CNR-SPIN (GENOVA)

FUNCTIONAL APPROACH TO HEAT EXCHANGE APPLICATION TO THE SPIN BOSON MODEL: FROM MARKOV TO QUANTUM NOISE REGIME

Matteo Carrega

In collaboration with:
Dr. P. Solinas
Dr. A. Braggio
Prof. M. Sassetti
Prof. U. Weiss

Outline

- Quantum Thermodynamics
- Path-integral approach to energy exchange
- Application to the spin-boson model
- Results for average heat and heat power

Introduction

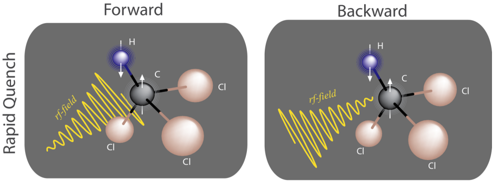
Esposito RMP '09, Campisi RMP '11

- Thermodynamics of small devices
- Definitions of work and heat at quantum level
- Precise measurement protocols

Recent experiments

Batalhao PRl '14

- Measurement of work distribution
- NMR study with RF field



Work distribution – Closed system

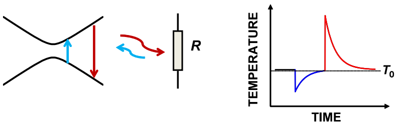
Verification of Jarzynski equality

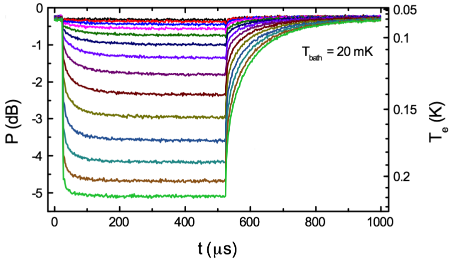
$$\langle e^{-\beta W} \rangle = \frac{Z(\tau)}{Z(0)}$$

Recent experiments

Pekola PRL '13, Gasparinetti Phys. Applied '15, Pekola Nat. Phys. '15

Measurement of dissipated heat





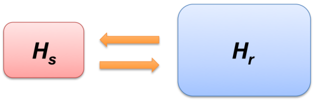
Hybrid electronic circuit

→ **Temperature measurement of the environment**

Measurement protocol

Tasaki ArXiv '00, Talkner PRE '07, Gasparinetti NJP '14

$$H_{\rm tot} = H_{\rm S}(t) + H_{\rm R} + H_{\rm I}$$



- Double measurement protocol
  - initial state $\rho_{\rm tot}(t=0)$
  - first projective measurement $\sum_{E_1} p_{E_1}$
  - time evolution $U(t)$ generated by $H_{\rm tot}$
  - second projective measurement $\sum_{E_2} p_{E_2}$

Heat statistics $P(Q, t)$

Probability distribution of energy exchange

$$P(Q,t) = \sum_{E_1,E_2} \delta(E_2 - E_1 - Q) P[E_2; E_1] P[E_1]$$

Conditional probability $P[E_2; E_1]$

$$P[E_2; E_1] P[E_1] = \mathrm{Tr}\left[U^\dagger(t) p_{E_2} U(t) p_{E_1} \rho_{\mathrm{tot}}(0) p_{E_1}\right]$$

- Characteristic function $G_\nu(t) = \int_{-\infty}^{+\infty} dQ e^{iQ\nu} P(Q,t)$

$$G_\nu(t) = \mathrm{Tr}\left[U^\dagger(t) e^{i\nu H_{\mathrm{R}}} U(t) e^{-i\nu H_{\mathrm{R}}} \rho_{\mathrm{tot}}(0)\right]$$

Heat statistics $G_\nu(t)$

- $G_\nu(t)$ Moment generating function

$$\langle Q^n(t)\rangle = (-i)^n \frac{d^n G_\nu(t)}{d\nu^n}\Big|_{\nu=0} = (-i)^n \frac{d^n \mathrm{Tr}\left[\rho_{\mathrm{tot}}^{(\nu)}(t)\right]}{d\nu^n}\Big|_{\nu=0}$$

Generalized time evolution $\rho_{\mathrm{tot}}^{(\nu)}(t) = U_{\nu/2}(t)\rho_{\mathrm{tot}}(0)U^\dagger_{\nu/2}(t)$ with
$U_\nu(t) = e^{i\nu H_{\mathrm{R}}}U(t)e^{-i\nu H_{\mathrm{R}}}$
Factorized initial condition

$$\rho_{\mathrm{tot}}(0) = \rho_{\mathrm{S}}(0)\otimes\rho_{\mathrm{R}}(0) = \rho_{\mathrm{S}}(0)\otimes\frac{e^{-\beta H_{\mathrm{R}}}}{Z_{\mathrm{R}}}$$

Standard approach: Master equation
$\rightarrow$ Born-Markov approximation

Functional integral

Feynman Ann. Phys. '63, Caldeira Phys. A '83, Weiss '99

Path integral approach − Reduced system dynamics

$$G_\nu(t) = \int d\eta_i \, \langle \eta_i | \, \rho_{\rm S}(0) \, | \eta_i \rangle \int d\eta_f \int \mathcal{D}\eta \int \mathcal{D}\xi \; e^{i \, S_{\rm S}[\eta,\xi]} \; \mathcal{F}_{\rm FV}[\eta,\xi] \cdot e^{i\Delta\Phi^{(\nu)}[\eta,\xi]}$$

Generalization of Feynman-Vernon influence functional for heat exchange

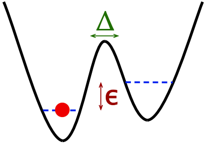
- $n-$th moment $\langle Q^n(t) \rangle$

$$\Phi^{(n)}[\eta,\xi] = (-i)^n \; \frac{d^n}{d\nu^n} e^{i\Delta\phi^{(\nu)}[\eta,\xi]} \Big|_{\nu=0}$$

Carrega NJP '15

Spin - boson model

- Dissipative two level system



$$H_{\mathrm{S}} = -\frac{\Delta}{2}\sigma_x - \frac{\epsilon(t)}{2}\sigma_z$$

state basis $|R/L\rangle$ with $\sigma_z|R/L\rangle = \pm|R/L\rangle$

- low energy state of a double well potential $v(q)$ $\quad q = q_0\sigma_z$

Δ tunneling amplitude $\quad$ external bias $\epsilon = \epsilon_0 + \epsilon_1(t)$

Weak coupling

Spectral function $j(\omega) \propto K\omega$ K coupling strength

- Weak damping regime $K \ll 1$ constant bias ϵ_0
- Total transferred heat

$$Q_\infty \equiv \lim_{t\to\infty} \langle Q(t) \rangle = \langle E_{\mathrm{ini}} \rangle - \langle E_{\mathrm{eq}} \rangle$$

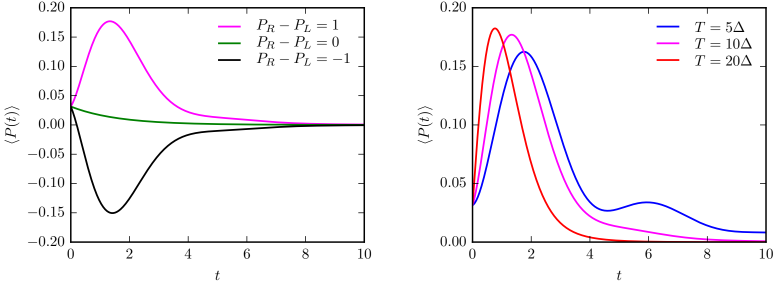
$$Q_\infty = \frac{(P_R - P_L)\epsilon_0}{2} + \frac{\sqrt{\delta^2 + \epsilon_0^2}}{2} \tanh\left(\frac{\sqrt{\delta^2 + \epsilon_0^2}}{2T} \right)$$

Markov regime

- High temperature regime $T > \sqrt{\Delta^2 + \epsilon_0^2}$

Average heat power $\langle P(t) \rangle = \langle \dot{Q}(t) \rangle$

$$\langle P(t) \rangle = \frac{\pi}{2} K \delta^2 - \pi K T \Delta \left[\langle \sigma_x(t) \rangle_{\mathrm{s}} - (P_R - P_L) \langle \sigma_x(t) \rangle_{\mathrm{a}} \right]$$



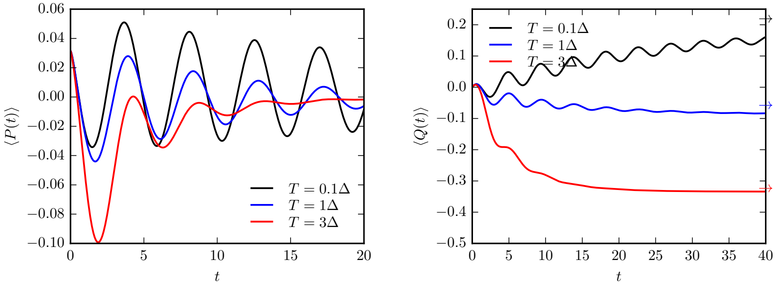
$K = 0.02 \qquad \Delta = \epsilon_0 = 1$

Quantum noise regime

- Low temperature $T < \sqrt{\Delta^2 + \epsilon_0^2}$

Average heat power − quantum noise contributions

$$\langle P(t)\rangle = -\frac{\Delta}{2}\int_0^t d\tau L'(\tau)\Big(\langle\sigma_x(t-\tau)\rangle_{\rm s}\langle\sigma_z(\tau)\rangle_{\rm s} - \langle\sigma_z(t-\tau)\rangle_{\rm a}\langle\sigma_x(\tau)\rangle_{\rm a}\Big)$$
$$+\frac{\Delta(P_R - P_L)}{2}\int_0^t d\tau L'(\tau)\Big(\langle\sigma_x(t-\tau)\rangle_{\rm a}\langle\sigma_z(\tau)\rangle_{\rm s} - \langle\sigma_z(t-\tau)\rangle_{\rm s}\langle\sigma_x(\tau)\rangle_{\rm a}\Big)$$



$K = 0.02 \qquad \Delta = \epsilon_0 = 1$

Conclusions

- Functional integral approach to energy exchange
- Application to the spin-boson model
- Average heat and heat power
- Quantum noise contribution at low temperature