Stochastic enhancement of absolute negative mobility

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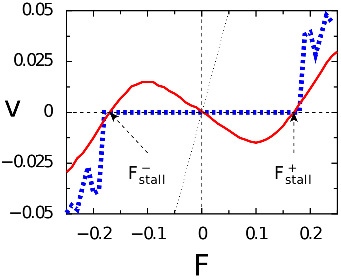
Motivation

$$m\ddot{x}(t) + \gamma\dot{x}(t) + \frac{dU(x)}{dx} = F + A\cos(\Omega t + \phi) + \xi(t)$$



LM, Kostur, Talkner, Łuczka, Hänggi
Phys. Rev. Lett. 98, 040601 (2007)

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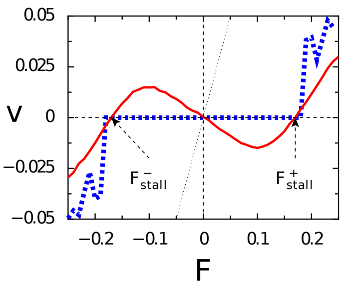

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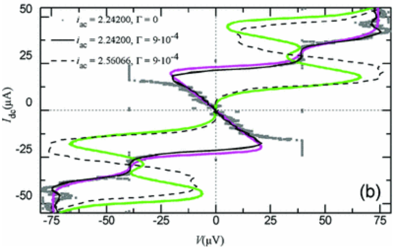

Nagel, Speer, Gaber, Sterck, Eichhorn, Reimann, Ilin, Siegel, Koelle, Kleiner
Phys. Rev. Lett. 100, 217001 (2008)

Main question

Is it possible to

- enlarge the mean velocity
- reduce the fluctuations
- improve the quality of transport

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or in general

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"New operating principle: consider replacing the constant force by nonequilibrium noise!"

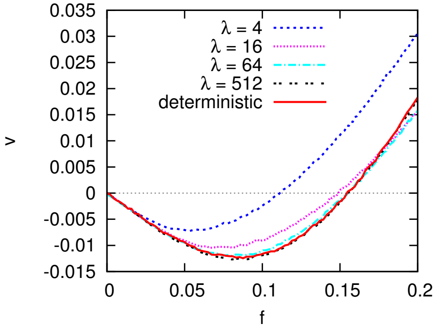
"Brownian motors in the micro-scale domain: Enhancement of efficiency by noise" $\rightarrow$ Monday talk by Jakub Spiechowicz

J. Spiechowicz, P. Hänggi and J. Łuczka, *Phys. Rev. E* **90**, 032104 (2014)

Main question

or in general

Is it possible to improve the transport properties?



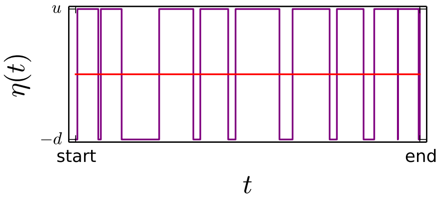
$$\ddot{x}(t) + \gamma\dot{x}(t) + \frac{dV(x)}{dx} = \eta(t) + a\cos(\omega t + \phi) + \xi(t)$$

dichotomic noise

$$\eta(t) = \{-d, u\}, \quad d, u > 0,$$

probabilities of transition per unit time from one state to another

$$Pr(-d \to u) = \mu_d = \frac{1}{\tau_d}, \quad Pr(u \to -d) = \mu_u = \frac{1}{\tau_u}$$



The idea:

deterministic $\rightarrow$ dichotomic

$$\ddot{x}(t) + \gamma \dot{x}(t) + \frac{dV(x)}{dx} = \eta(t) + a\cos(\omega t + \phi) + \xi(t)$$

dichotomic noise

$$\eta(t) = \{-d, u\}, \quad d, u > 0,$$

probabilities of transition per unit time from one state to another

$$Pr(-d \rightarrow u) = \mu_d = \frac{1}{\tau_d}, \quad Pr(u \rightarrow -d) = \mu_u = \frac{1}{\tau_u}$$

with one essential condition

$$\langle \eta(t) \rangle = F$$

Let's see...

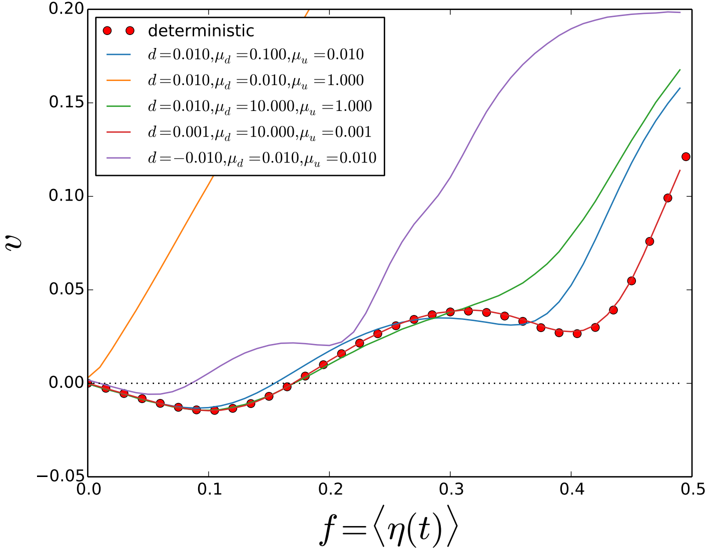
Comprehensive and fast numerical simulations (for poor)

- CUDA environment on a modern desktop GPU
- speed–up of order 3000x

GPU accelerated Monte Carlo simulation of Brownian motors dynamics with CUDA

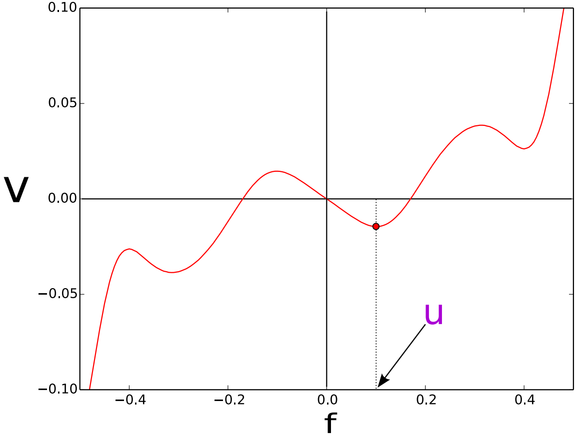
J. Spiechowicz, M. Kostur and LM, Comput. Phys. Commun. 191, 140 (2015)

simulations



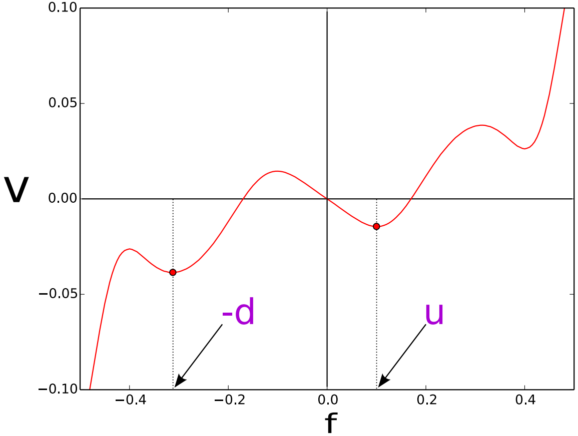
a simple idea, $T = 2\pi/\omega \simeq 1.28$

$$\ddot{x}(t) + \gamma\dot{x}(t) + \frac{dV(x)}{dx} = u + a\cos(\omega t + \phi) + \xi(t)$$



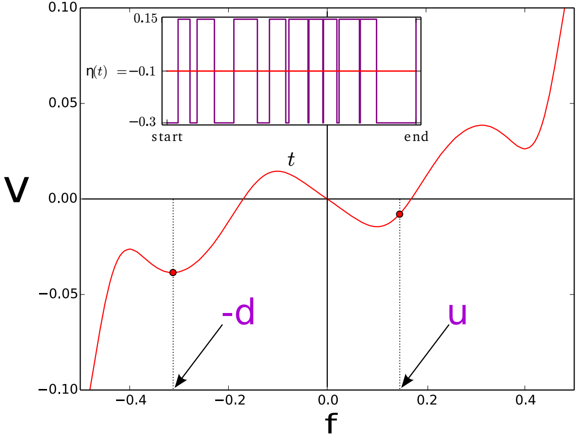
a simple idea, $T = 2\pi/\omega \simeq 1.28$

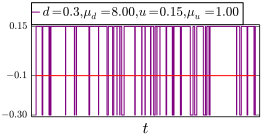
$$\ddot{x}(t) + \gamma\dot{x}(t) + \frac{dV(x)}{dx} = -d + a\cos(\omega t + \phi) + \xi(t)$$

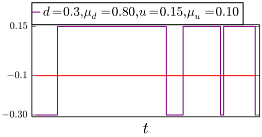


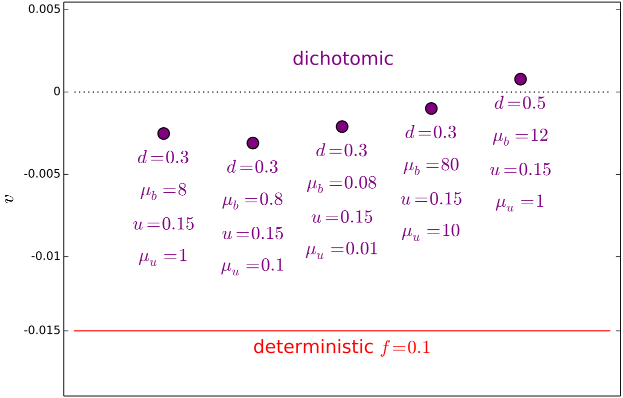
a simple idea, $T = 2\pi/\omega \simeq 1.28$

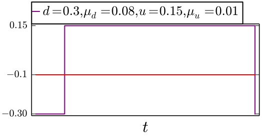
state $u = 0.15$: average residence time $\tau_u = 1 \simeq 0.78\,T$
state $d = 0.3$: average residence time $\tau_d = 1/8 \simeq 0.1\,T$
$\rightsquigarrow \langle \eta(t) \rangle = 0.1$

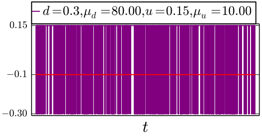


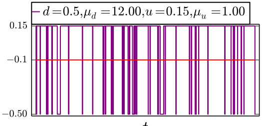

$d=0.3,\mu_d=8.00,u=0.15,\mu_u=1.00$
0.15
−0.1
−0.30
t


$d=0.3,\mu_d=0.80,u=0.15,\mu_u=0.10$
0.15
−0.1
−0.30
t


0.005
0
-0.005
-0.01
-0.015
v
dichotomic
$d=0.3$
$\mu_b=8$
$u=0.15$
$\mu_u=1$
$d=0.3$
$\mu_b=0.8$
$u=0.15$
$\mu_u=0.1$
$d=0.3$
$\mu_b=0.08$
$u=0.15$
$\mu_u=0.01$
$d=0.3$
$\mu_b=80$
$u=0.15$
$\mu_u=10$
$d=0.5$
$\mu_b=12$
$u=0.15$
$\mu_u=1$
deterministic $f=0.1$

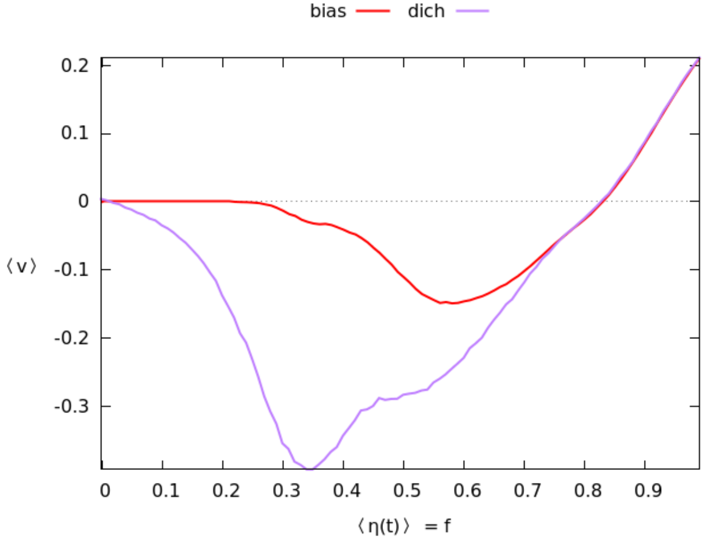

$d=0.3,\mu_d=0.08,u=0.15,\mu_u=0.01$
0.15
−0.1
−0.30
t


$d=0.3,\mu_d=80.00,u=0.15,\mu_u=10.00$
0.15
−0.1
−0.30
t

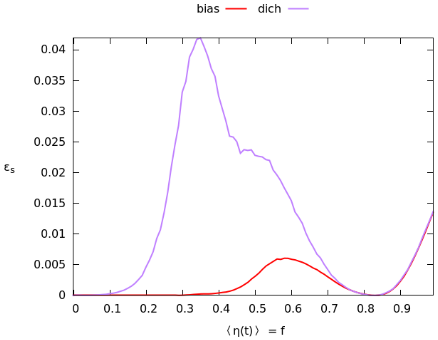

$\mu_d=12.00,u=0.15,\mu_u=1.00$
0.15
−0.1
−0.50

Two state noise can also be better than deterministic force

$d = 0.85, \mu_d = 8.7, \mu_u = 0.708, a = 8.95, \omega = 3.77, \gamma = 1.546, D = 0.001$



H. Linke, Appl. Phys. A 75, 167 (2002), Kostur, et. al, Physica A 371, 20, (2006)

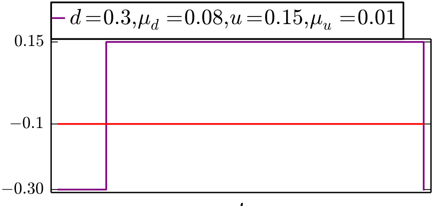


Stokes efficiency (against the friction force γv)

$$\varepsilon_S = \frac{P_{out}}{P_{in}} = \frac{\gamma\langle v\rangle^2}{\gamma(\langle v\rangle^2 + \sigma_v^2 - D)}$$

Efficiency of the energy conversion

$$P_{out} = ?$$



Thank you for your attention!