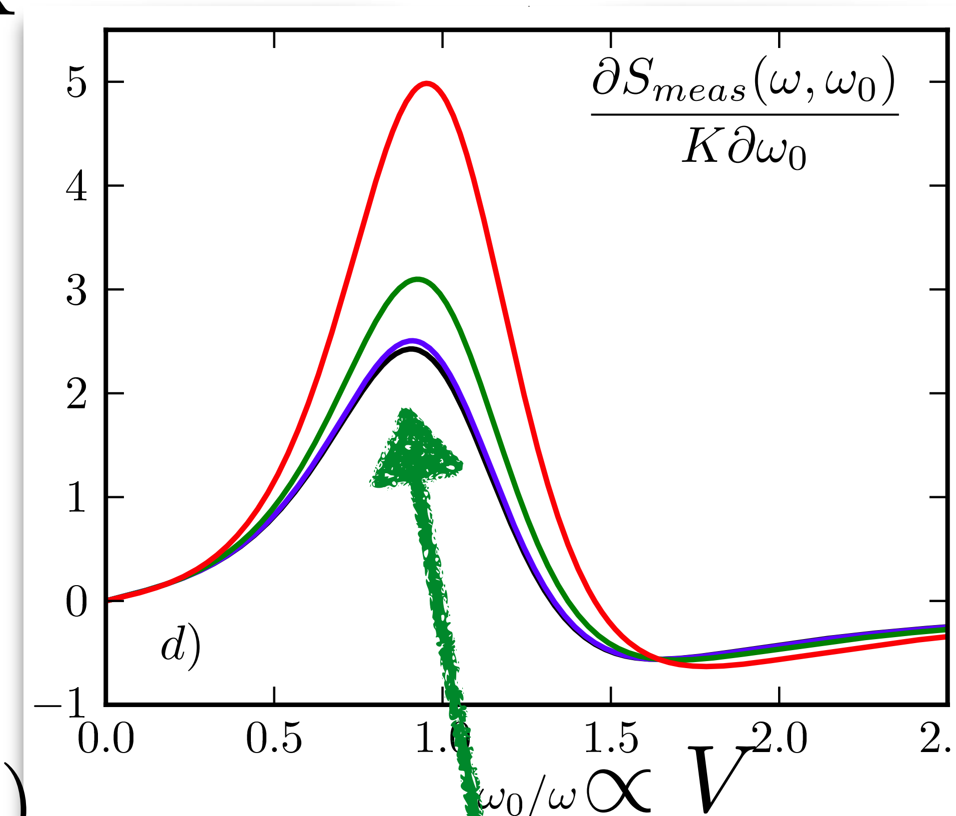
Safi & Sukhorukov EPL10

$T_c = 5, 15, 30, 60$ mK



$\nu = 1/3$





$T_c = 15\text{mK}$

$\omega = 7.9\text{GHz}(60\text{mK})$

$\omega_c = 660\text{GHz}(5\text{K})$

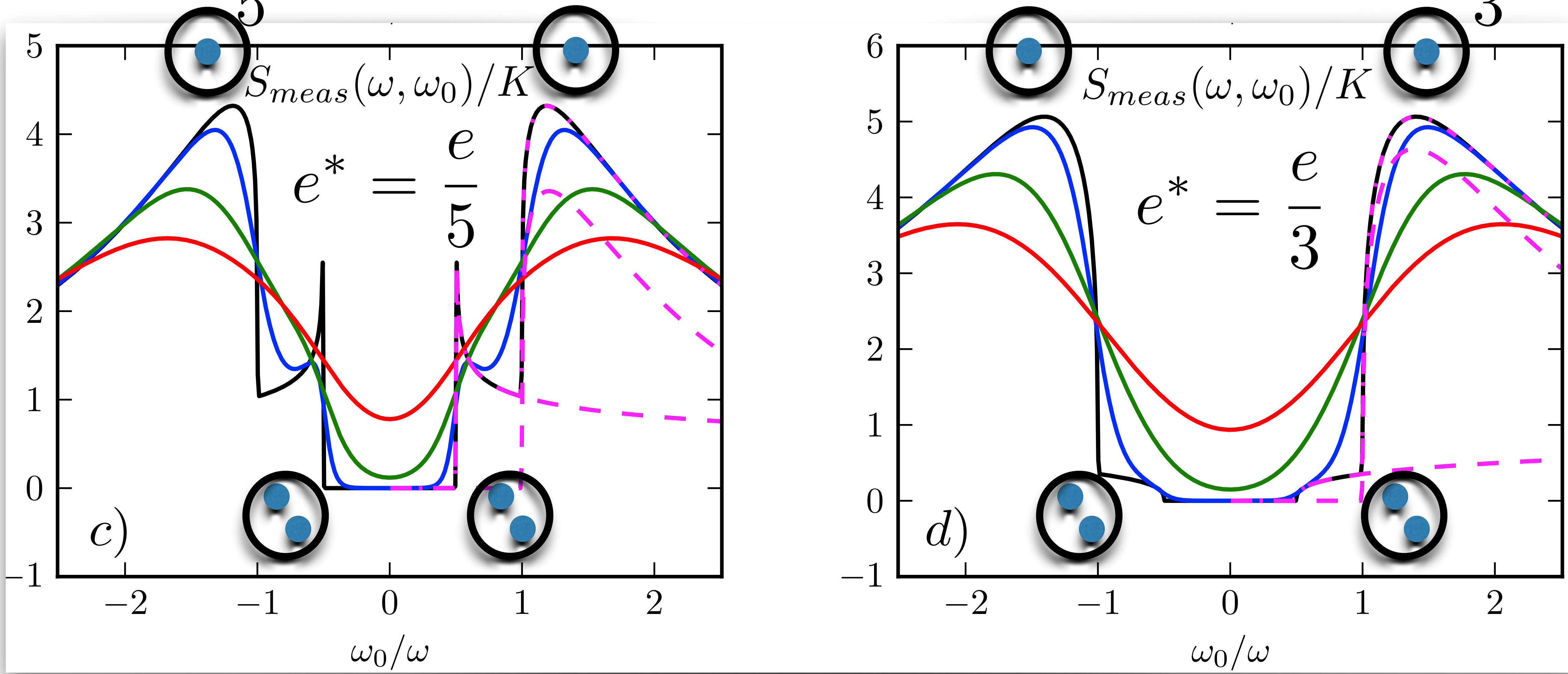
The QPC cannot excite detector modes only absorptive

The QPC excites detector

The combined effect is an enhancement of jump/peak

Multiple-qp spectroscopy: S_{meas}

$\nu = \frac{2}{5}$ $T = 0.1, 5, 15, 30 [\mathrm{mK}]$ $\nu = \frac{2}{3}$



Note that $S_{meas} \equiv S_{ex}$ $T = T_c$

$$S_{meas}(\omega, \omega_0) \approx \alpha_1 \mathbf{\Gamma}^{(1)}(\omega_0 - \omega) + \alpha_2 \mathbf{\Gamma}^{(2)}(2\omega_0 - \omega)$$

- Rates are directly fitted: scaling dimensions at finite T
- Multiple-qps are observed in different window

Conclusion

- QPC+LC resonator is a powerful tool
- f.f. noise resolve the presence of multiple qps
- Multiple-qp spectroscopy can be done at realistic T
- Information on qps by analysing bias behaviour
- Changing detector temperature increases the sensibility
- Validate composite edge model theories
- This techniques can be used in other systems

New. J. Phys. 16 043018 (2014)

Topological order in IQHE

- Topological invariant 2+1D under magnetic field (Kubo)

$$\sigma_{xy} = -ie^2\hbar \sum_{E_\alpha < E_F < E_\beta} \frac{(v_y)_{\beta\alpha}(v_x)_{\alpha\beta} - (v_x)_{\alpha\beta}(v_x)_{\beta\alpha}}{(E_\alpha - E_\beta)^2}$$

- Magnetic Brillouin zone (torus)

$$\sigma_{xy}^{(\alpha)} = \frac{e^2}{2\pi i}\int d^2k \int d^2r \left(\frac{\partial u_{k_1,k_2}^{\alpha *}}{\partial k_2}\frac{\partial u_{k_1,k_2}^{\alpha}}{\partial k_1} - \frac{\partial u_{k_1,k_2}^{\alpha *}}{\partial k_1}\frac{\partial u_{k_1,k_2}^{\alpha}}{\partial k_2} \right)$$

$$\sigma_{xy}^{(\alpha)} = n\frac{e^2}{h}$$ Topological invariant

Thouless, Kohmoto, Nightingale,den Nijs PRL'82; Kohmoto Ann. Phys. 160, 343 (1985)

Edge states & Multiple-qp

- ϕ_i Chiral Luttinger liquids   Wen, Kane & Fisher ,….

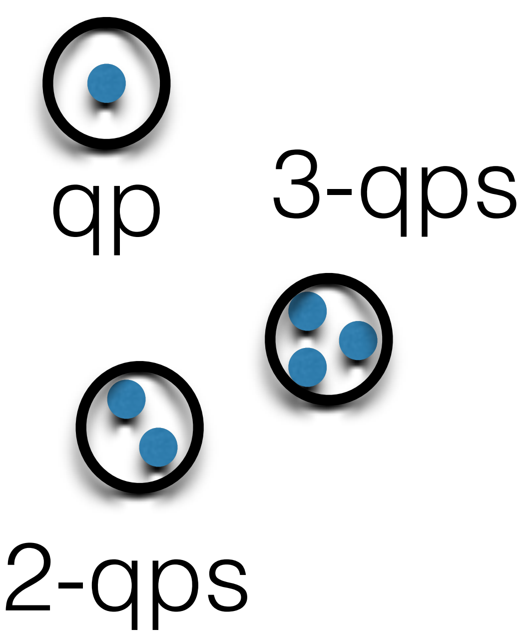
$$\mathcal{L} = \frac{1}{4\pi}(K_{ij}\partial_x\phi_i\partial_t\phi_j + V_{ij}\partial_x\phi_i\partial_x\phi_j + 2\epsilon^{\mu\nu}t_j\partial_\mu\phi_j A_\nu)$$

- Multiple-qps excitations $\Psi_l(x) \propto e^{l^T \cdot K \cdot \phi}$

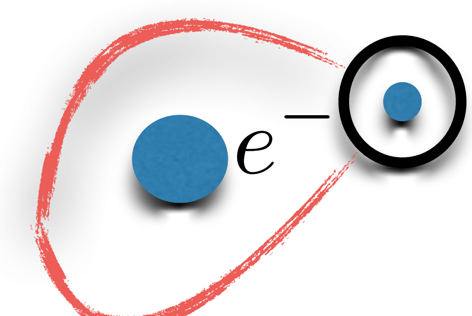
- Filling factor $\nu = t^T \cdot K^{-1} \cdot t$

- Fractional charges $q_l = \frac{1}{2\pi}\, l^T \cdot K^{-1} \cdot t = me^*$

- Fractional statistics $\theta_l = 2\pi\; l^T \cdot K^{-1} \cdot l$



- Monodromy: qp aquires 2π phase in a loop around e^-



Wen & Zee PRB 92, J. Fröhlich et al JSTAT 97

TFT for FQHE: CS (Laughlin)

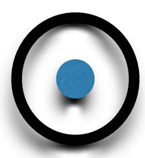
- Fractional qp. $\mathcal{L} = \mathcal{L}_{CS} + a_\mu j^\mu_{qp}$ $\nu = \frac{1}{2np+1} = 1, \frac{1}{3}, \frac{1}{5}, \cdots$

$j^0_{qp} = el\delta(r - r_0)$ $\frac{J^0}{e} = \nu \frac{B}{\phi_0} + l\nu\delta(r - r_0)$ $e^*_l = e\nu l$

- Multiple qps

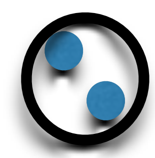
$\nu = \frac{1}{3}$

$l = 1$



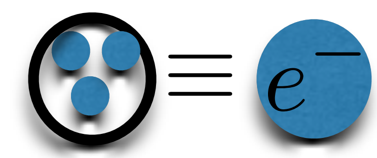
Single-qp

$l = 2$



2-agglomerate

$l = 3$



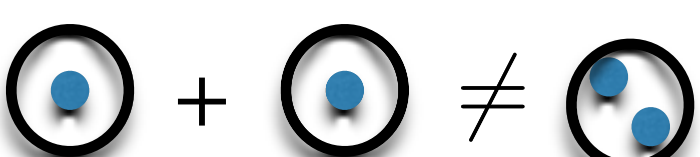
electron

- Abelian fractional statistics

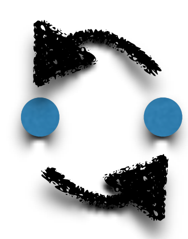
$$\Psi^{(m)}(x)\Psi^{(m)}(y) = \Psi^{(m)}(y)\Psi^{(m)}(x)e^{-i\theta_m \mathrm{sgn}(x-y)}$$

Laughlin PRL 83, Arovas, Schrieffer & Wilczek PRL 84

- Non-Abelian extension Wen Adv. Phys. 95, Read&Rezayi PRB99



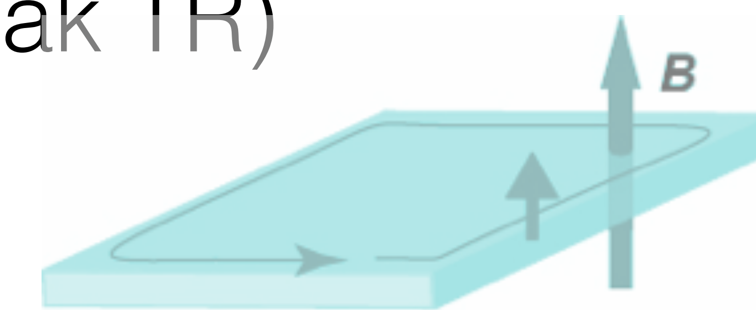
Fusion Rules



$e^{i\theta_m} \to e^{i\hat{\Theta}_{mn}}$

TFT for FQHE: CS

- Electron in a flatland 2+1D under B (break TR)



- Gapped bulk state
- Chiral edge states (no backscattering) $\sigma_{xy} = \nu \frac{e^2}{2\pi}$

$$J^\mu = \sigma_{xy}\epsilon^{\mu\nu\rho}\partial_\nu A_\rho$$

$$\frac{J^0}{e} = \nu \frac{e}{2\pi} B = \nu \frac{B}{\phi_0} \Rightarrow \nu = \frac{N}{N_\Phi}$$

$$J^i = \sigma_{xy}\epsilon^{ij}E_j$$

- Low energy effective theory TFT: Chern-Simons $\nu = \frac{1}{k}$

$$J^\mu = \frac{e}{2\pi}\epsilon^{\mu\nu\rho}\partial_\nu a_\rho$$

Current cons. $\partial_\mu J^\mu = 0$

$$\mathcal{L}_{CS} = -\frac{k}{4\pi}\epsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho + \frac{e}{2\pi}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu a_\rho$$

Froehlich & Zee, NPB 91; Wen, Adv.Phys. 95

TFT for FQHE: CS

$$\nu = \frac{p}{2np+1} = \frac{2}{5}, \frac{2}{3}, \dots$$

- Abelian Hierarchical models $a^I = (a^1, ..., a^n)$

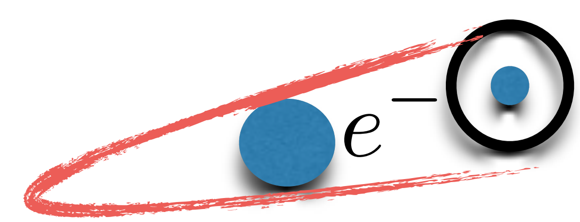
$$\mathcal{L}_{CS} = -\frac{K_{IJ}}{4\pi}\epsilon^{\mu\nu\rho} a^I_\mu \partial_\nu a^J_\rho + \frac{e}{2\pi} t_I \epsilon^{\mu\nu\rho} A_\mu \partial_\nu a^I_\rho$$

Filling factor $\nu = t^T \cdot K^{-1} \cdot t$

Fractional charges $q_l = \frac{1}{2\pi}\, l^T \cdot K^{-1} \cdot t = me^*$

$l^{qp}_I = (l_1, .., l_n)$

Fractional statistics $\theta_l = 2\pi\, l^T \cdot K^{-1} \cdot l$



Monodromy: qp aquires 2π phase in a loop around e^-

Wen & Zee PRB 92, J. Fröhlich et al JSTAT 97

CS with boundary

- Breaking of gauge invariance

- Requiring full gauge invariance bulk+boundary

$$S_{tot} = S_{bulk} + S_{bd} \qquad \delta S_{tot} = 0$$

Wen Adv Phys. 95, Zee 95

- Local observable locate at the boundary: 1+1D chiral currents Kač-Moody algebra WZW model

$$[a(z), a(z')] = \frac{2\pi}{k}\delta(z - z') \qquad z = (x+t)/\sqrt{2}$$

- 1+1D chiral boson χ-Luttinger liquid $a_\mu = \partial_\mu \phi$

$$S_{edge} = \frac{k}{2\pi}\int dtdx\partial_x\phi(\partial_t - \partial_x)\phi$$

$$[\phi(x), \phi(x')] = i\pi\nu \text{sgn}(x - x')$$

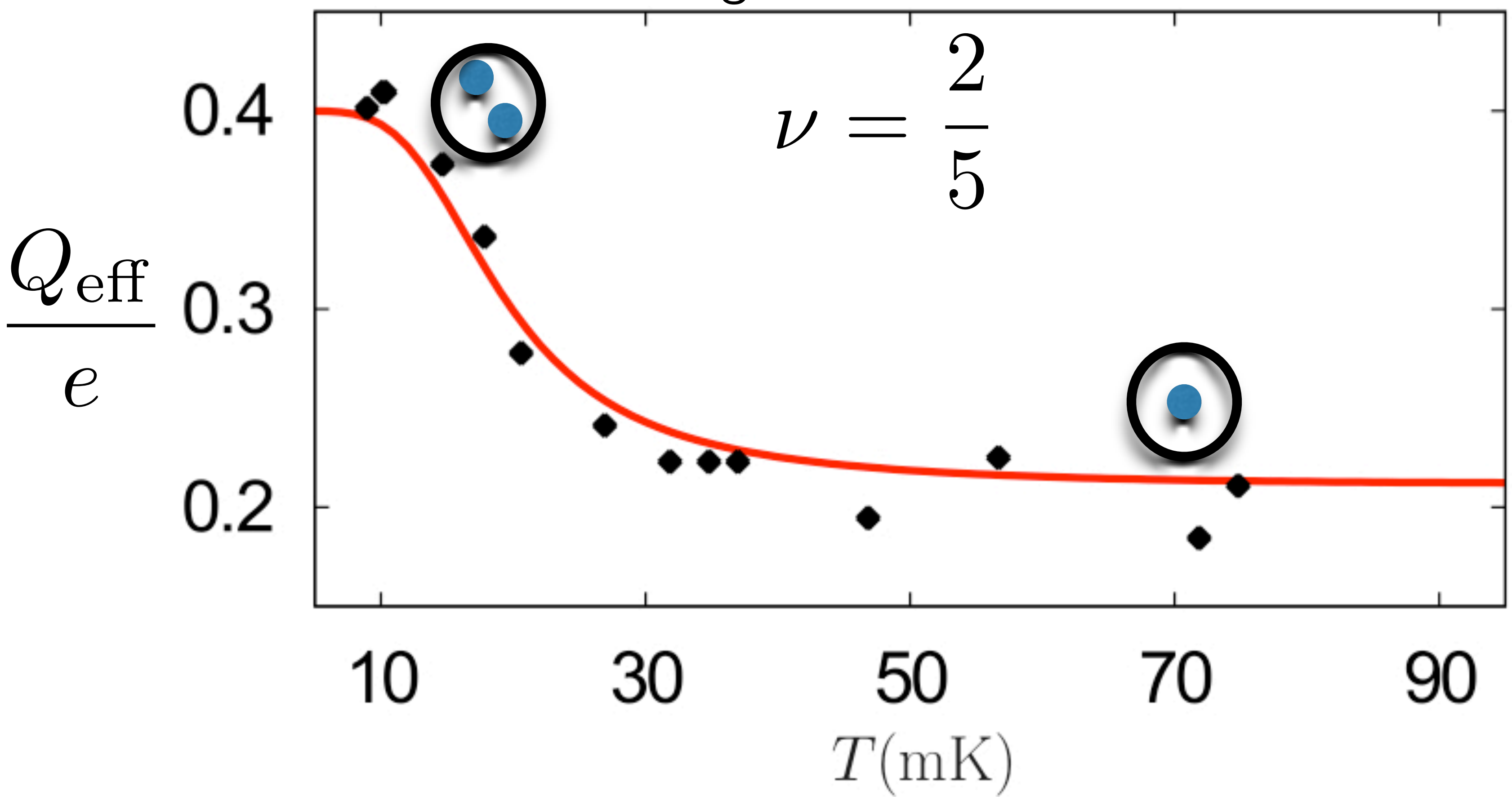
$$\Psi_l(x) \propto e^{l^T \cdot K \cdot \phi}$$

Bosonization

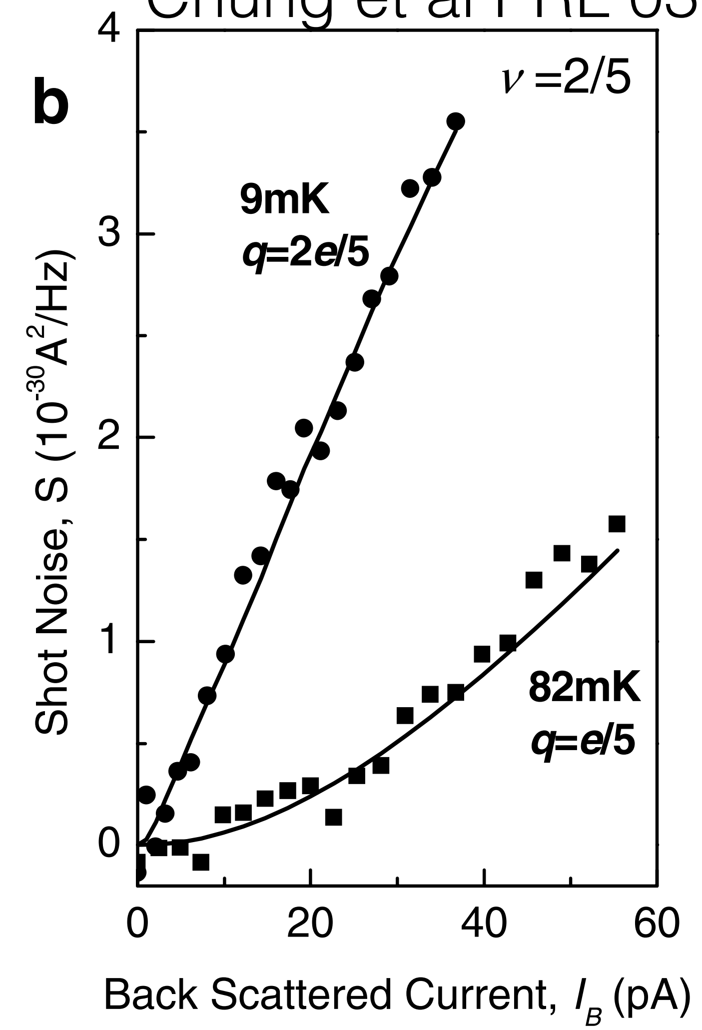
Wen ; Fisher&Kane

Theoretical explanations 2

M. Heiblum data from Chung et al PRL 03



Chung et al PRL 03



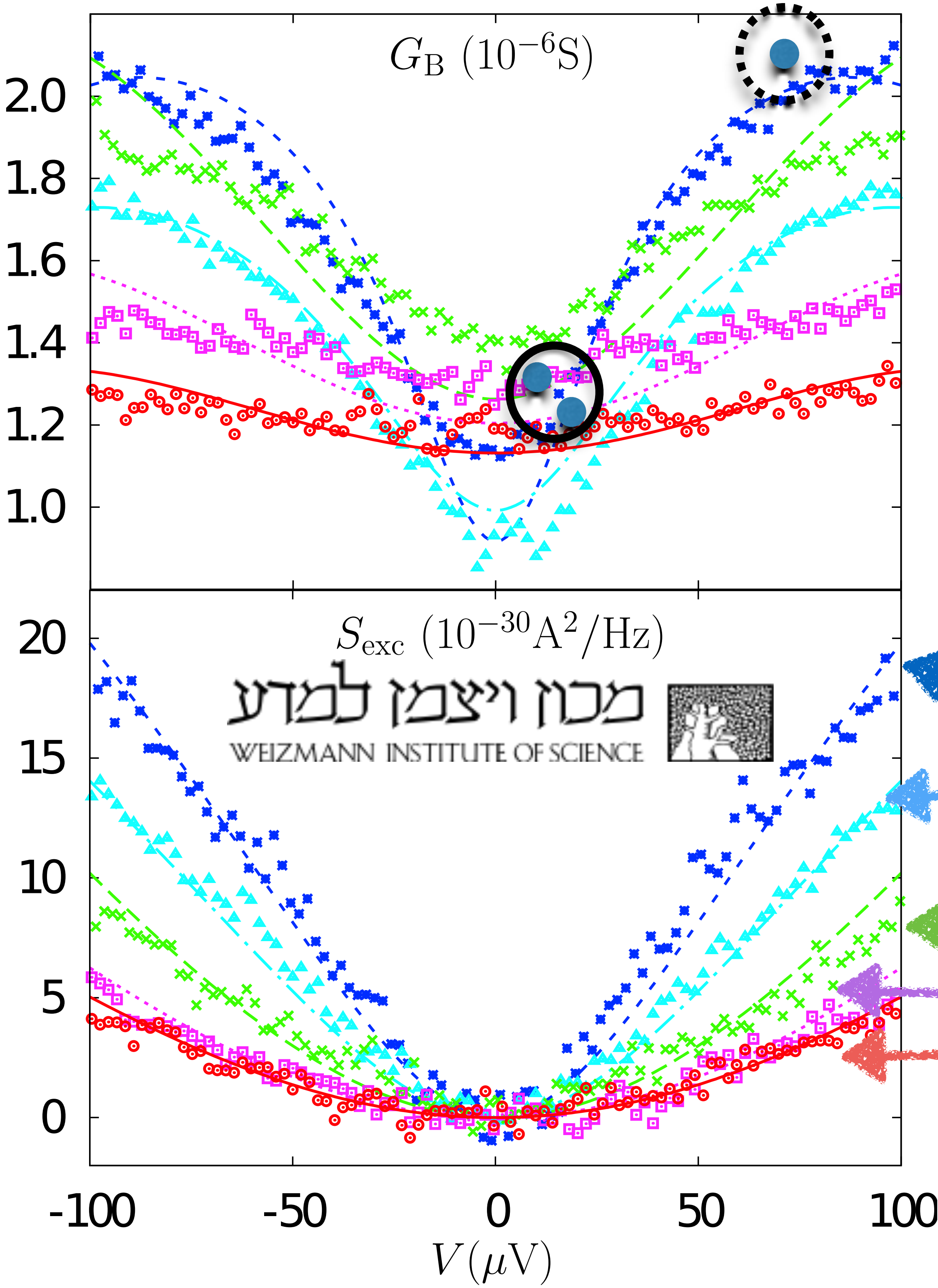
D. Ferraro, A. B., N. Magnoli, M. Sassetti, PRB10
D. Ferraro, A. B., N. Magnoli, M. Sassetti, NJP10

$$S_{\text{exc}} = Q_{\text{eff}}(T)\coth\left[\frac{Q_{\text{eff}}(T)V}{2T}\right] I_{\text{B}}(V,T) - 2TG_{\text{B}}(T)$$

$$Q_{\text{eff}}(T) = \left[\frac{3T}{G_{\text{B}}^{(\text{tot})}}\left(\frac{d^2 S_{\text{exc}}}{dV^2} - \frac{2}{3}T\frac{d^3 I_{\text{B}}}{dV^3}\right)\right]^{\frac{1}{2}}_{V\to 0} \approx \left[\frac{\langle I_{\text{B}}\rangle_3}{(e^2 I_{\text{B}})}\right]^{\frac{1}{2}}_{V\to 0}$$

Theoretical explanations 2

Data from M. Dolev and M.Heiblum

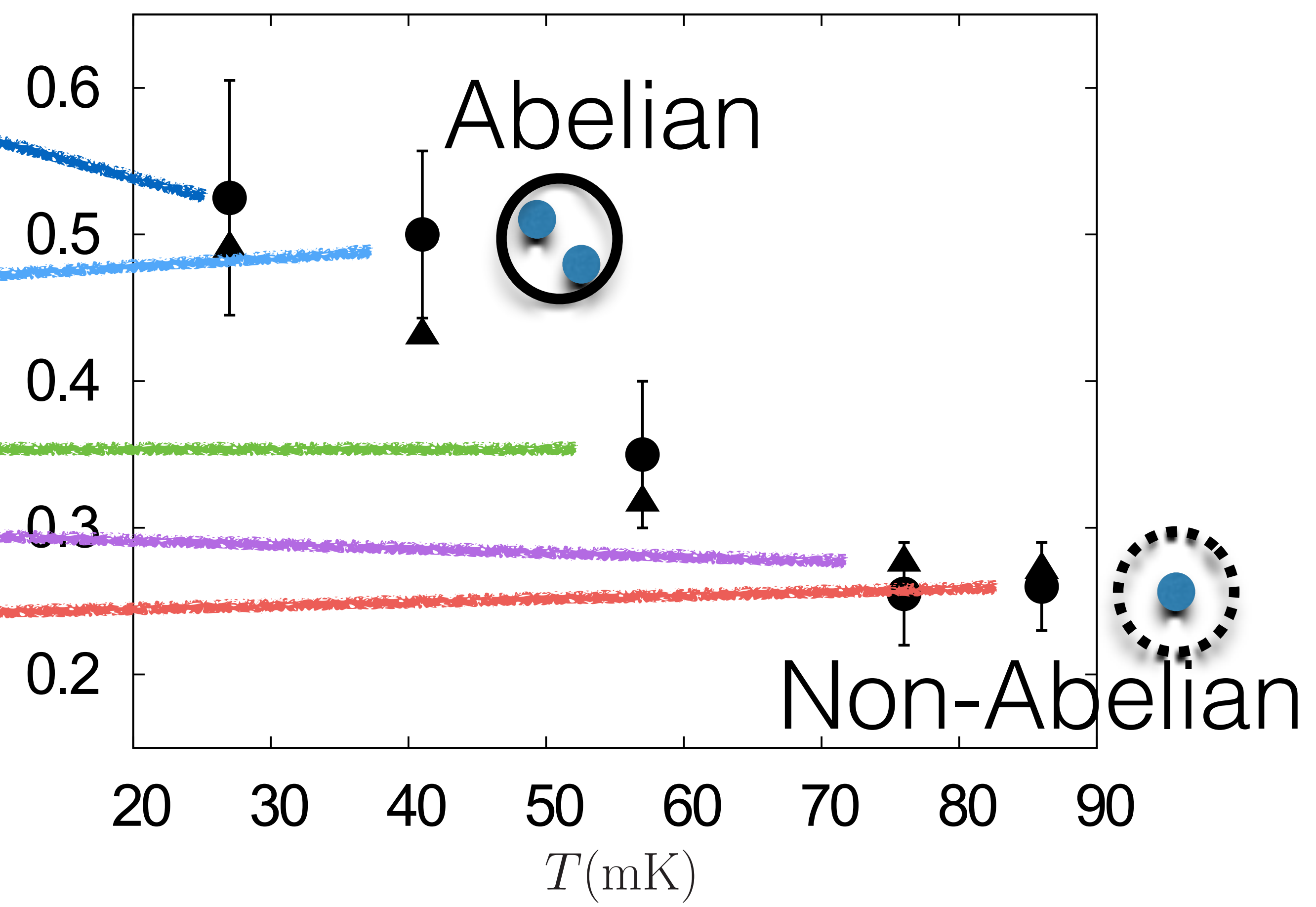


Valid also for $\nu = 5/2$

Non-Abelian theory

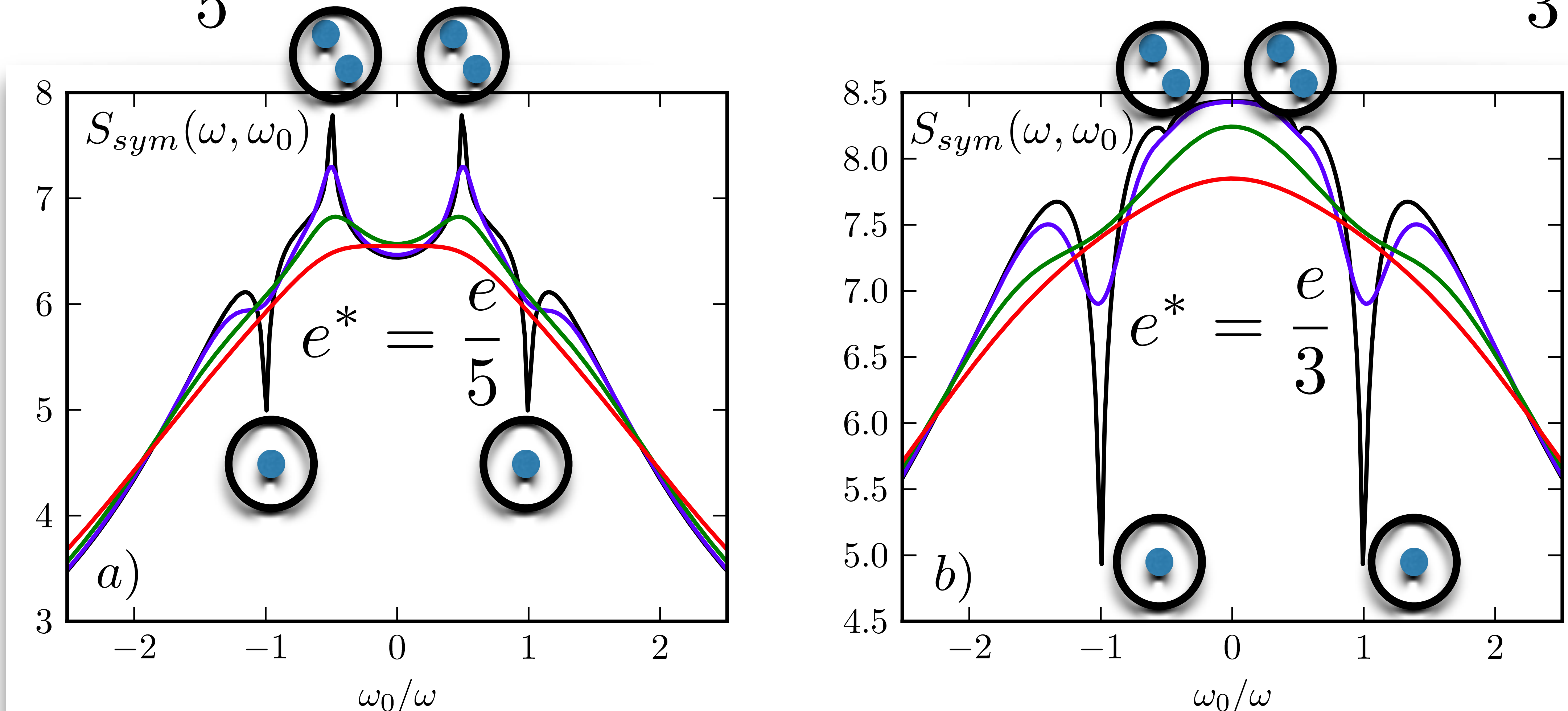
$$\Psi_\sigma^{(1)} + \Psi_\sigma^{(1)} \neq \Psi_I^{(2)}$$

M. Carrega, D. Ferraro, A. B., N. Magnoli, M. Sassetti, PRL 107, 146404 (2011)



Resolving m-qp scalings? S_{sym}

$\nu = \frac{2}{5}$ $\quad T = 0.1, 5, 15, 30 [\mathrm{mK}]$ $\quad \nu = \frac{2}{3}$



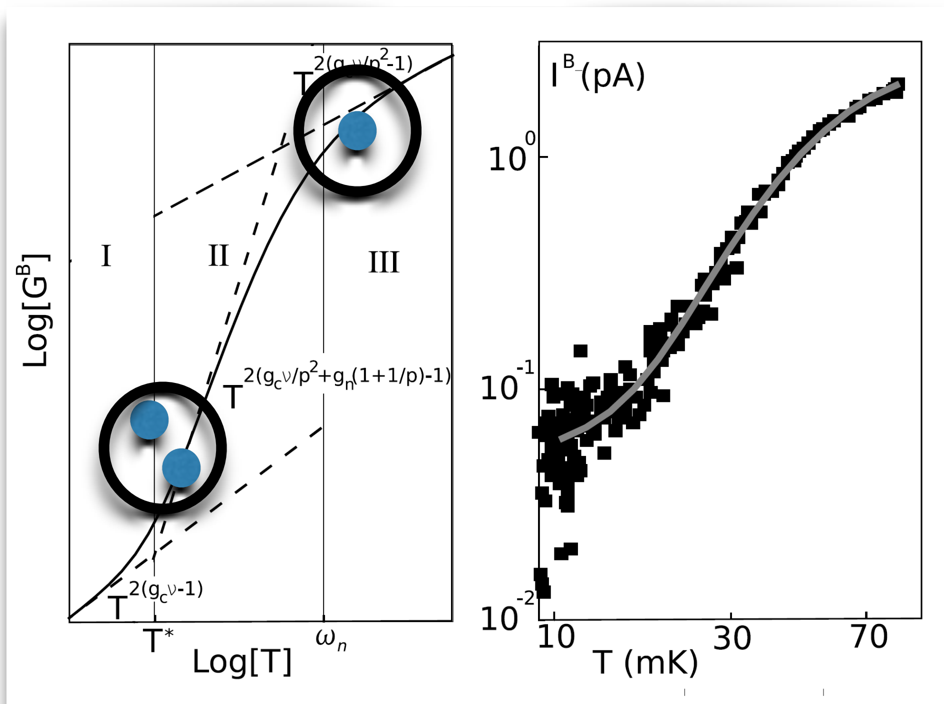
$$S_{sym}^{(m)}(\omega,\omega_0) \approx |\omega - \omega_0|^{4\Delta_\nu^{(m)}-1}$$

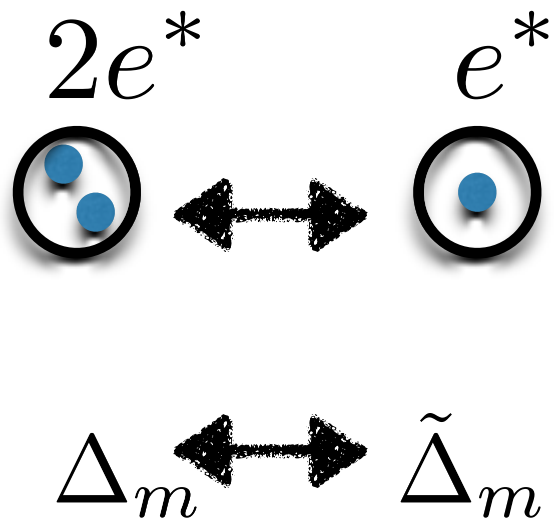
Chamon, Freed & Wen PRB95,PRB96

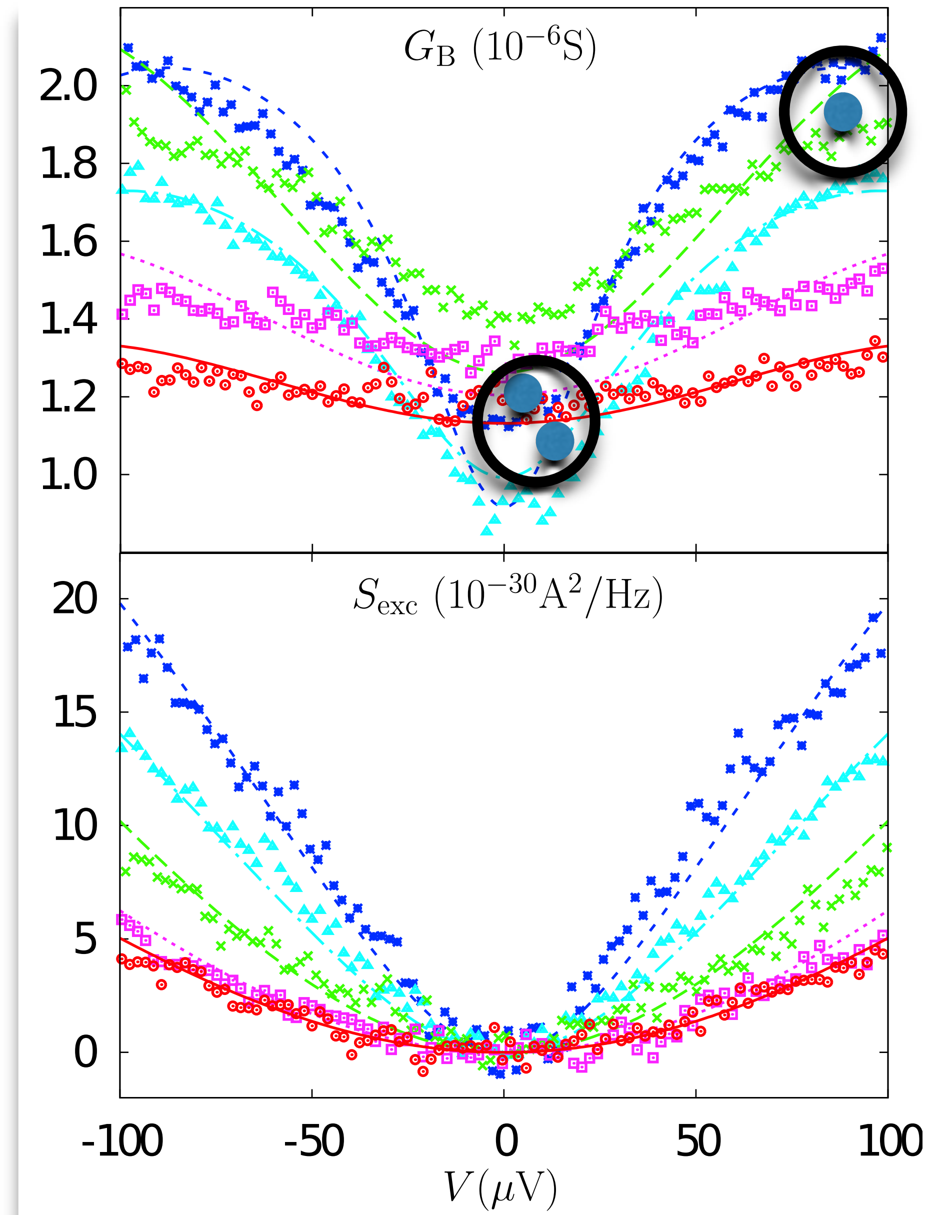
- $\omega \approx m\omega_0$ Josephson resonances
- Peaks ($\Delta_\nu^{(m)} < 1/4$) or dips ($\Delta_\nu^{(m)} > 1/4$)
- Thermal effect spoil the signatures

Is enough?

- Single-qp and multiple-qp crossover

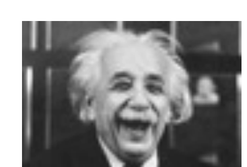






D. Ferraro, A. B, M. Merlo, N. Magnoli, M. Sassetti PRL 08
D. Ferraro, A. B., N. Magnoli, M. Sassetti, NJP10
D. Ferraro, A. B., N. Magnoli, M. Sassetti, PRB10
M. Carrega, D. Ferraro, A. B., N. Magnoli, M. Sassetti PRL11
M. Carrega, D. Ferraro, A. B., N. Magnoli, M. Sassetti, NJP12
A. B., D. Ferraro, M. Carrega, N. Magnoli, M. Sassetti NJP12

Good agreement with many observations,
simple & coherent explanations



Theorist

Exp
?

Some properties of QPC-LC

- Detail balance $S_+^{(m)}(\omega) = e^{-\hbar\omega/k_B T} S_+^{(m)}(-\omega)$
- QPC absorption $S_+^{(m)}(\omega) - S_-^{(m)}(\omega) = -\omega\, \Re e\left[G_{ac}^{(m)}(\omega)\right]$
- Diff, conductance $G_{ac}^{(m)}(\omega) = \int dt \frac{e^{i\omega t} - 1}{\omega} \langle [\delta I_B^{(m)}(t), \delta I_B^{(m)}(0)] \rangle$
- Detector quantum limit $k_B T_c \ll \omega$
  $$S_{meas}^{(m)}(\omega) \approx K S_+^{(m)}(\omega) + \mathcal{O}(e^{-\hbar\omega/k_B T_c})$$
- Absorbitive QPC limit (Emissive detector) $k_B T_c \gg \omega$
  $$S_{meas}^{(m)}(\omega) \approx K \left\{ S_+^{(m)}(\omega) - k_B T_c \Re e\left[G_{ac}^{(m)}(\omega)\right] \right\}$$
- Is it measurable?
  $$S_{ex}(\omega, \omega_0) = S_{meas}(\omega, \omega_0) - S_{meas}(\omega, \omega_0 = 0)$$
  $\omega_0 = e^* V/\hbar$
- Lowest order in the tunnelling $|t_m|^2$ (purely additive)

$$S_{sym}(\omega) = \sum_m S_{sym}^{(m)}(\omega) \qquad S_{meas}(\omega) = \sum_m S_{meas}^{(m)}(\omega)$$

A Fermi-golden rule result

- Standard Keldysh formalism blow up in Fermi's rule
- Tunnelling rate $\mathbf{\Gamma}^{(m)}(E) = |t_m|^2 \int_{-\infty}^{+\infty} d\tau \; e^{iEt} \mathcal{G}^{<}_{m,-}(-t) \mathcal{G}^{>}_{m,+}(t)$
- Green functions $\mathcal{G}^{>}_{m,\pm}(t) = \langle \Psi^{(m)}_{\nu,\pm}(t) \Psi^{(m)\dagger}_{\nu,\pm}(0) \rangle$
- Non-sym Noise

$$S^{(m)}_{+}(\omega, \omega_0) = \frac{(me^*)^2}{2} \left[\mathbf{\Gamma}^{(m)} \left(-\omega + m\omega_0 \right) + \mathbf{\Gamma}^{(m)} \left(-\omega - m\omega_0 \right) \right]$$

- Symmetrized noise

$$S^{(m)}_{sym}(\omega, \omega_0) = \frac{(me^*)^2}{2} \sum_{j,k=\pm} \mathbf{\Gamma}^{(m)} \left(j\omega + km\omega_0 \right)$$

- Differential conductance (diss.)

$$\Re e \left[G^{(m)}_{ac}(\omega) \right] = \frac{(me^*)^2}{2\omega} \sum_{j,k=\pm} j \mathbf{\Gamma}^{(m)} \left(j\omega + km\omega_0 \right)$$