HEAT AND CHARGE CURRENT FLUCTUATIONS IN A THERMOELECTRIC QUANTUM DOT

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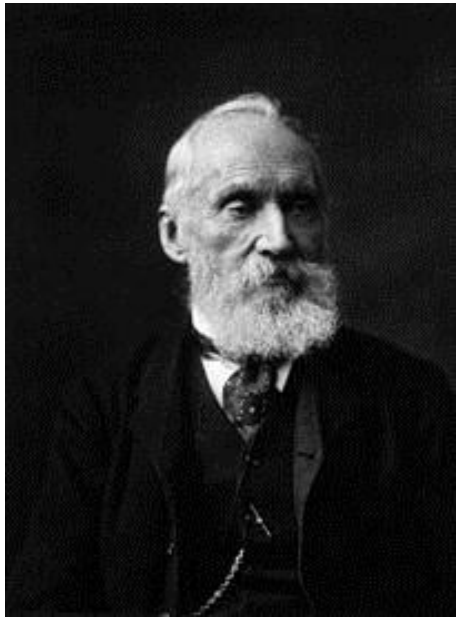
THERMOELECTRICITY

Seebeck effect
1821

Peltier effect
1834



Thomson effect
1851

ELECTRICITY ↔ HEAT

APPLICATION

Thermocouple

Cool water fountain

Thermoelectric generator

$\eta \approx 6\%$

APPLICATION

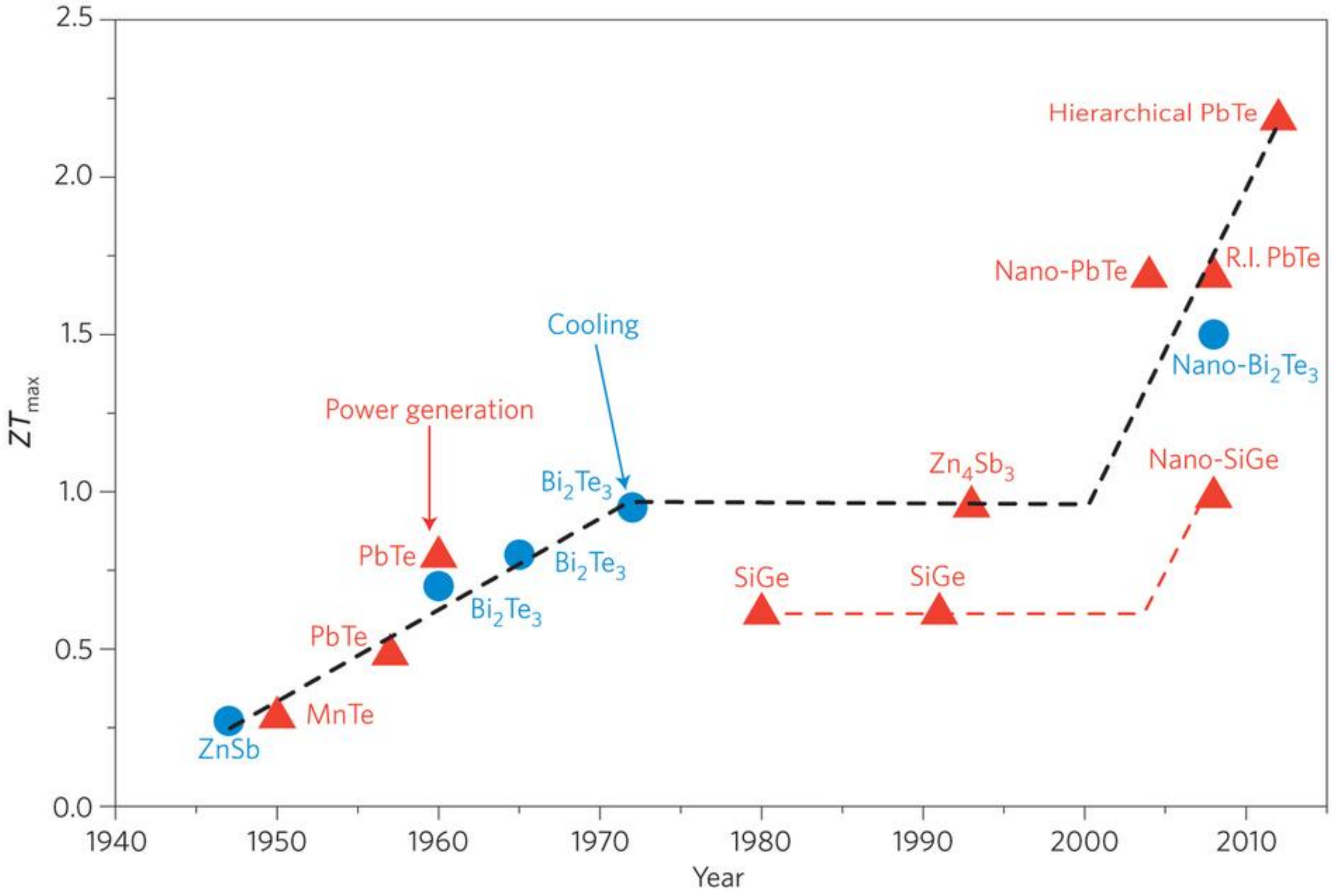
A KEROSENE LAMP GENERATOR.

In 1959, the Russians announced construction of a junction-type, 2 watt, thermo-electric generator. It converted heat into electricity. While providing a light - it also generates 80-90 volts to operate a portable Rdo. However, with Transistors we don't require that much voltage these days. It was invested by Prof. Ioffe of the Institute of semi-conductors in Leningrad.

$\eta < 2\%$

APPLICATION

FIGURE OF MERIT



HEREMANS et al.
Nature Nanotechnologies 8, 471 (2013)

→ **New fields of research : Nanothermoelectricity / Quantum Thermoelectricity**

LINEAR RESPONSE

Charge current → I

Heat current → J

$$\begin{pmatrix} I \\ J \end{pmatrix} = \begin{pmatrix} G & SG \\ \Pi G & \tilde{\kappa} \end{pmatrix} \begin{pmatrix} \Delta V \\ \Delta T \end{pmatrix}$$

$$S = -\frac{\Delta V}{\Delta T}\bigg|_{I=0} \qquad \Pi = \frac{J}{I}\bigg|_{\Delta T=0}$$

$$\kappa = \tilde{\kappa} - S^2 T_0 G$$

Onsager relation

$$\Pi = T_0\, S$$

Wiedemann-Franz law

$$\frac{\kappa}{G T_0} = \frac{\pi^2 k_B^2}{3e^2}$$

$$\eta_{\max} = \eta_C \frac{\sqrt{1+ZT_0} - 1}{\sqrt{1+ZT_0} + 1}$$

$$ZT_0 = \frac{S^2 G T_0}{\kappa}$$

OUTSIDE THE LINEAR RESPONSE

$$\begin{pmatrix} I \\ J \end{pmatrix} \neq \begin{pmatrix} G & SG \\ \Pi G & \tilde{\kappa} \end{pmatrix} \begin{pmatrix} \Delta V \\ \Delta T \end{pmatrix}$$

$$\eta_{\max} \neq \eta_C \frac{\sqrt{1+ZT_0} - 1}{\sqrt{1+ZT_0} + 1}$$

The figure of merit is no longer the adequate quantity to quantify thermoelectricity

QUESTION

Can noise quantifies the thermoelectric conversion ?

MIXED NOISE

$$S^{IJ}_{pq} = \int_{-\infty}^{\infty} \left\langle \delta \hat{I}_p(0)\, \delta \hat{J}_q(t) \right\rangle dt$$

CHARGE CURRENT

$$\delta \hat{I}_p(t) = \hat{I}_p(t) - \left\langle \hat{I}_p \right\rangle$$

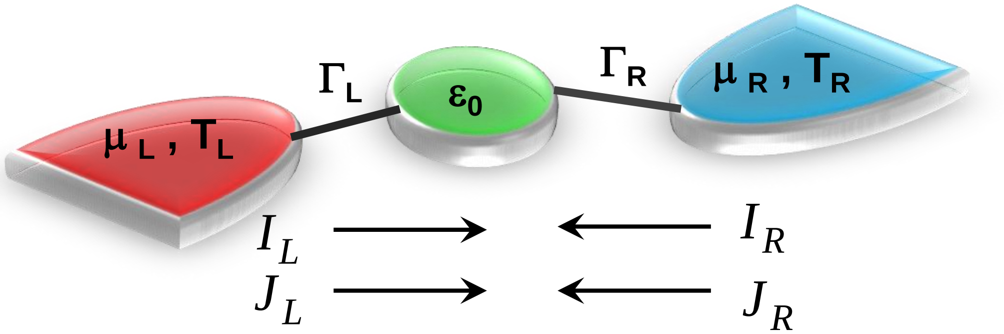
$$\hat{I}_p(t) = -e\,\dot{N}_p$$

HEAT CURRENT

$$\delta \hat{J}_p(t) = \hat{J}_p(t) - \left\langle \hat{J}_p \right\rangle$$

$$\hat{J}_p(t) = \hat{I}^E_p(t) - \frac{\mu_p}{e}\hat{I}_p(t)$$

$$dQ_p = dE_p - \mu_p dN_p$$



VERY FEW STUDIES ON MIXED NOISE

REVIEWS OF MODERN PHYSICS, VOLUME 78, JANUARY 2006

Opportunities for mesoscopics in thermometry and refrigeration: Physics and applications

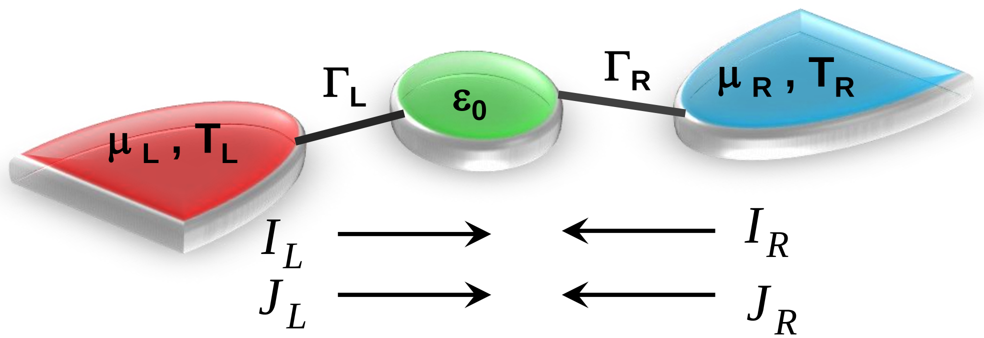
Francesco Giazotto, Tero T. Heikkilä, Arttu Luukanen, Alexander M. Savin, and Jukka P. Pekola

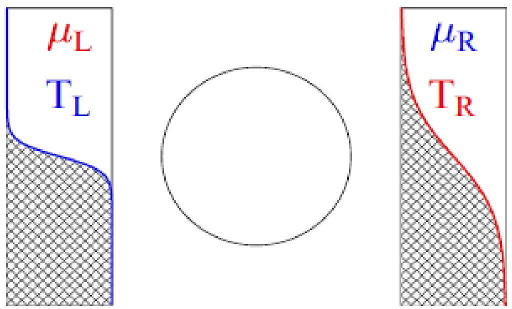
New Journal of Physics **15** (2013) 125001

Correlations of heat and charge currents in quantum-dot thermoelectric engines

Rafael Sánchez[1,4], Björn Sothmann[2], Andrew N Jordan[3] and Markus Büttiker[2]

SYSTEM





$$H = \underbrace{\sum_{k\in L} \varepsilon_k c_k^+ c_k}_{\text{reservoir L}} + \underbrace{\sum_{k\in R} \varepsilon_k c_k^+ c_k}_{\text{reservoir R}} + \underbrace{\varepsilon_0 d^+ d}_{\text{dot}} + \underbrace{\sum_{p=L,R} \sum_{k\in p} V_k c_k^+ d + h.c.}_{\text{dot -- reservoir transfer}}$$

METHOD AND ASSUMPTIONS

- Mixed noise expressed in terms of two-particles Keldysh Green's functions
- Non-interacting system → Wick's theorem
- Dyson equation of motion for the dot Green's function
- Fourier transform
- Wide-band approximation

RESULTS

LANDAUER-LIKE EXPRESSIONS

$$I_L = \frac{e}{h}\int_{-\infty}^{+\infty} d\varepsilon \;\; [\; f_L(\varepsilon) - f_R(\varepsilon)\;]\, \mathcal{T}(\varepsilon)$$

$$J_L = \frac{1}{h}\int_{-\infty}^{+\infty} d\varepsilon \; (\varepsilon - \mu_L) \;\; [\; f_L(\varepsilon) - f_R(\varepsilon)\;]\, \mathcal{T}(\varepsilon)$$

$$\mathrm{f}_{\mathrm{L,R}}(\varepsilon) = \text{Fermi} - \text{Dirac distribution function}$$

$$\mathcal{T}(\varepsilon) = \text{transmission coefficient}$$

BUTCHER, JPCM 2, 4869 (1990)

ZERO-FREQUENCY NOISES

Charge noise $$\mathrm{S}_{pq}^{\mathrm{II}} = \pm\frac{e^2}{h}\int_{-\infty}^{\infty} F(\varepsilon)\, d\varepsilon$$

Mixed noise $$\mathrm{S}_{pq}^{\mathrm{IJ}} = \pm\frac{e}{h}\int_{-\infty}^{\infty} (\varepsilon - \mu_q) F(\varepsilon)\, d\varepsilon$$

Heat noise $$\mathrm{S}_{pq}^{\mathrm{JJ}} = \pm\frac{1}{h}\int_{-\infty}^{\infty} (\varepsilon - \mu_p)(\varepsilon - \mu_q) F(\varepsilon)\, d\varepsilon$$

$$F(\varepsilon) = \mathcal{T}(\varepsilon)\;\; [\, f_L(\varepsilon)(1 - f_L(\varepsilon)) + f_R(\varepsilon)(1 - f_R(\varepsilon))\,] + \mathcal{T}(\varepsilon)\;\; [\,1 - \mathcal{T}(\varepsilon)\,][\, f_L(\varepsilon) - f_R(\varepsilon)\,]^2$$

- **Conservation rules**
- **Mixed noise at equilibrium**
- **Mixed noise far from equilibrium**

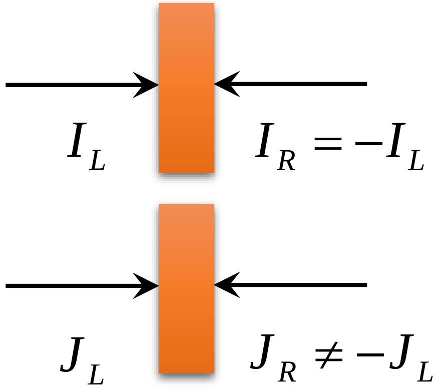
CONSERVATION RULES

NUMBER OF ELECTRONS IS CONSERVED

$$\langle \hat{N}_L \rangle + \langle \hat{N}_T \rangle = Cste \;\Rightarrow\; I_L + I_R = 0$$

TOTAL HEAT IS NOT CONSERVED

$$\langle \hat{Q}_L \rangle + \langle \hat{Q}_R \rangle \neq Cste \;\Rightarrow\; J_L + J_R \neq 0$$



Contact resistance dissipation

POWER CONSERVATION

$$J_L + J_R = V\, I_R \Rightarrow \langle \hat{P}^{th} \rangle = \langle \hat{P}^{el} \rangle$$

TOTAL CHARGE AND MIXED NOISES

$$\sum_{p,q} S_{pq}^{II} = \sum_{p,q} S_{pq}^{IJ} = \sum_{p,q} S_{pq}^{JI} = 0$$

POWER FLUCTUATIONS CONSERVATION

$$\sum_{p,q} S_{pq}^{JJ} = V^2 S_{LL}^{II} \Rightarrow \int_{-\infty}^{\infty} \langle \hat{P}^{th}(t)\, \hat{P}^{th}(0) \rangle\, dt = \int_{-\infty}^{\infty} \langle \hat{P}^{el}(t)\, \hat{P}^{el}(0) \rangle\, dt$$

AT EQUILIBRIUM (linear response)

RELATIONS BETWEEN NOISES AND CONDUCTANCES

$$\mathrm{S}_{\mathrm{pp}}^{\mathrm{II}} = 2k_B T_0 G$$

$$\mathrm{S}_{\mathrm{pp}}^{\mathrm{IJ}} = \mathrm{S}_{\mathrm{pp}}^{\mathrm{JI}} = -2k_B T_0^2 S G$$

$$\mathrm{S}_{\mathrm{pp}}^{\mathrm{JJ}} = 2k_B T_0^2 \tilde{\kappa}$$

G = electrical conductance
S = Seebeck coefficient
κ = thermal conductance
T_0 = average temperature

$$\kappa = \tilde{\kappa} - S^2 T_0 G$$

→ Fluctuation-Dissipation Theorem applies for any kind of noises

KUBO et al., J. Phys. Soc. Jpn. 12, 1203 (1957)

FIGURE OF MERIT

$$ZT_0 = \frac{S^2 T_0 G}{\kappa} = \frac{\left(\mathrm{S}_{\mathrm{pq}}^{\mathrm{IJ}}\right)^2}{\mathrm{S}_{\mathrm{pq}}^{\mathrm{II}} \mathrm{S}_{\mathrm{pq}}^{\mathrm{JJ}} - \left(\mathrm{S}_{\mathrm{pq}}^{\mathrm{IJ}}\right)^2}$$

Independent of p and q

CREPIEUX / MICHELINI, JPCM 27, 015302 (2015)

FAR FROM EQUILIBRIUM

SCHOTTKY REGIME $\mathcal{T}(\varepsilon) << 1$

NOISES

$$\mathsf{S}_{LR}^{II} = C\,e\,I_R$$
$$\mathsf{S}_{LR}^{IJ} = C\,e\,J_R = C(\varepsilon_0 - \mu_R)\,I_R$$
$$\mathsf{S}_{LR}^{JJ} = C(\varepsilon_0 - \mu_L)\,J_R$$

$$C = \coth\left(\frac{\varepsilon_0-\mu_R}{2k_BT_R} - \frac{\varepsilon_0-\mu_L}{2k_BT_L}\right) = 1 \text{ when } T_{L,R} = 0$$

→ Noises are proportional to currents

EFFICIENCY

$$\eta = \frac{P^{th}}{P^{el}} = \left|\frac{J_R}{V\,I_R}\right|$$

$$I_R = \frac{\mathsf{S}_{LR}^{II}}{e\,C} \qquad J_R = \frac{\mathsf{S}_{LR}^{IJ}}{e\,C} \qquad eV = \frac{\mathsf{S}_{LR}^{IJ}}{I_L\,C} - \frac{\mathsf{S}_{LR}^{JJ}}{J_R\,C}$$

EQUIVALENTLY

$$\eta = \frac{\left(\mathsf{S}_{LR}^{IJ}\right)^2}{\left|\left(\mathsf{S}_{LR}^{IJ}\right)^2 - \mathsf{S}_{LR}^{II}\mathsf{S}_{LR}^{JJ}\right|}$$

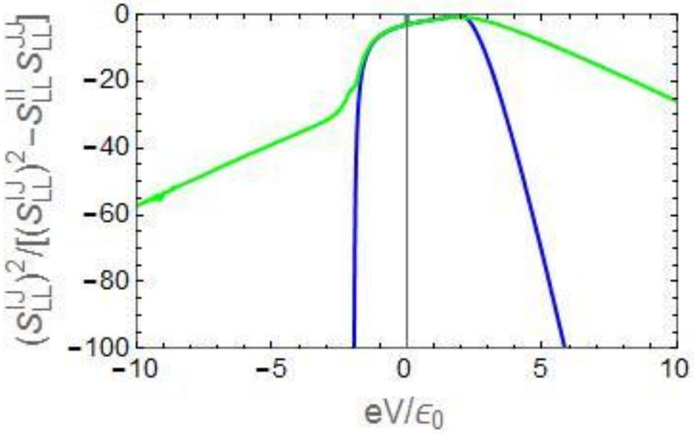
Does not depend on C

→ Thermoelectric efficiency can be written as a ratio of noises

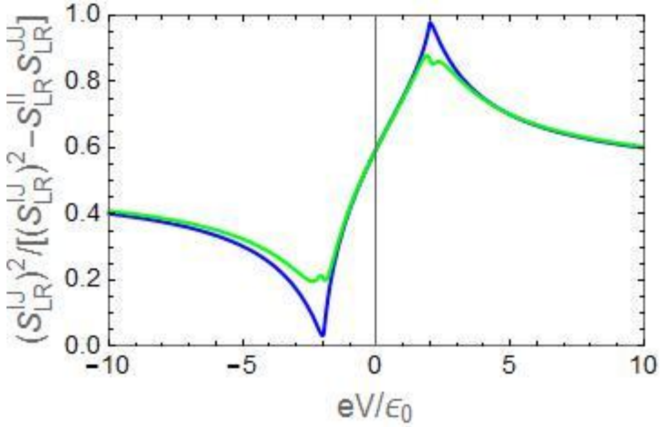
CREPIEUX / MICHELINI, JPCM 27, 015302 (2015)

NUMERICAL TEST

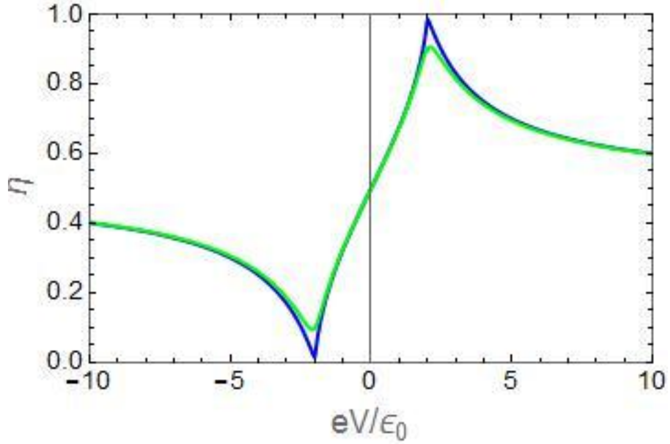
AUTO-RATIO



CROSS-RATIO



EFFICIENCY



— $\Gamma/\epsilon_0=0.01$

— $\Gamma/\epsilon_0=0.1$

$k_B T/\varepsilon_0 = 0$

$k_B T_0/\varepsilon_0 = 0.001$

→ The efficiency fits with the cross-ratio !
It has no relation with the auto-ratio

CONCLUSION

MIXED NOISE $\mathsf{S}_{\mathrm{pq}}^{\mathrm{IJ}} = \int_{-\infty}^{\infty} \left\langle \delta\, \hat{I}_p(0)\, \delta\, \hat{J}_q(t) \right\rangle dt$

allows to quantify thermoelectric conversion

In the linear response regime

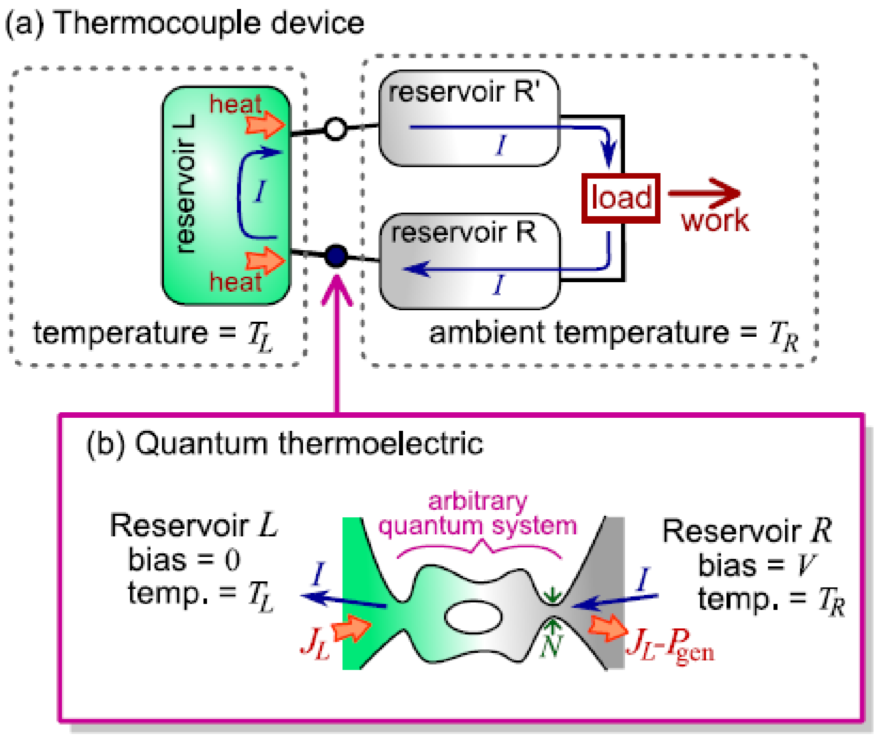
$$ZT_0 = \frac{\left(\mathsf{S}_{\mathrm{pq}}^{\mathrm{IJ}}\right)^2}{\mathsf{S}_{\mathrm{pq}}^{\mathrm{II}}\mathsf{S}_{\mathrm{pq}}^{\mathrm{JJ}} - \left(\mathsf{S}_{\mathrm{pq}}^{\mathrm{IJ}}\right)^2}$$

In the Schottky regime

$$\eta = \frac{\left(\mathsf{S}_{\mathrm{LR}}^{\mathrm{IJ}}\right)^2}{\left|\left(\mathsf{S}_{\mathrm{LR}}^{\mathrm{IJ}}\right)^2 - \mathsf{S}_{\mathrm{LR}}^{\mathrm{II}}\mathsf{S}_{\mathrm{LR}}^{\mathrm{JJ}}\right|}$$

OPEN PROBLEMS

- ❑ Coulomb interactions, phonons
- ❑ Mixed noise for ac-driven
- ❑ Efficiency fluctuations
- ❑ Mixed noise in a 3-terminals thermoelectric systems
- ❑ Measurement of mixed noise



WHITNEY, PRB 91, 115425 (2015)

Thank you for your attention !