Elementary events and probabilities in time-dependent quantum transport

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Collaboration with
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UPoN, Barcelona (2015)

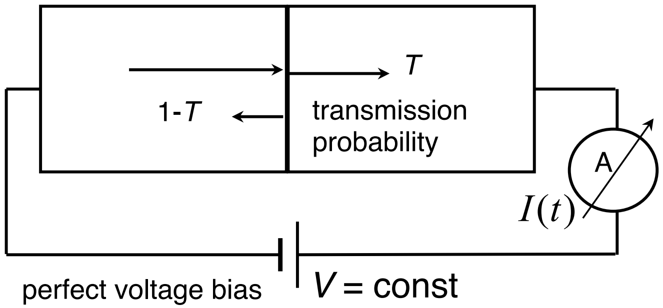
Content

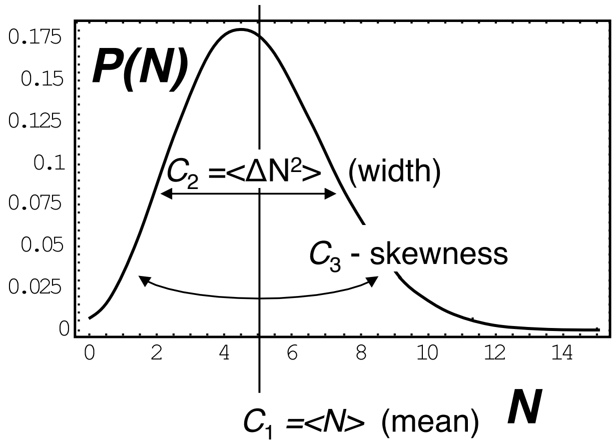
- Current noise, photon-assisted transport and Levitons
- Full counting statistics analysis and elementary events
- Noise minimization by pulse shaping
- Non-equilibrium wave function

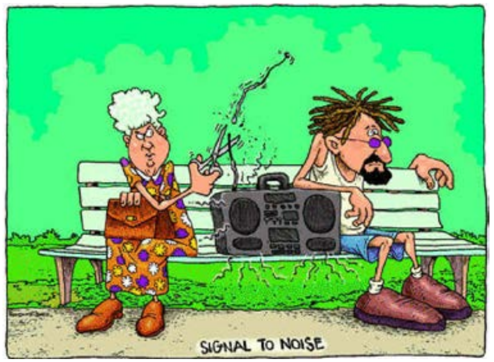
Photon-assisted noise and Levitons

Quantum shot noise

The Noise *is* the Signal !
(R. Landauer)







$$S^{(2)}(\omega) = \left\langle \left\{ \Delta I(t), \Delta I \right\} \right\rangle_{\omega}$$

$$F = \frac{S^{(2)}(\omega = 0)}{2eI}$$

$$S^{(3)}(\omega,\omega') = \left\langle \Delta I(t')\Delta I(t)\Delta I \right\rangle_{\omega,\omega'}$$

Cumulant generating function (CGF):

$$S(\chi) = \ln\left\langle e^{i\chi N} \right\rangle = i\chi \bar{N} - \chi^2 \left\langle \Delta N^2 \right\rangle / 2 + \ldots$$

		For a simple QPC
Average current and conductance	$I = \frac{e\bar{N}}{t_0} = V\frac{2e^2}{h}\sum_n T_n$	
Current noise power and the Fano factor	$S_2(0) = \frac{2e^2}{t_0}\overline{(N-\bar{N})^2} = 2eIF$	$F = \frac{T(1-T)}{T}$
The third correlator and the 'skewness' C	$S_3(0,0) = \frac{e^3}{t_0}\overline{(N-\bar{N})^3} = e^2 IC$	$C = \frac{T(1-T)(1-2T)}{T}$

Conductance = transmision

Shot noise suppression (for Fermions)

Sign change at T=1/2 (for Fermions)

Blanter , Büttiker (2000)

Photon-assisted noise

VOLUME 72, NUMBER 4 PHYSICAL REVIEW LETTERS 24 JANUARY 1994

Noise in an ac Biased Junction: Nonstationary Aharonov-Bohm Effect

G. B. Lesovik[1,*] and L. S. Levitov[2,†]

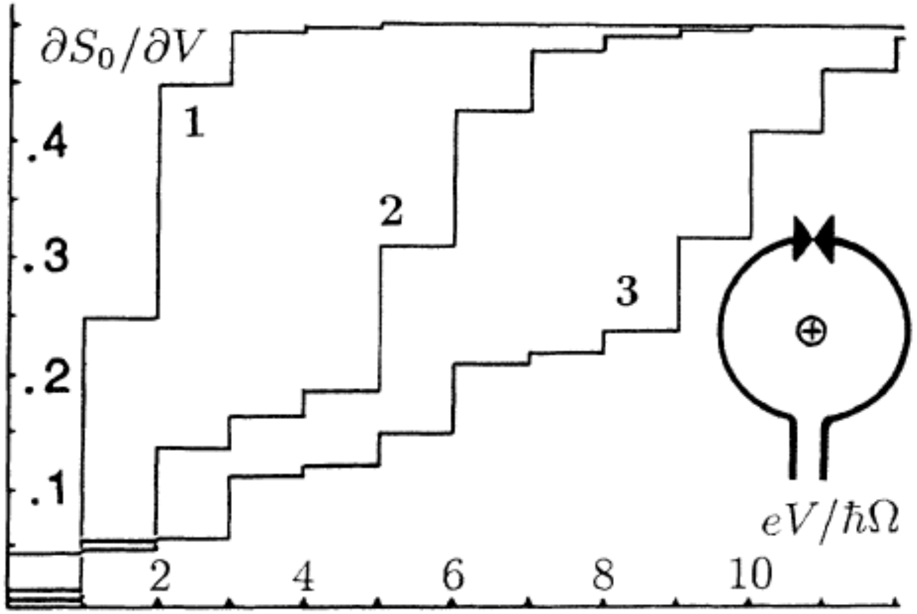


FIG. 1. Differential noise $\partial S_0/\partial V$ at $T = 0$ given by Eqs. (4) and (6) is plotted against V for three flux amplitudes: (1) $\Phi_a = 5\Phi_0/4\pi$; (2) $\Phi_a = 7\Phi_0/2\pi$; (3) $\Phi_a = 23\Phi_0/4\pi$. Inset: Junction with leads bent in a loop through which alternating magnetic flux is applied.

Harmonic drive:

$V(t)=V_{dc}+V_{ac}\cos(\Omega t)$

$\Phi_a=\Phi_0 eV_{ac}/\Omega$

$$\frac{\partial S}{\partial V_{dc}} = eGF \times \sum_n J_n^2\left(\frac{eV_{ac}}{\hbar\Omega}\right)\theta\left(eV_{dc} - n\hbar\Omega\right)$$

Steps at $eV_{dc}=n\hbar\Omega$ in the noise at quantized

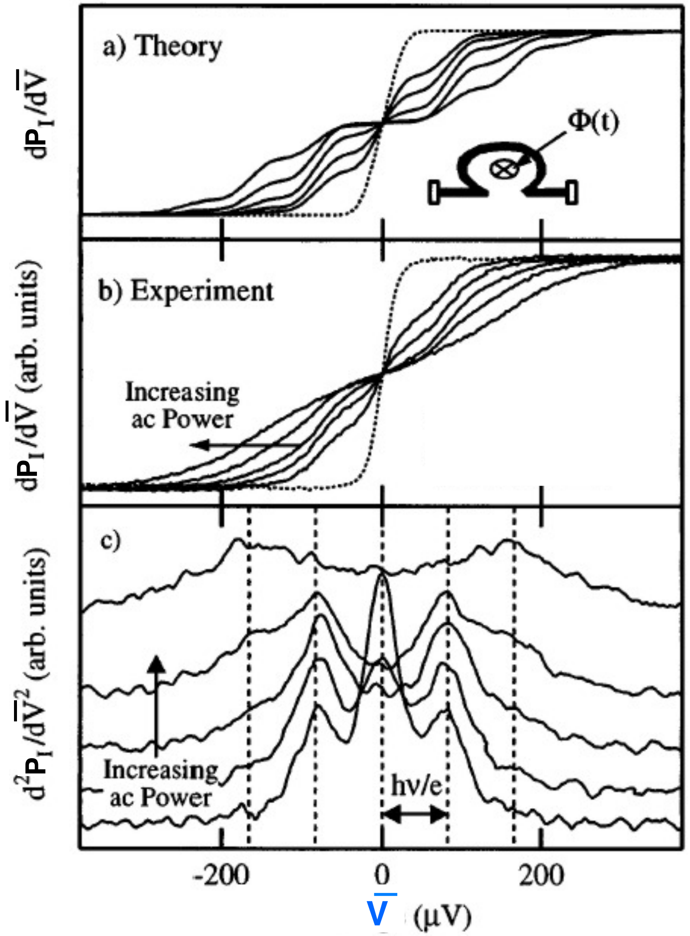
Transport in the presence of harmonic driving V(t): experiments

$$V(t) = \bar{V} + V_0 \cos(\omega t)$$

$$S_2 = \frac{2e^2}{\pi}\left[T^2 2T_e + T(1-T)\sum_n J_n^2\left(\frac{eV_0}{\hbar\omega}\right)(e\bar{V}+n\omega)\coth\left(\frac{e\bar{V}+n\omega}{2T_e}\right)\right]$$

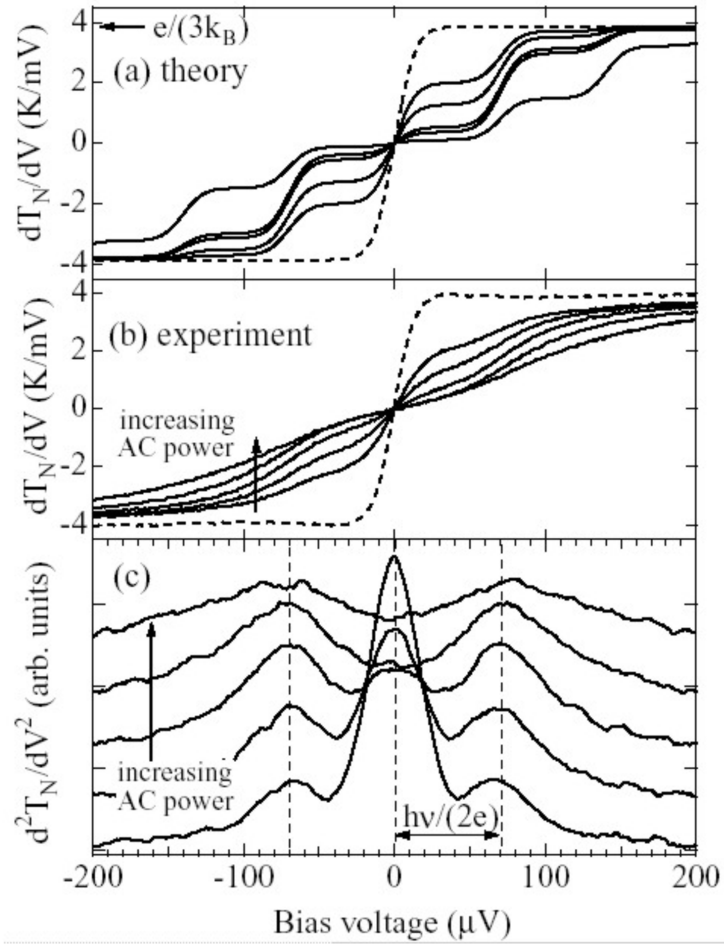
Normal coherent conductors

Schoelkopf et al. PRL 98; Reydellet et al. PRL 03



Diffusive S / N junction (e* = 2e)

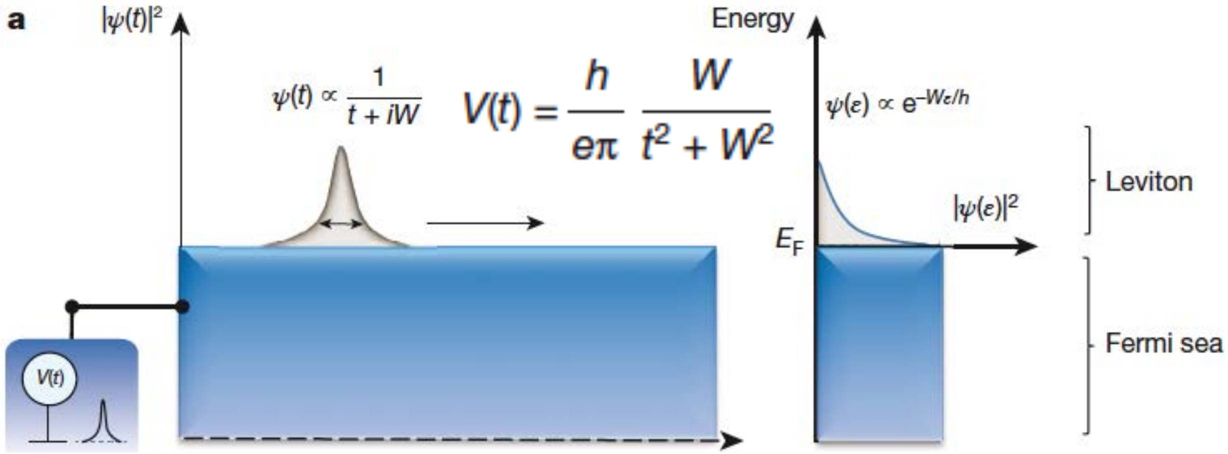
Kozhevnikov et al PRL 00

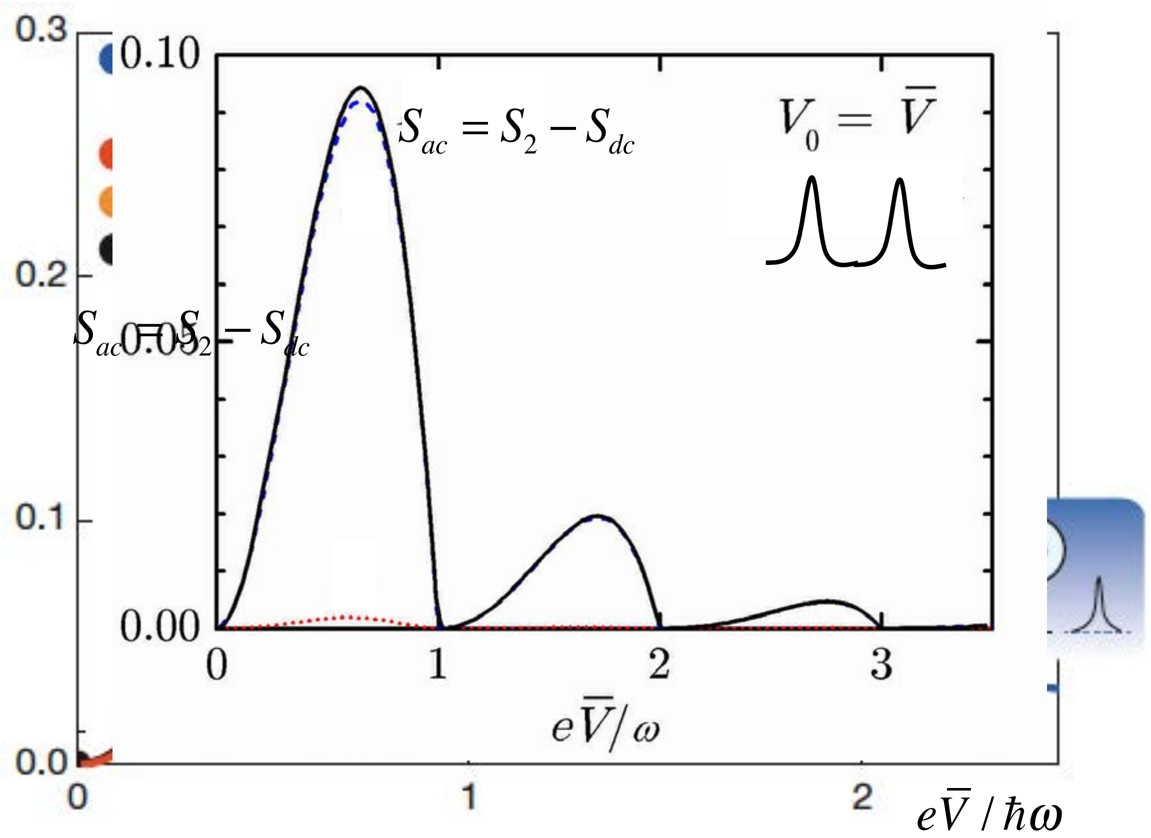


Levitons

Quantization of Lorentzian voltage pulses

$$e\int dt V(t) = 2\pi n\hbar$$





- Quantized Lorentzian pulses emit exactly one electron (at exactly $k_BT=0$)
- Noise should be minimized for integer pulses
- In general (non-integer, non-Lorentzian, finite temperature) additional electron-hole pairs are created and enhance the noise

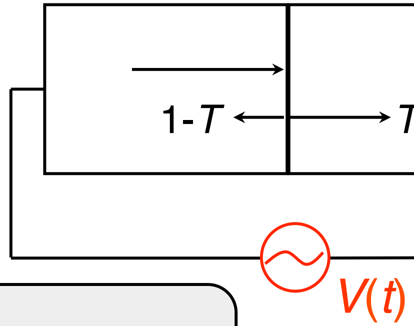
Theory: Levitov et al. 1995-
Experiment: Dubois et al., Nature 2013

Elementary events analysis of mesoscopic transport

(U)PoN: What are the elementary events and what is their statistics in a general time-dependent transport problem?

FCS for arbitrary (periodic) driving V(t)

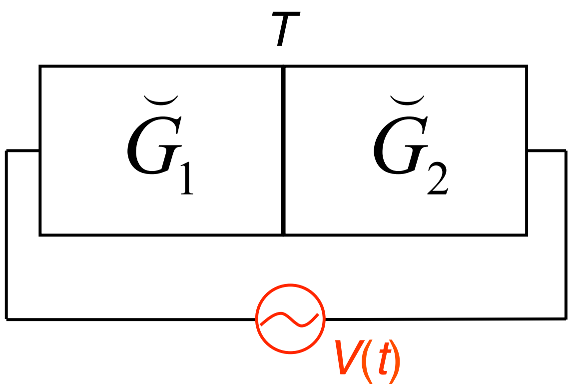
M. Vanevic, Yu. V. Nazarov, W. Belzig
Phys. Rev. Lett. **99**, 076601 (2007)



Q: $S(\chi) = ?$
and what are the (independent) elementary processes?

Previous results:

Scattering approach can give S_2, S_3, … but does not provide an expression for the CGF.
The approach for calculating S_2 for Levitons essentially depends on analytical properties of Lorentzian pulses and can not be simply generalized to arbitrary V(t)
→ we use the **extended Keldysh-Green's function formalism + circuit-theory**
Nazarov, Ann. Phys. (Leipzig) 1999; Belzig, Nazarov PRL 2001



$$S(\chi) = Tr\ Ln\left[\breve{1} + \frac{T}{2}\left(\frac{\{\breve{G}_1, \breve{G}_2\}}{2} - \breve{1}\right)\right]$$

$$\breve{G}_1 = e^{-i\chi\breve{\tau}_1/2}\begin{pmatrix} 1 & 2UhU^\dagger \\ 0 & -1 \end{pmatrix} e^{i\chi\breve{\tau}_1/2} \qquad \breve{G}_2 = \begin{pmatrix} 1 & 2h \\ 0 & -1 \end{pmatrix}$$

$$h(E) = \tanh(E/2T_e) \qquad U(t) = e^{i\varphi(t)} = e^{-ie\int_0^t dt' V(t')}$$

Trace is in Keldysh x „time“ indices

$$S(\chi) = Tr\,\ln\left[\breve{1} + \frac{T}{2}\left(\frac{\{\breve{G}_1, \breve{G}_2\}}{2} - \breve{1}\right)\right] = Tr\ln Z(t,t')$$

For periodic functions:

$$\breve{G}(E,E') = \int dt\,dt' e^{iEt - E't'}\breve{G}(t,t') = \sum_n \breve{G}_{n0}(E)\delta(E - E' - n\hbar\omega)$$

Convolutions become usual matrix products:

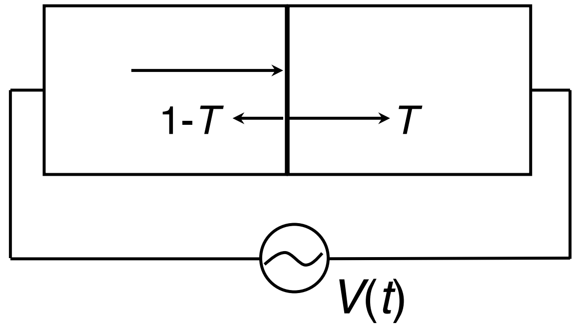
$$Tr\ln Z(t,t') = \frac{t_0}{h}\int_0^{\hbar\omega} dE\,tr\ln Z$$

Special property at zero temperature $h(E) = \tanh(E/2T_e) = \pm 1$

-> Z(E) is piecewise constant and energy integration can be done analytically

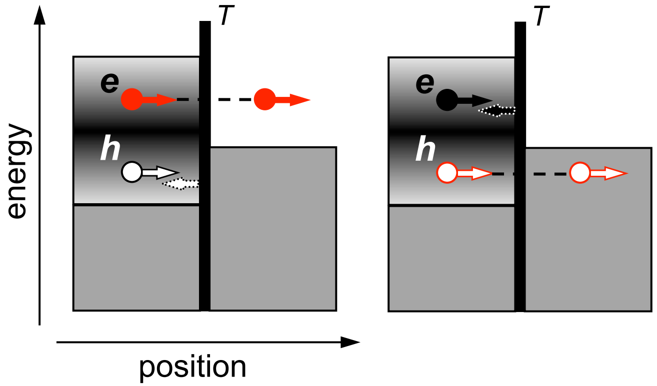
FCS for arbitrary periodic driving V(t)

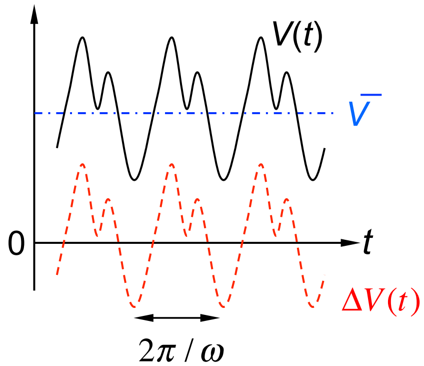
Obtained from the diagonalization of Z_{nm}



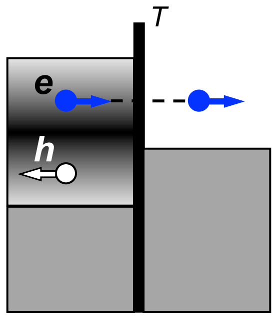
$$S(\chi)=\frac{t_0}{h}\sum_i \Delta E_i tr \ln Z_i = S_{1p}(\chi)+S_{eh}(\chi)$$

2-particle electron-hole processes





1-particle processes



$$S_{eh}(\chi)=M\sum_k \ln\left[1+TR\,p_k\left(e^{i\chi}+e^{-i\chi}-2\right)\right]$$

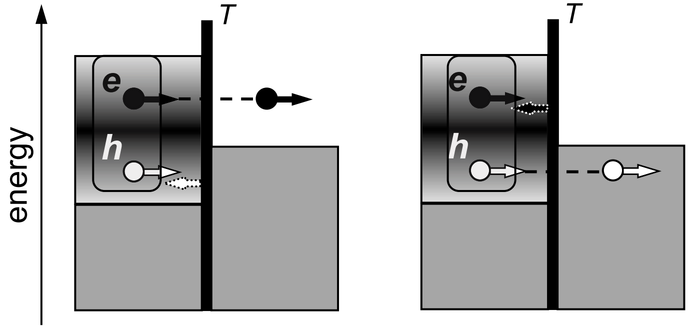
$$S_{1p}(\chi)=\frac{2t_0 e\bar{V}}{h}\ln\left[1+T\left(e^{-i\chi}-1\right)\right]$$

contributes only to the noise and even-order cumulants

depend of V_{dc} and pulse shape

Gives the usual binomial statistics

Examples: harmonic drive



$$S_{eh} \propto \sum_k \ln\left[1 + TR\, \boxed{p_k} \left(e^{i\chi} + e^{-i\chi} - 2\right)\right]$$

$$V(t) = V_0 \cos(\omega t)$$

Scattering approach:

$$S_2 \propto \sum_n |n| J_n^2\left(\frac{eV_0}{\hbar\omega}\right)$$

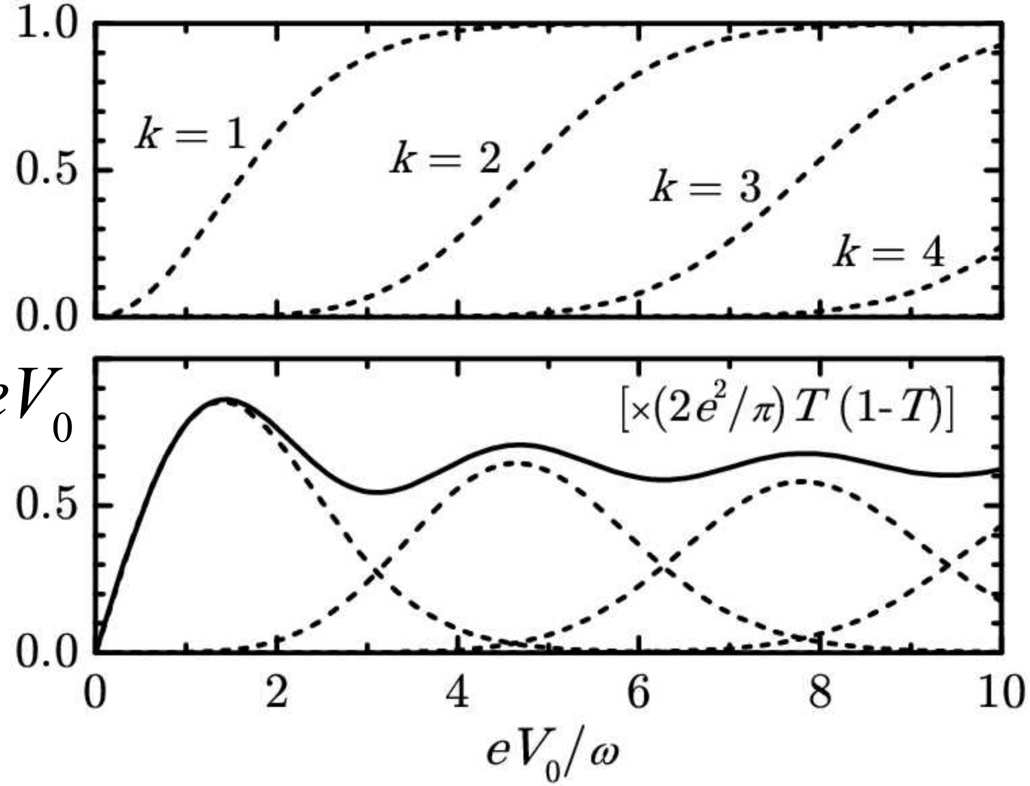
This is not a decomposition into elementary events.

The proper decomposition is obtained from the CGF:

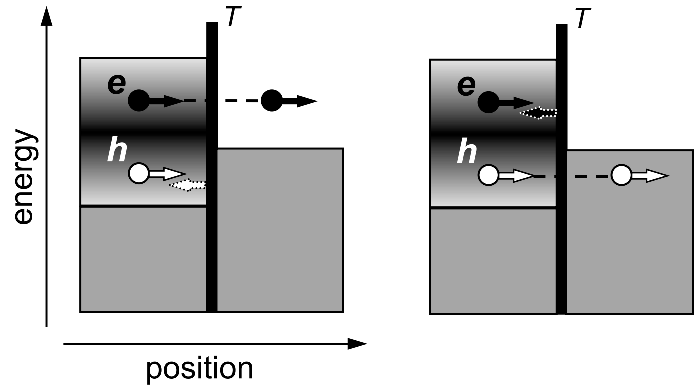
$$S_2 \propto \sum_k p_k$$

$\boxed{p_k}$

$\partial S_2 / \partial e V_0$



Examples: different pulse shapes

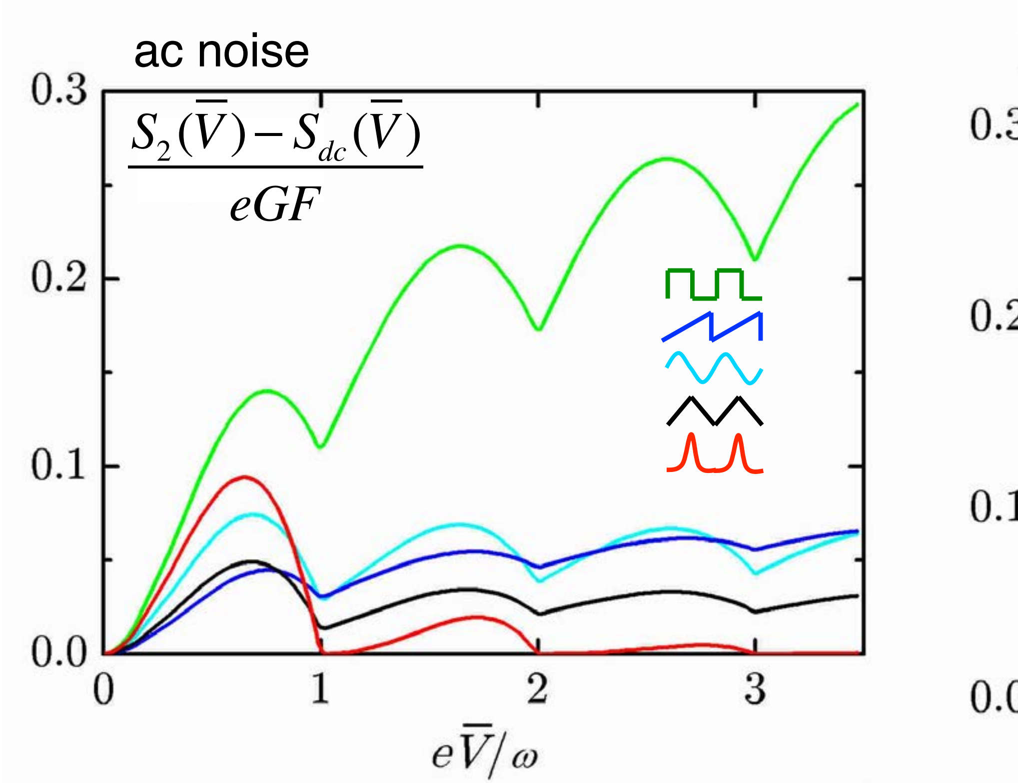


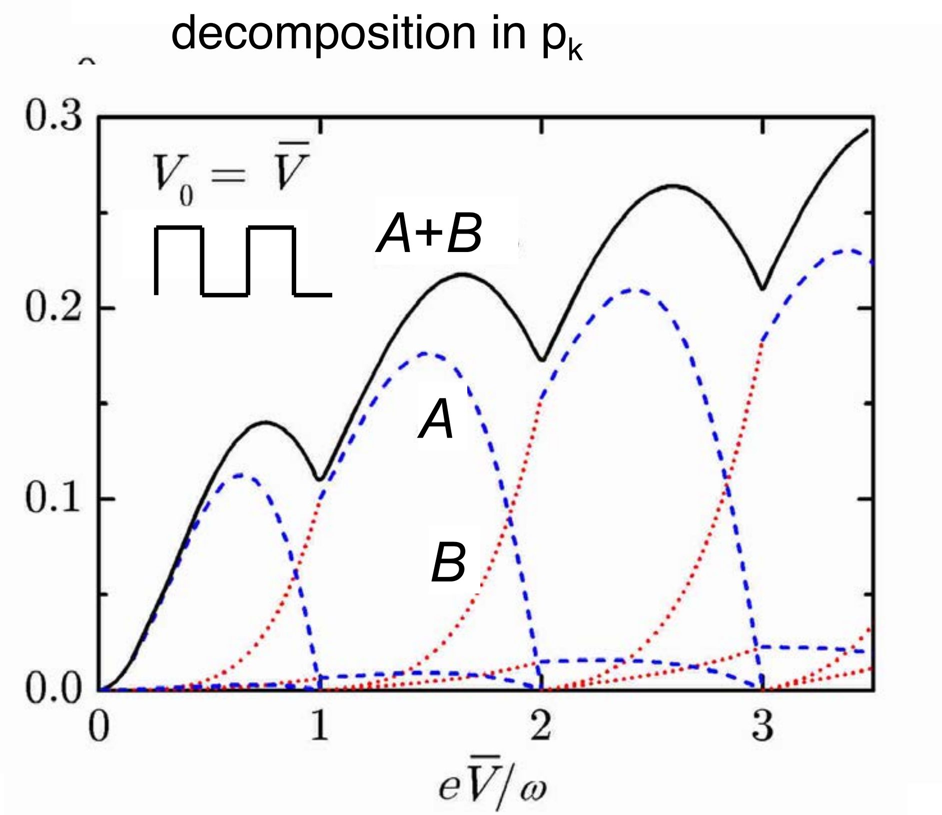
Excess noise contribution
of bidirectional 2-particle processes

$$S_{ac} = S_2(\bar{V}) - S_{dc}(\bar{V}) \sim \partial_\chi^2 S_{eh}(\chi)\Big|_{\chi=0}$$

$$S_{eh}(\chi) = M \sum_k \ln\left[1 + TR\, p_k \left(e^{i\chi} + e^{-i\chi} - 2\right)\right]$$

Unidirectional driving: $V(t) = \bar{V} v(\omega t)$ $\quad v(t) > 0$ and $\bar{v} = 1$





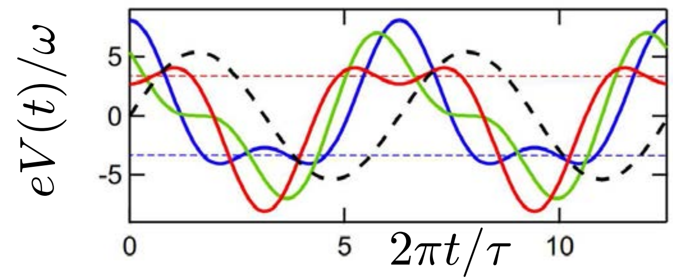
Full counting statistics analysis of zero frequency noise minimization

(U)PoN: What voltage drive will lead to minimal noise for given temperature, dc-voltage, frequency,….?

PHYSICAL REVIEW B **87**, 075403 (2013)

Shaping a time-dependent excitation to minimize the shot noise in a tunnel junction

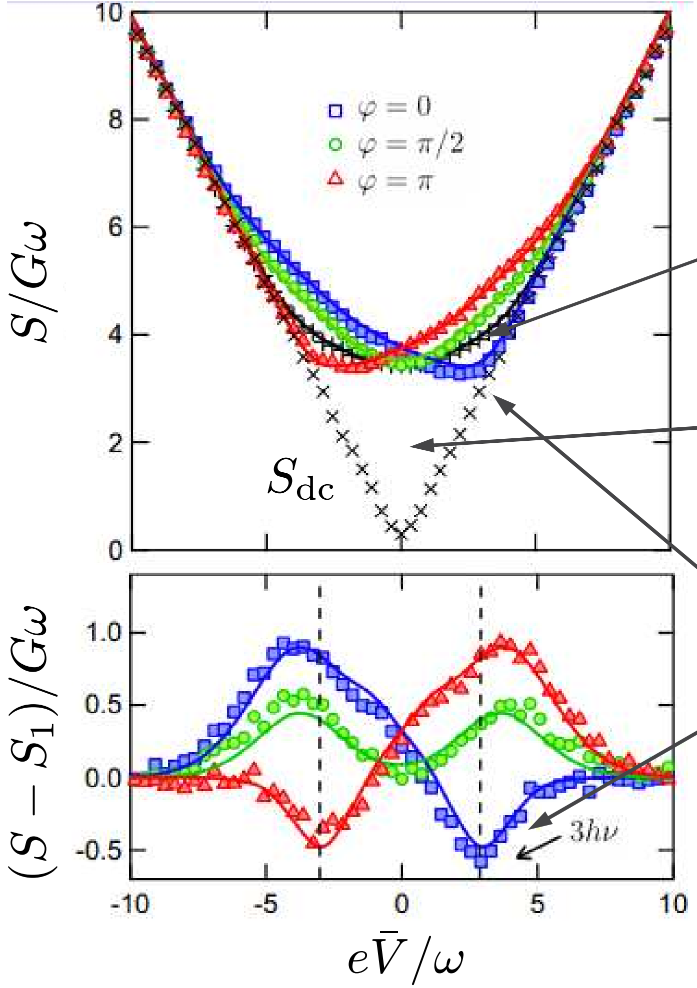
Julien Gabelli[1] and Bertrand Reulet[1,2]



$$V(t) = \bar{V} + V_1 \cos(\omega t) + V_2 \cos(2\omega t + \varphi)$$

$$eV_1 = 2eV_2 = 5.4\omega$$

$$T = 0.14\omega = 75\,\mathrm{mK}$$



S_1: noise in the presence of a single harmonic

1) $S_{ac} = S - S_{dc}$ excess photon-assisted noise

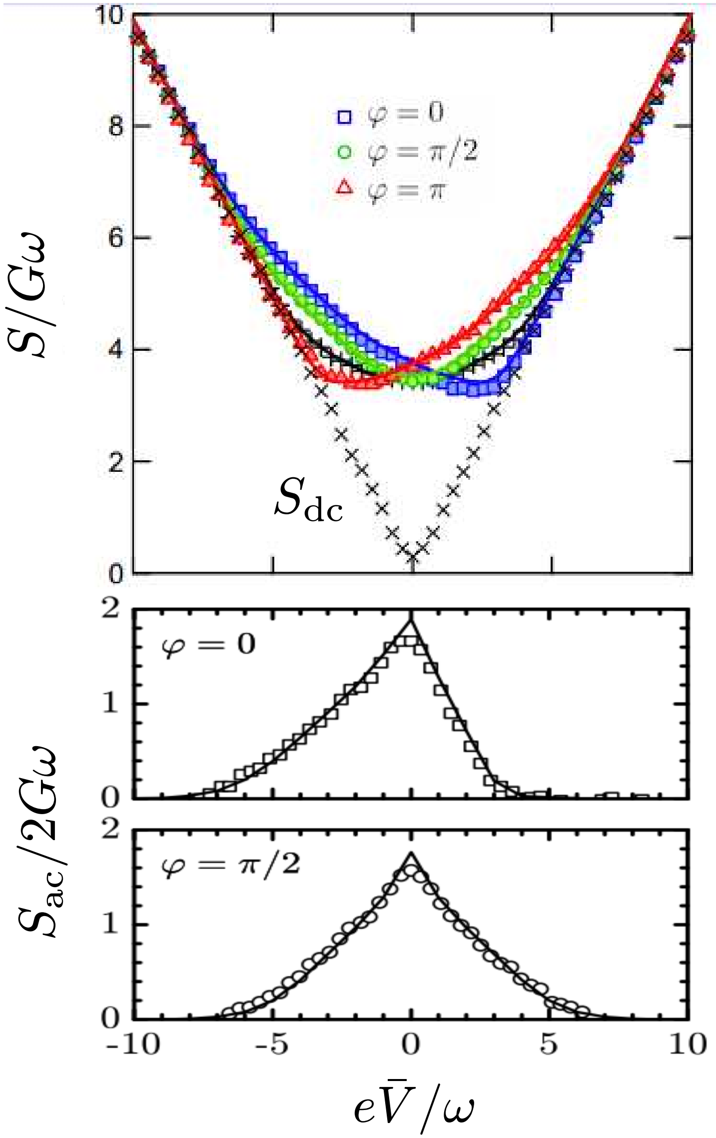
2) reduction of the noise in the presence of an in-phase second harmonic

$$S_{ac}(\bar{V}) = \sum |c_n|^2 \left| e\bar{V} - n\hbar\omega \right|$$

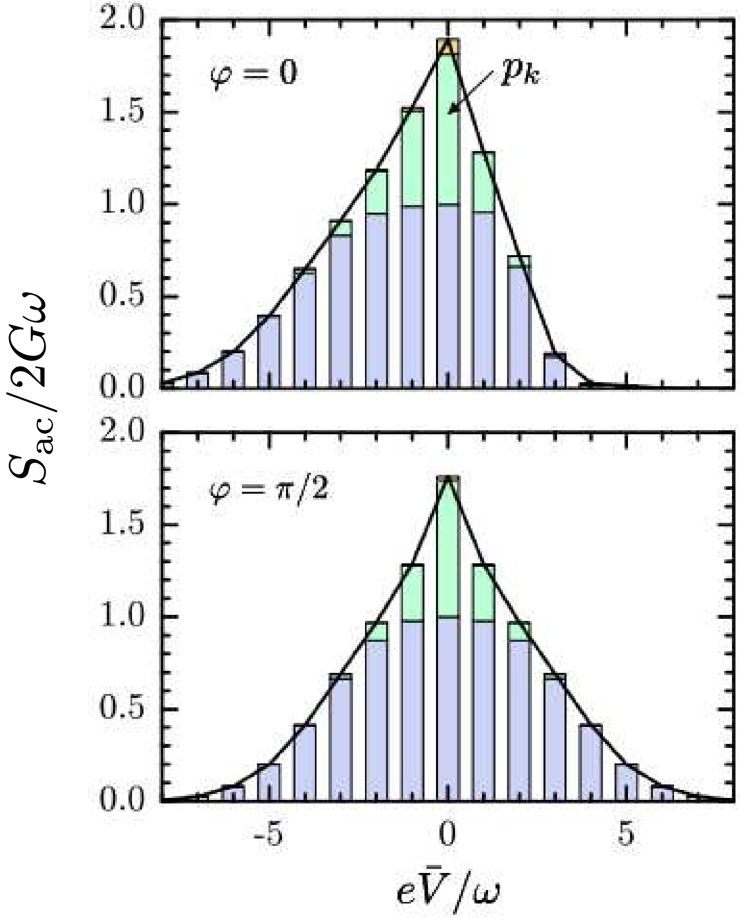
$$c_n = \sum_{m=-\infty}^{\infty} e^{-im\varphi} J_{n-2m}\left(\frac{eV_1}{\hbar\omega}\right) J_m\left(\frac{eV_2}{2\hbar\omega}\right)$$

Analysis in terms of the e-h pairs created

$$S_{\mathrm{ac}} = S - S_{\mathrm{dc}}$$



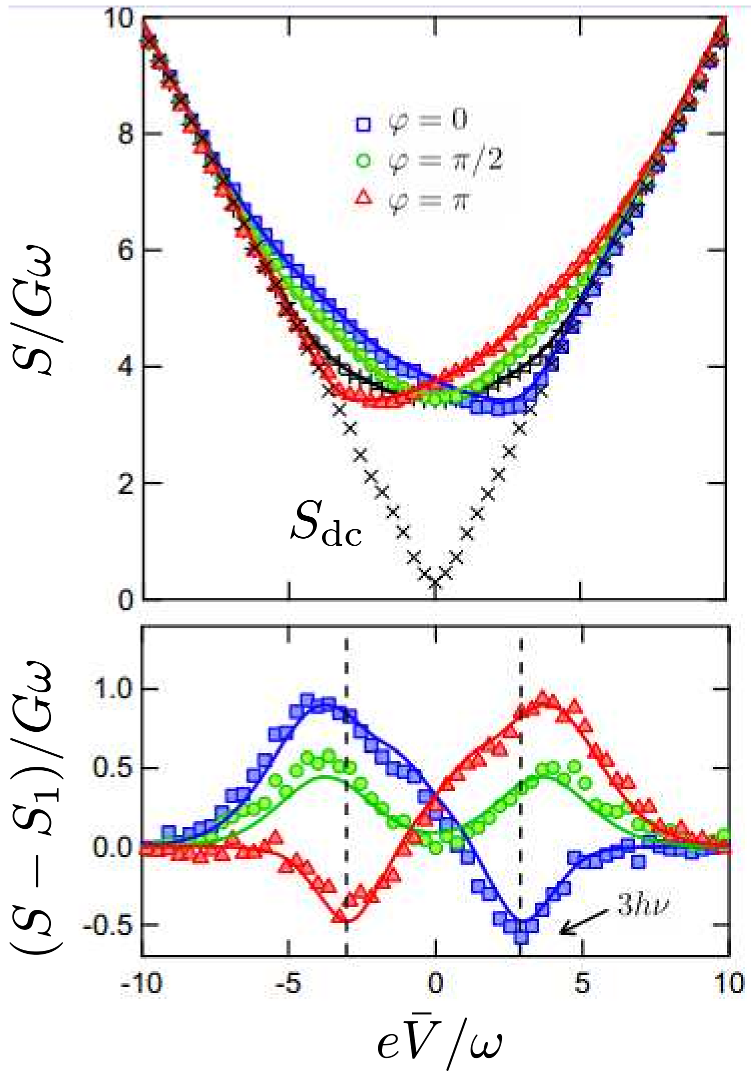
$$S_{\mathrm{ac}}\left(\frac{e\bar{V}}{\omega} = N\right) = 2GF\omega \sum_k p_k^{(N)}$$



[Gabelli & Reulet, PRB (2013)]

[Vanevic & Belzig, PRB (2012)]

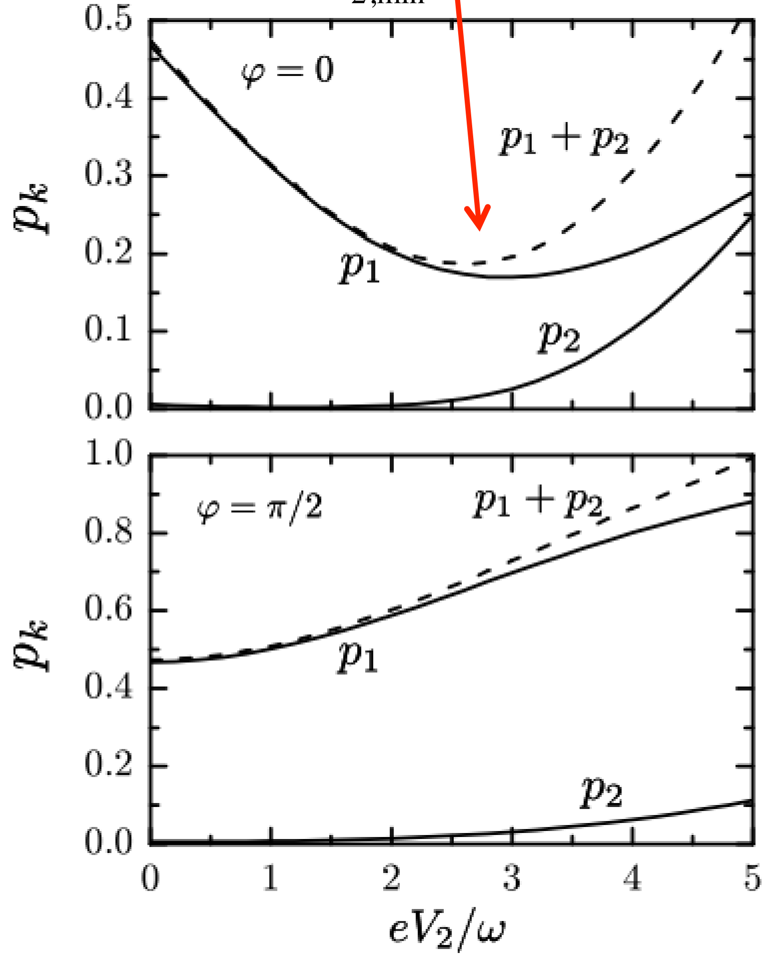
Noise reduction by an in-phase second harmonic



$e\bar{V}/\omega = 3 \qquad eV_1/\omega = 5.4$

e-h pair creation is minimized

$eV_{2,\mathrm{min}} \approx 2.7\omega$

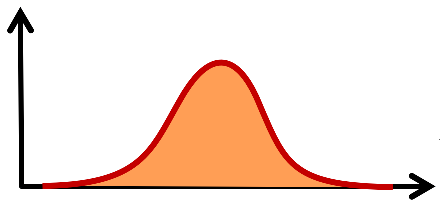


[Gabelli & Reulet, PRB (2013)]

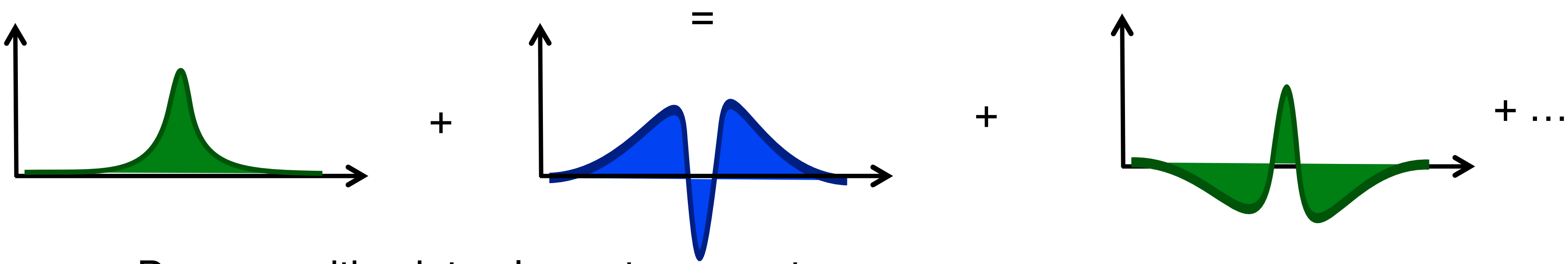
[Vanevic & Belzig, PRB (2012)]

Non-equilibrium many-body wave function probed by noise

Voltage pulse V(t)



Average current <I(t)>=GV(t)



Decomposition into elementary events

UPoN: What is the wave function created by an arbitrary voltage pulse?

M. Vanevic, J. Gabelli, W. Belzig, and B. Reulet, arxiv:1506.03878

Decomposition of FCS in the time domain

The CGF (Belzig, Nazarov 2001) for contacts described by scattering matrix with (energy-independent) transmission probabilities T

$$S(\chi) = Tr\ Ln\left[\breve{1} + \frac{T}{2}\left(\frac{\{\breve{G}_1, \breve{G}_2\}}{2} - \breve{1}\right)\right]$$

$$\breve{G}_1 = e^{-i\chi\breve{\tau}_1/2}\begin{pmatrix} 1 & 2h_1 \\ 0 & -1 \end{pmatrix} e^{i\chi\breve{\tau}_1/2} \qquad \breve{G}_2 = \begin{pmatrix} 1 & 2h_2 \\ 0 & -1 \end{pmatrix} \qquad h_2(E) = 1 - 2f(E)$$

$$h_1 = U h_2 U^{\dagger} \qquad U(t', t'') = \exp[-i\int_0^{t'} dt\ eV(t)]\,\delta(t' - t'')$$

$$S(\chi) = \mathrm{Tr}\ln\left[\breve{1} + T\breve{f}_1(1 - \breve{f}_2)\left(e^{i\chi} - 1\right) + T\breve{f}_2(1 - \breve{f}_1)\left(e^{-i\chi} - 1\right)\right]$$

Eigenvalues and Eigenvectors of the operators: $\breve{f}_1\breve{f}_2$ or $\breve{h}_1\breve{h}_2$

At k_BT=0: $\breve{h}_1^{\,2} = \breve{h}_2^{\,2} = \breve{1}$ Eigenvalues: $1, e^{\pm i\alpha}$ $p_k = \sin^2(\alpha_k/2)$

single electrons electron-hole pairs

$$S(\chi) = e\bar{V}t_0 \ln\left[1 + T(e^{i\chi} - 1)\right] + \omega t_0 \sum_k \ln\left[1 + TRp_k(e^{i\chi} + e^{-i\chi} - 2)\right]$$

Decomposition of FCS in the time domain

$$S(\chi) = e\bar{V}t_0 \ln\left[1 + T(e^{i\chi} - 1)\right] + \omega t_0 \sum_k \ln\left[1 + TRp_k(e^{i\chi} + e^{-i\chi} - 2)\right]$$

Eigenvalues and Eigenvectors

$$h_1 h_2 |v\rangle = h_2 h_1 |v\rangle = |v\rangle \qquad h_1 h_2 |v_\alpha\rangle = e^{i\alpha} |v_\alpha\rangle \qquad p_k = \sin^2\left(\frac{\alpha_k}{2}\right)$$

single electrons

$$|u_\pm\rangle = \frac{|v_\alpha\rangle \pm |v_{-\alpha}\rangle}{\sqrt{2}} = \begin{cases} \text{electron} \\ \text{hole} \end{cases}$$

Single particle density matrix $\quad f_1(E, E') = \langle \psi | c_E^\dagger c_{E'} | \psi \rangle$

$$\boxed{|\psi\rangle = \hat{C}^\dagger \prod_k \left(\sqrt{1 - p_k} + i\sqrt{p_k}\, \hat{A}_k^\dagger \hat{B}_k \right) |F\rangle}$$

fermi sea of left reservoir

$$\hat{C}^\dagger = \int dE v^*(E) c_E^\dagger \qquad \hat{A}^\dagger = \int_0^\infty dE u_+^{k*}(E) c_E^\dagger \quad \hat{B} = \int_{-\infty}^0 dE u_-^k(E) c_E$$

single electron

electron-hole pair

M. Vanevic, J. Gabelli, W. Belzig, and B. Reulet, arxiv:1506.03878

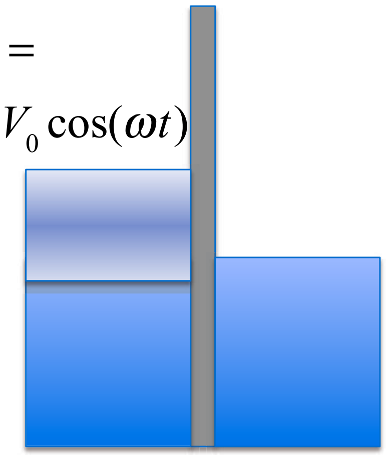
Results

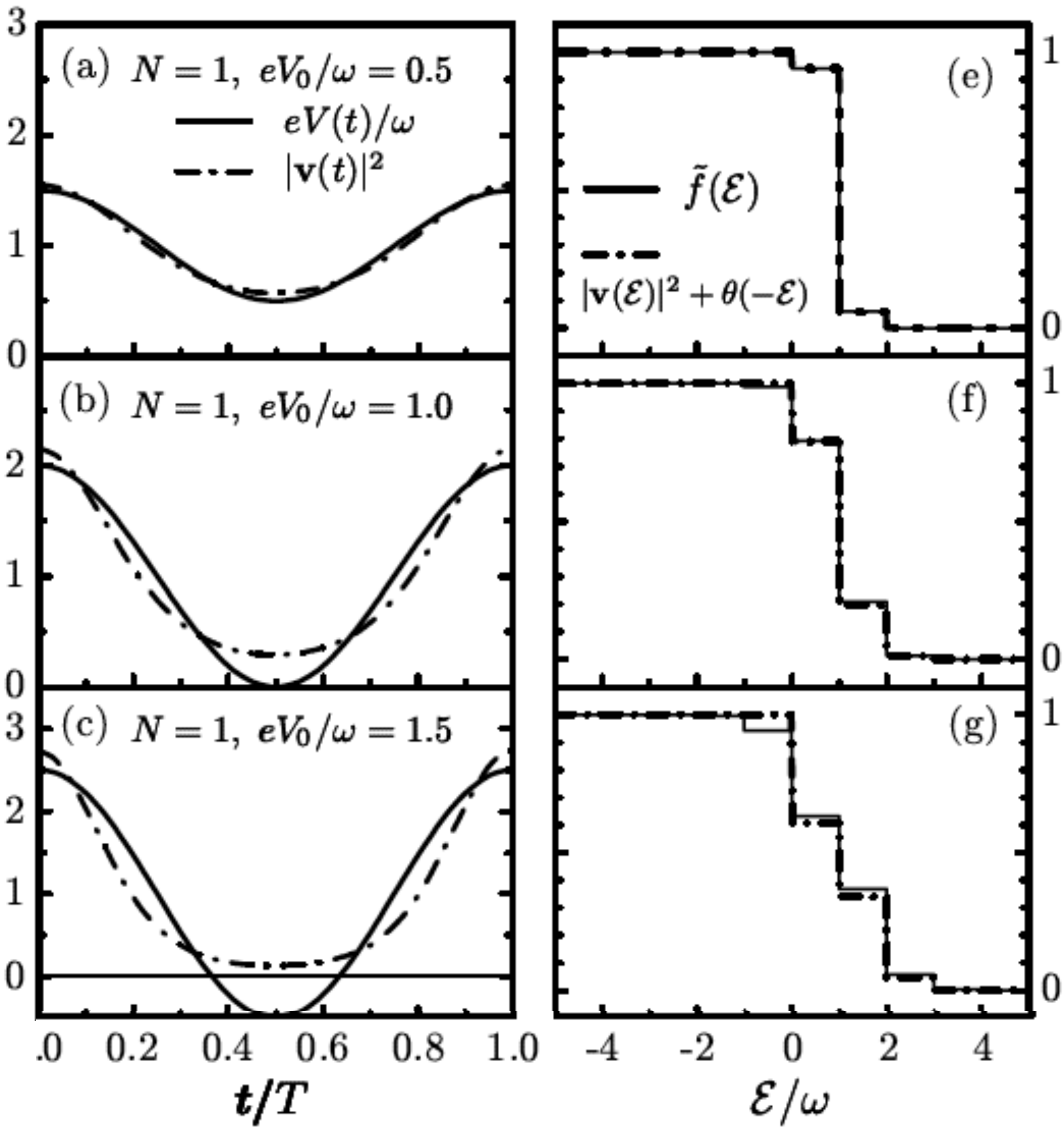
Comparison of
Voltage profile
(=current profile)
and the single electron
wave function

$$V(t) = V_{dc} + V_0 \cos \omega t$$

$$N = \frac{eV_{dc}}{\hbar\omega} = 1$$

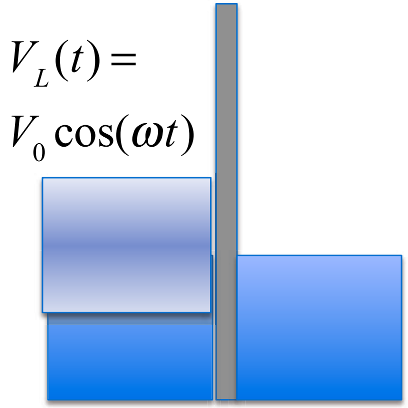
$V_L(t) =$
$V_{dc} + V_0 \cos(\omega t)$

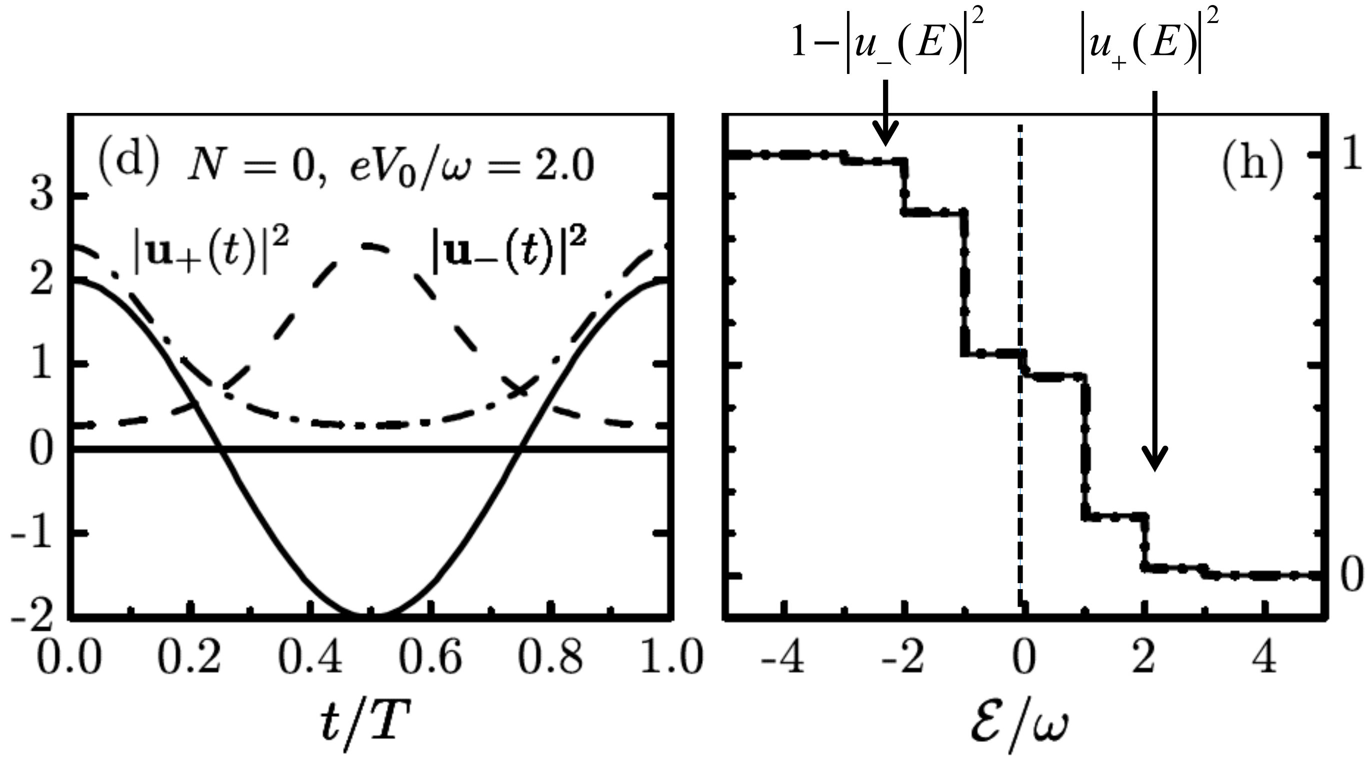




M. Vanevic, J. Gabelli, W. Belzig, and B. Reulet, arxiv:1506.03878

Results: electron-hole pair wave functions





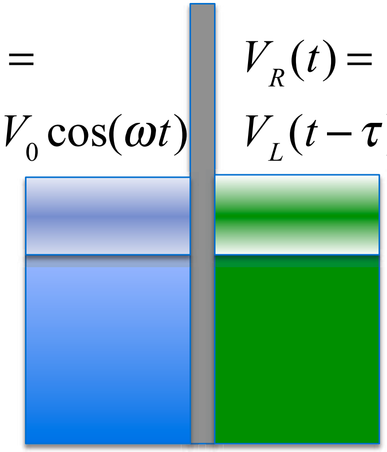
Electron- and hole-wave functions are identical, shifted by half a period and located at the respective maximum or minimum of the voltage

M. Vanevic, J. Gabelli, W. Belzig, and B. Reulet, arxiv:1506.03878

Test of the wave function: Hong-Ou-Mandel interferometry

$V_L(t) = V_{dc} + V_0 \cos(\omega t)$

$V_R(t) = V_L(t-\tau)$



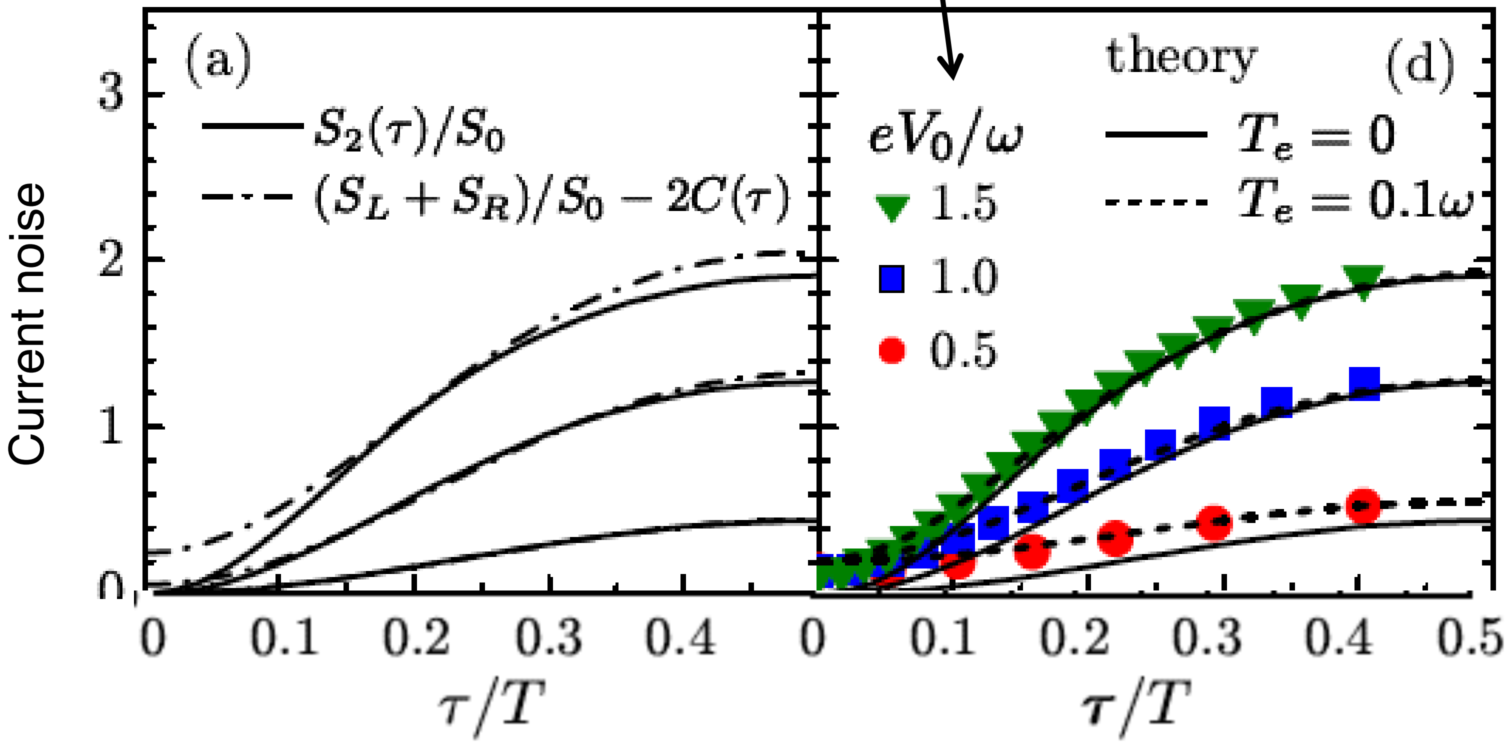
The $\boldsymbol{\tau}$-dependent noise probes the single-electron wave function

$$S_2(\tau) = \underbrace{S_L}_{\text{due to } V_L} + \underbrace{S_R}_{\text{due to } V_R} - 2S_0 C(\tau) \qquad S_0 = e^2 \omega G F$$

wave function overlap

$$C(\tau) = \left| \langle v(t) | v(t+\tau) \rangle \right|^2 = \left| \int_0^T \frac{dt}{T} v^*(t) v(t-\tau) \right|^2$$

experiment



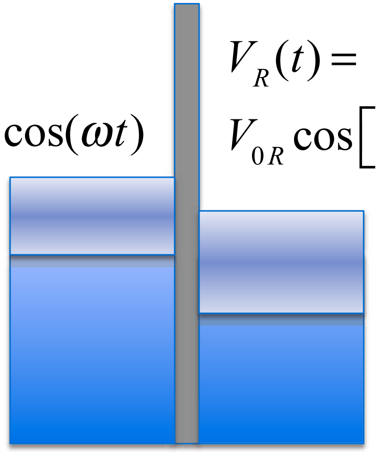
Test of the wave function: Hong-Ou-Mandel interferometry

$V_L(t) =$

$V_{dc} + V_{0L}\cos(\omega t)$

$V_R(t) =$

$V_{0R}\cos\left[\omega(t-\tau)\right]$

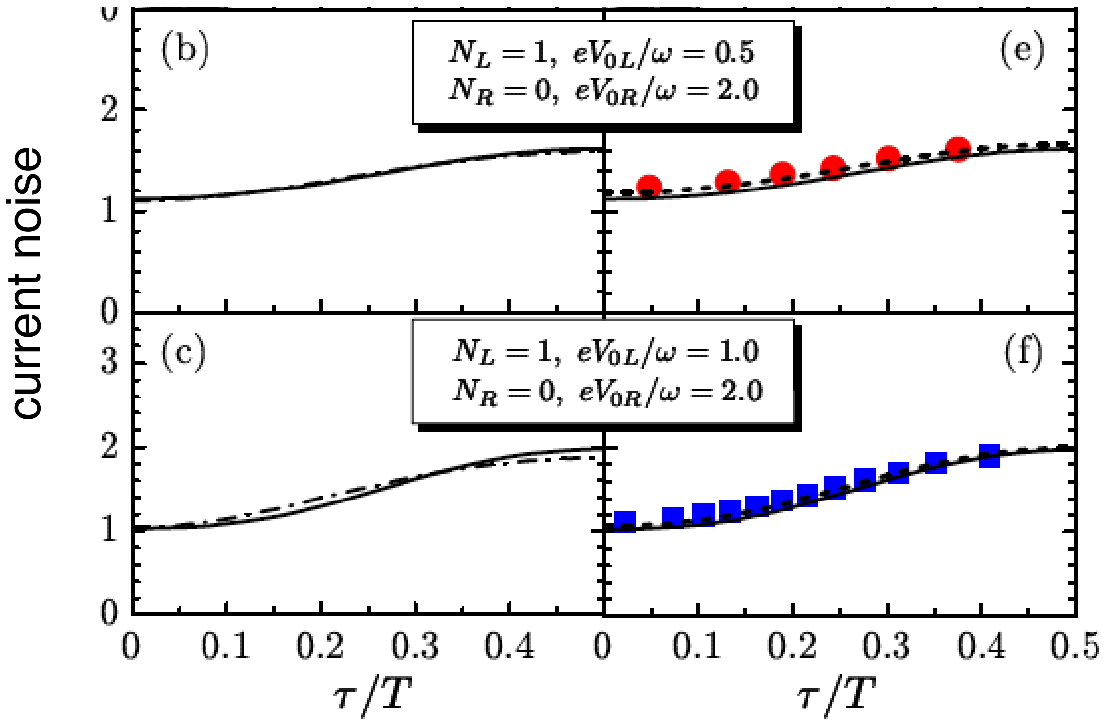


Left contact: injects single electrons
Right contact: injects electron-hole pairs

$$S_2(\tau) = S_L + S_R - 2S_0 C(\tau)$$

$$C(\tau) = p_R\left|\left\langle v(t)\middle|u_+(t-\tau)\right\rangle\right|^2 \quad p_R \approx 0.63$$

wave function overlap measured noise



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Summary

- Current noise, photon-assisted transport and Levitons
  - Playground for coherent single electron transport (electron quantum optics)
- Full counting statistics analysis and elementary events
  - Statistical analysis to fully identify the elementary events
  - UPoN: application to other systems: superconducting, interacting, dots,…
- Noise minimization by pulse shaping
  - Time-dependent voltages tailor the voltage-dependent noise
  - UPoN: minimal noise (not necessary Levitons)
- Non-equilibrium wave function
  - Proposal of a manybody wave function created by time-dependent voltages
  - UPoN: Test of wave function by accessing high-frequency or higher-order correlators, entanglement

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