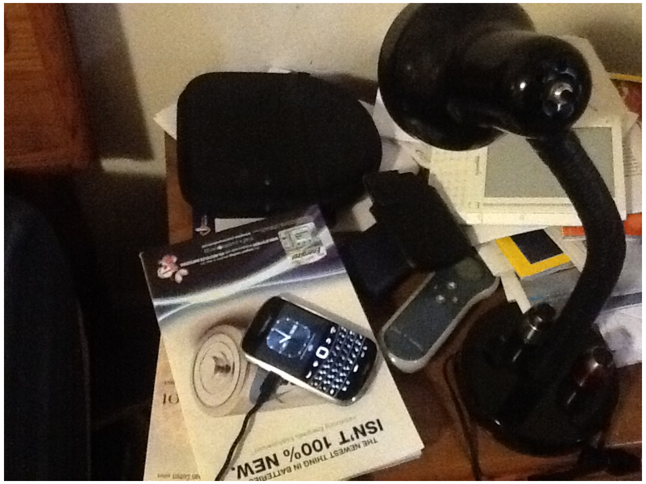
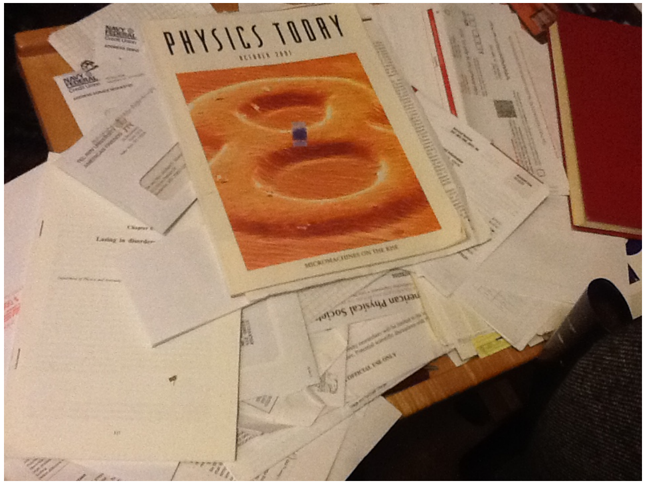
Is There an Optimal Search Strategy, When Time is Limited?





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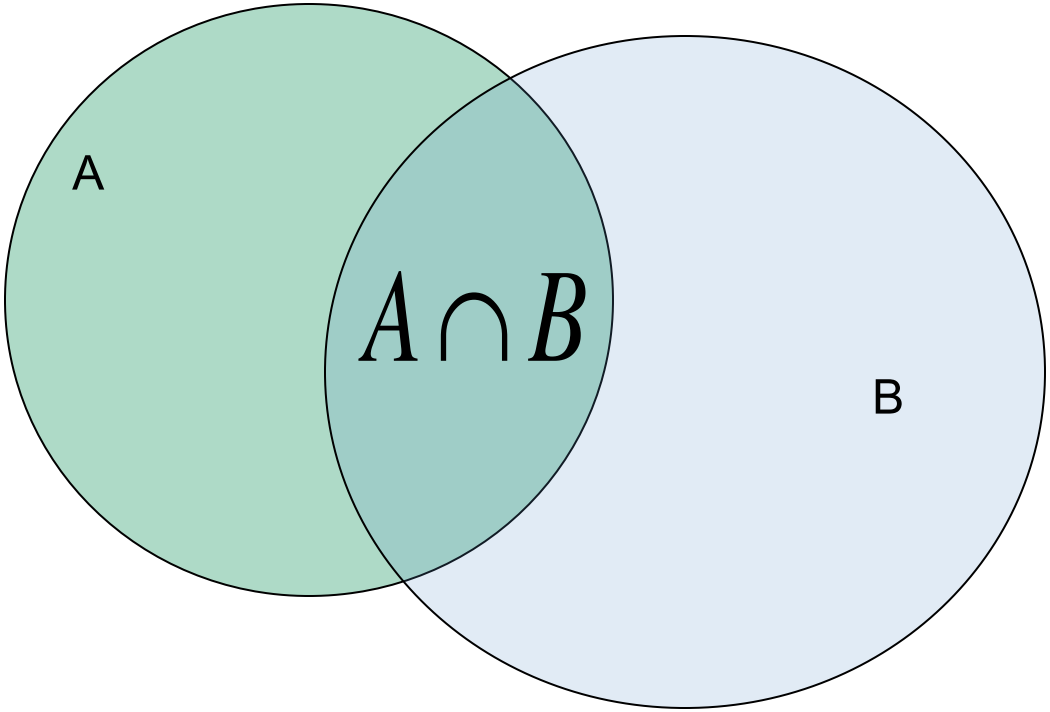
Unsolved Problems of Noise
Barcelona July 13-17, 2015

Reverend Bayes (1701-1761)

Bayes' Theorem(1763)

$$p\left(A \bigcap B\right) = p\left(A|B\right)p(B) = p\left(B|A\right)p(A)$$



The Sceptical Bayesian's Unfair Coin: STATISTICS VS. PROBABILITY

(when to stop searching)

$$p(nT) = \left(\frac{1}{2}\right)^n \quad \text{for a fair coin}$$

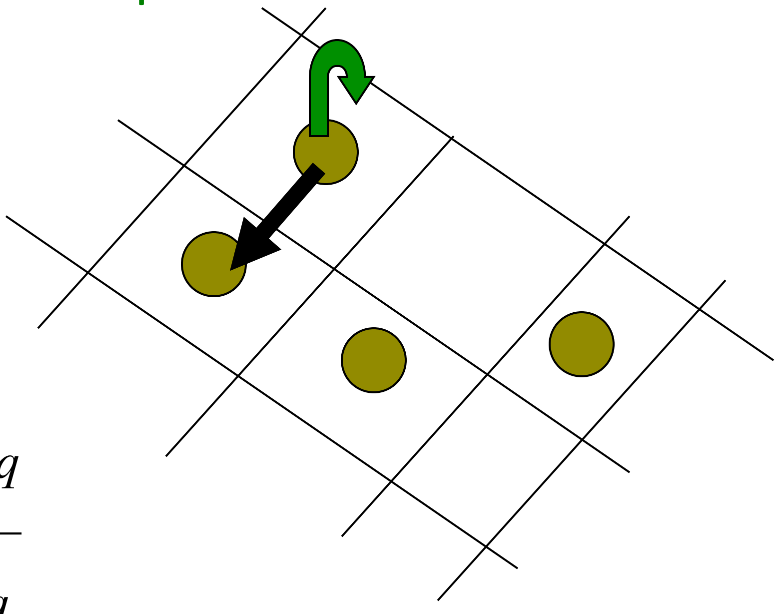
heads = found target

Flip a coin n times and get n TAILS

What can you infer about the fairness of the coin?

Use data to estimate q

$$p(q|nT) = \frac{p(nT|q)\,p(q)}{\int_0^1 p(nT|q)\,p(q)\,dq}$$



$$\langle q \rangle = \frac{\int_0^1 q\,p(nT|q)\,p(q)\,dq}{\int_0^1 p(nT|q)\,p(q)\,dq} = \frac{\int_0^1 q q^n\,p(q)\,dq}{\int_0^1 q^n\,p(q)\,dq}$$

$$\langle q \rangle = \frac{n+1}{n+2}$$

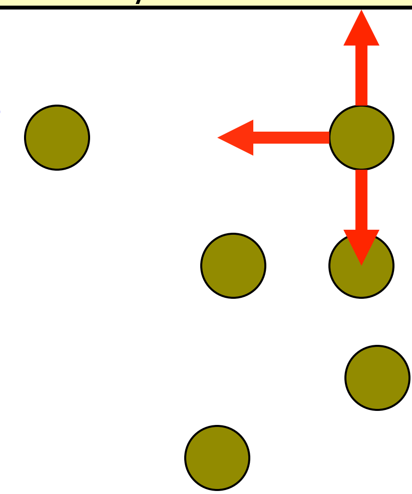
$$\langle q(n=0) \rangle = 1/2$$

$$\langle q(n \to \infty) \rangle \to 1$$

When to switch coins: Stick with same coin or switch coins?

outcome	estimate <T>	estimate <H>	switch if	same as
no coin toss	1/2	1/2		
T	2/3	1/3	1/2-cost>1/3	cost<1/6
TT	3/4	1/4	1/2-cost>1/4	cost<1/4
TTT	4/5	1/5	1/2-cost>1/5	cost<3/10
TTTT	5/6	1/6	1/2-cost>1/6	cost<1/3

Flipping a coin and getting a HEAD, then win one coin.



w=prob(next space has a coin)

N=number of TAILS in a row

C=cost to switch coins

One flip T case estimated gain by switching =(1/2)w-C

Switch coins if w/2-C>1/3

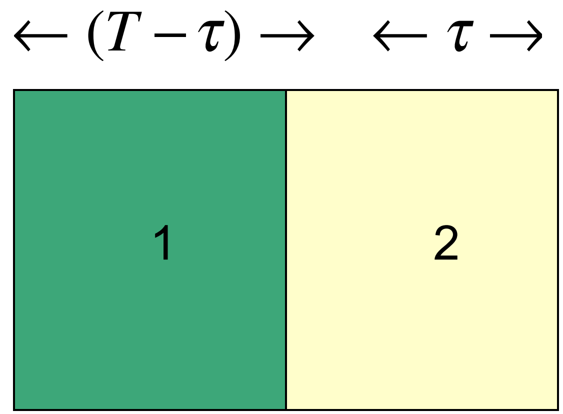
N tails TTT…T switch if (1/2)w-C>1/(N+2)

switch if cost<(w/2)-(1/(N+2))

What if w=w(t)=exp(-bt)?

Finite Time T Searching Two Regions

λ=rate of success to find target in a search if a target is present
K= rate of success to retrieve/capture the target
G= gain (payoff of search)
C=cost of search/unit time
p=prob that a target is present
N= net return

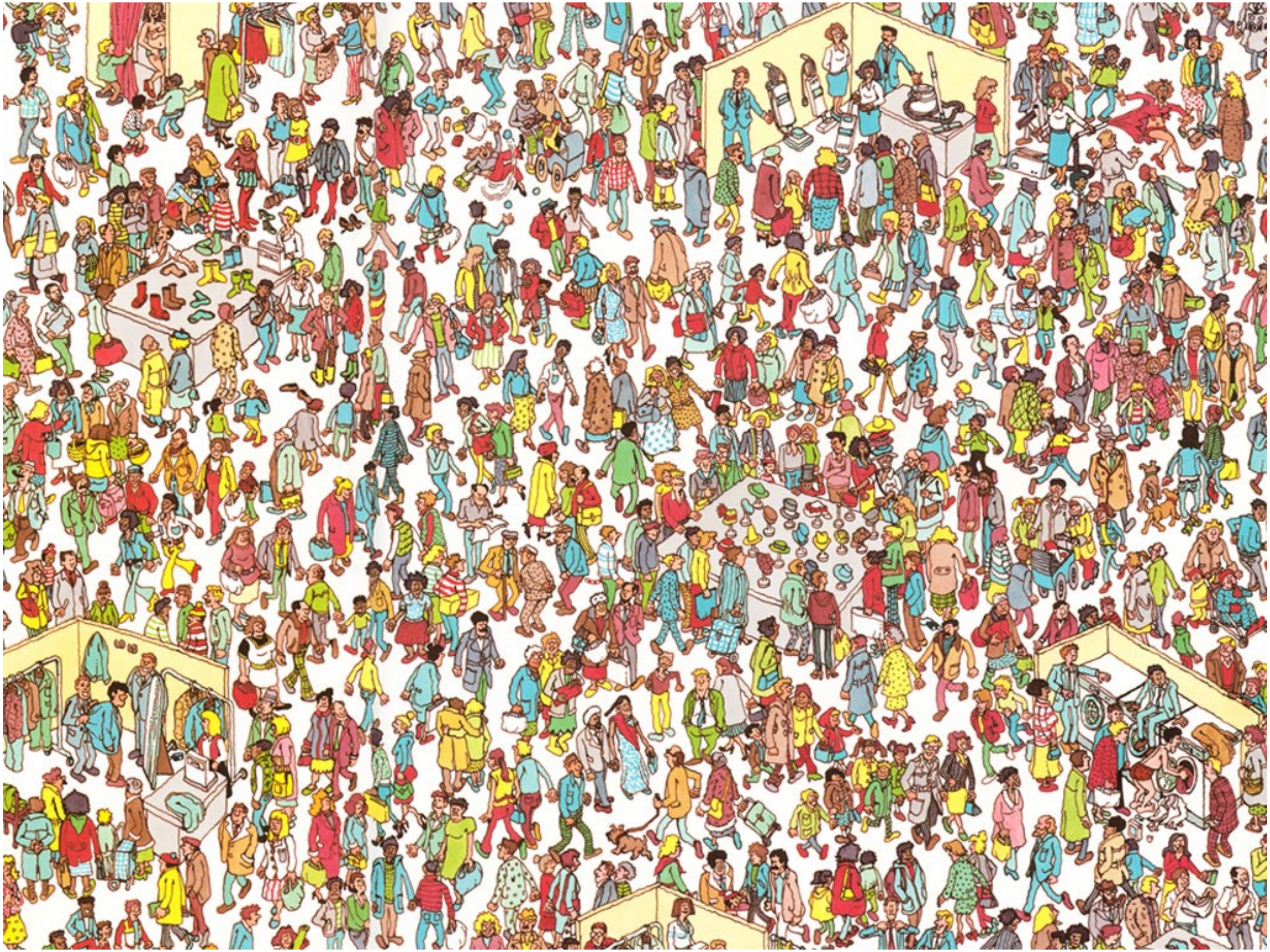


$$N(T,\tau) = G_1 K_1 (T-\tau) p_1 \left(1-\exp\left(-\lambda_1 (T-\tau)\right)\right) + G_2 K_2 (\tau) p_2 \left(1-\exp\left(-\lambda_2 \tau\right)\right)$$
$$-C_1 (T-\tau) - C_2 \tau$$

Choose τ to maximize N

$$\frac{\partial N(T,\tau)}{\partial \tau} = 0$$

Rate Limited or Reaction Limited



Getting to the right location does not mean the target is found

For Brownian motion with diffusion constant D and average target spacing d

$$\lambda \propto \frac{D}{\langle d \rangle^2}$$

For ballistic motion with velocity v and sighting range R

$$\lambda \propto \frac{\mathrm{v}R}{\langle d \rangle^2} \qquad \text{(2D)}$$

$$\lambda \propto \frac{\mathrm{v}R^2}{\langle d \rangle^3} \qquad \text{(3D)}$$

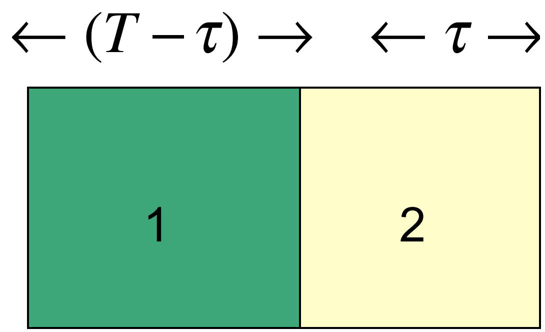
$$\text{cost} \propto m\mathrm{v}^2 t$$

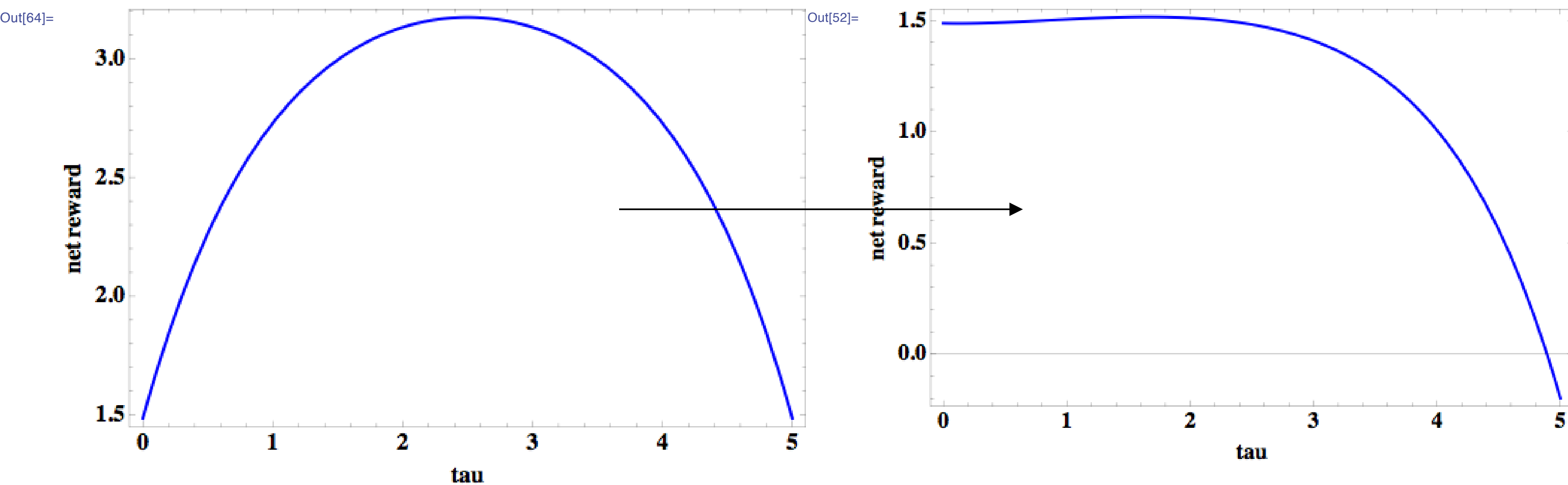
$$\text{gain G} = \begin{cases} \text{constant} \\ 1/\lambda \\ f(\lambda) \end{cases}$$

$$d \rightarrow d^{1/\alpha}$$

For fractal set of targets with dimension α

In the figure on the left both regions are the same;

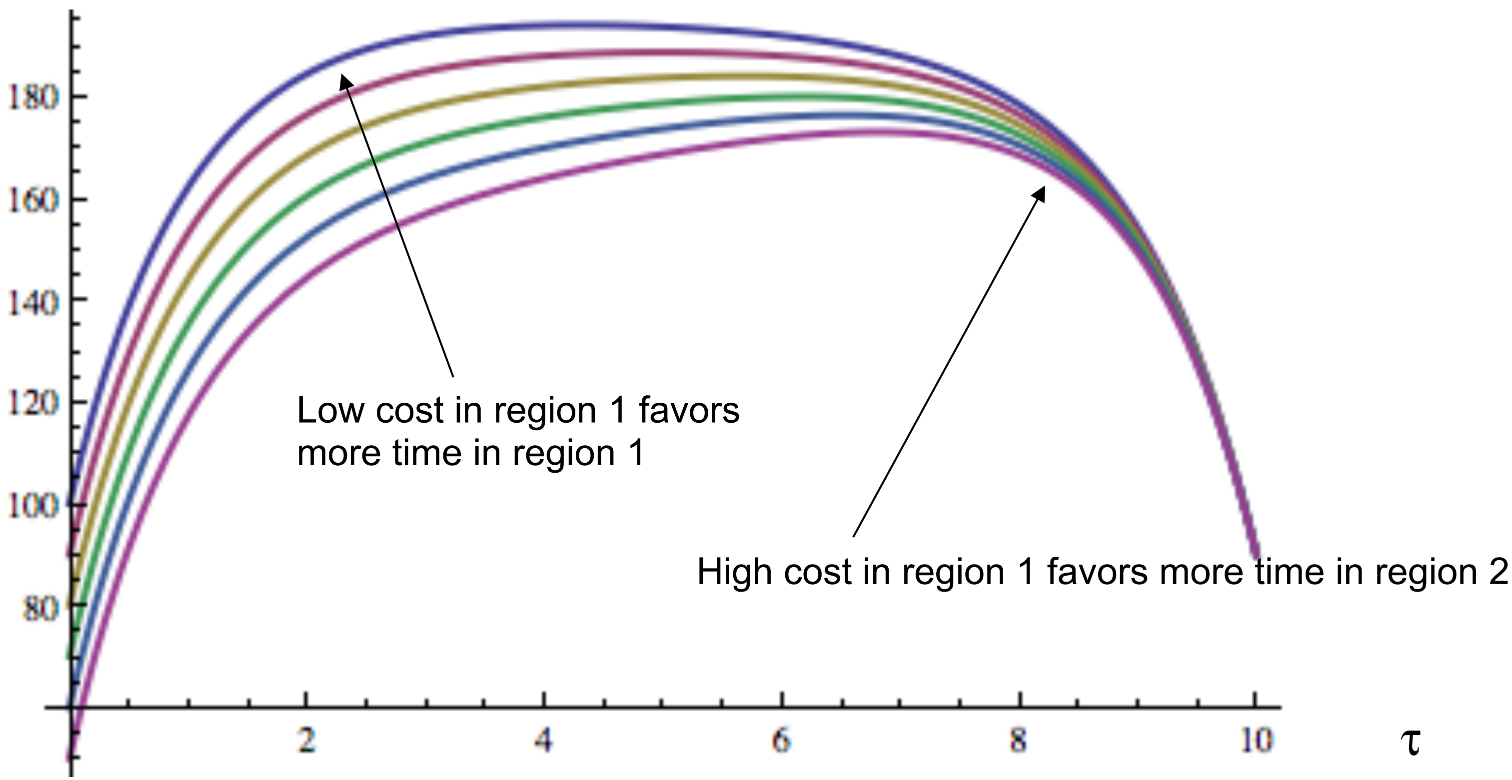




in the figure on the right atime penalty was imposed to move from region 1 to region 2 so the max net return shifted to spending more time in region 1

Higher cost in region 1 shows better to spend more time in region 2

Payoff versus time in region 2



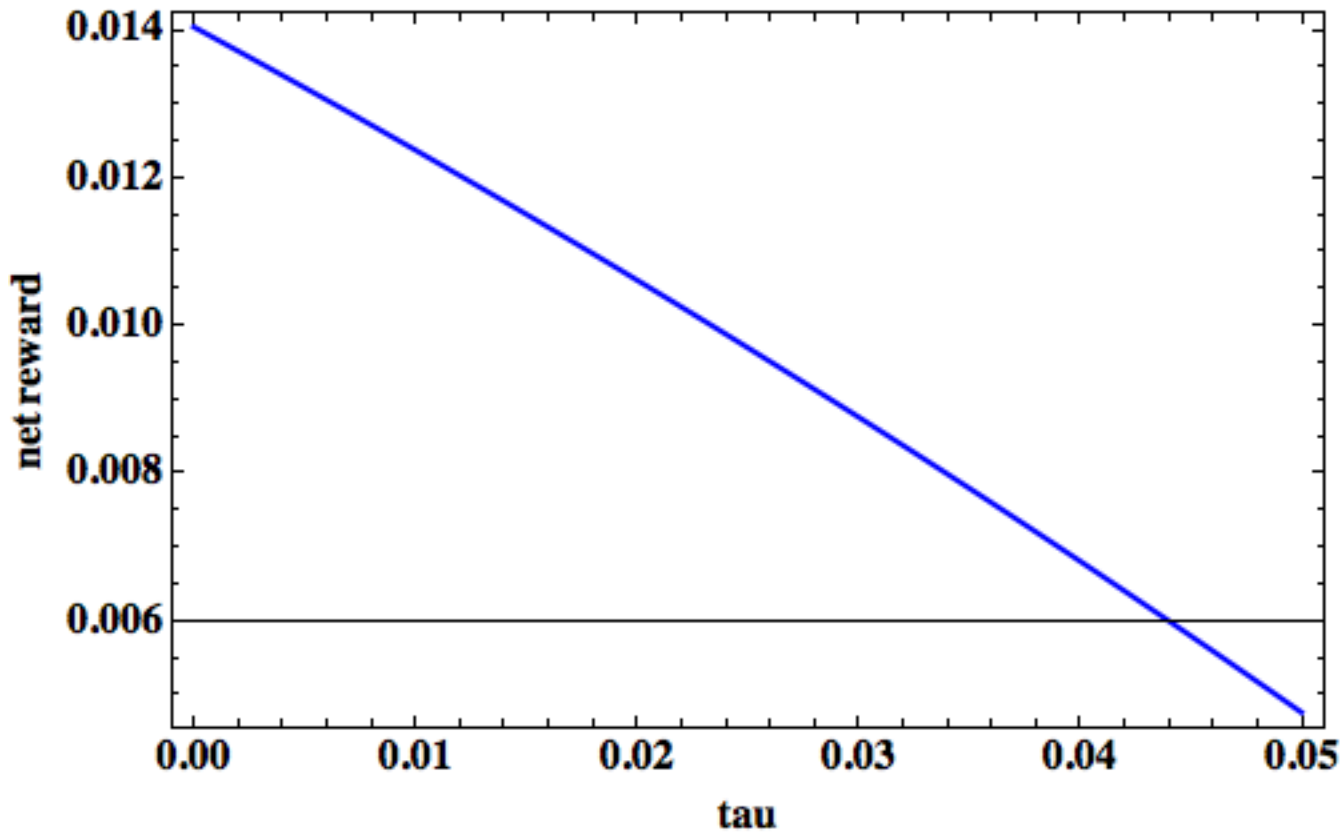
Cost in region 1 highest in bottom curve lowest in top curve

Hunter chooses between going to the plains (gain G_1)or to the river (gain G_2)

Search time T=.05

success rate in region 1 =2.0

success rate in region 2 =1.0



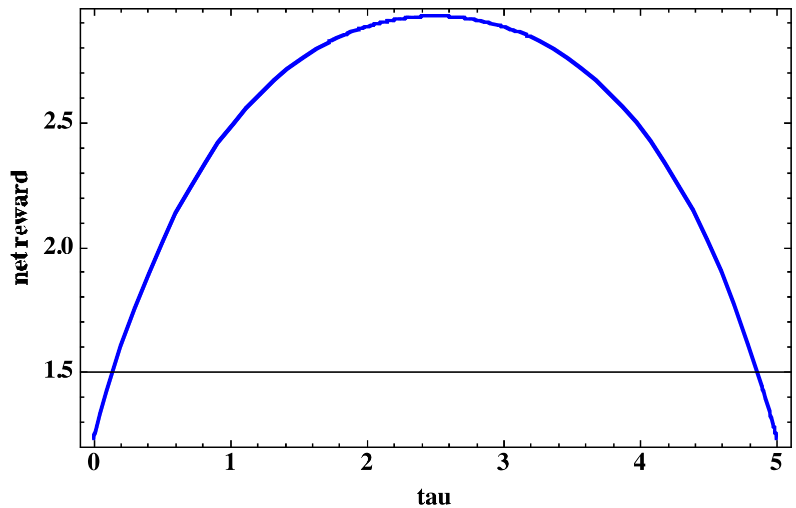
Better to stay in region 1 as search time is very short

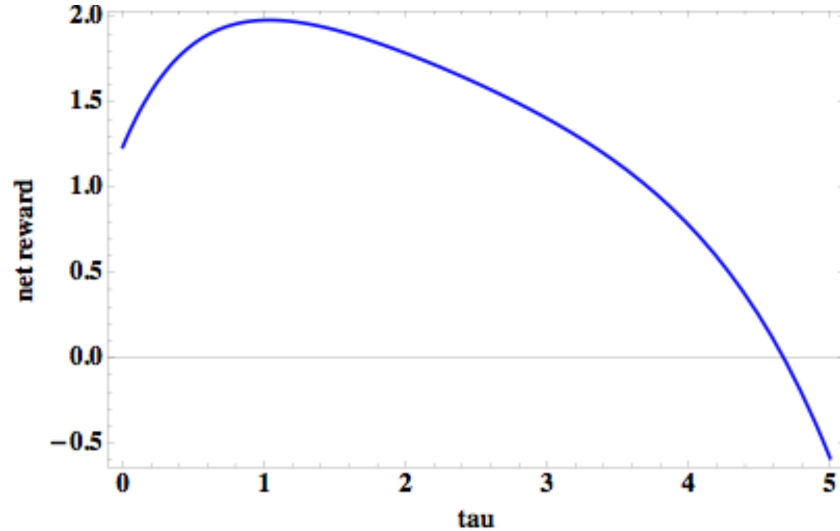
Generalizations:scare away the target in region 2

(the longer the search the more likely the target has left)

$$N(T,\tau) = G_1 p_1\left(1-\exp\left(-\lambda_1 (T-\tau)\right)\right) + G_2 p_2(\tau)\left(1-\exp\left(-\lambda_2 \tau\right)\right)$$

$$-C_1(T-\tau) - C_2\tau$$



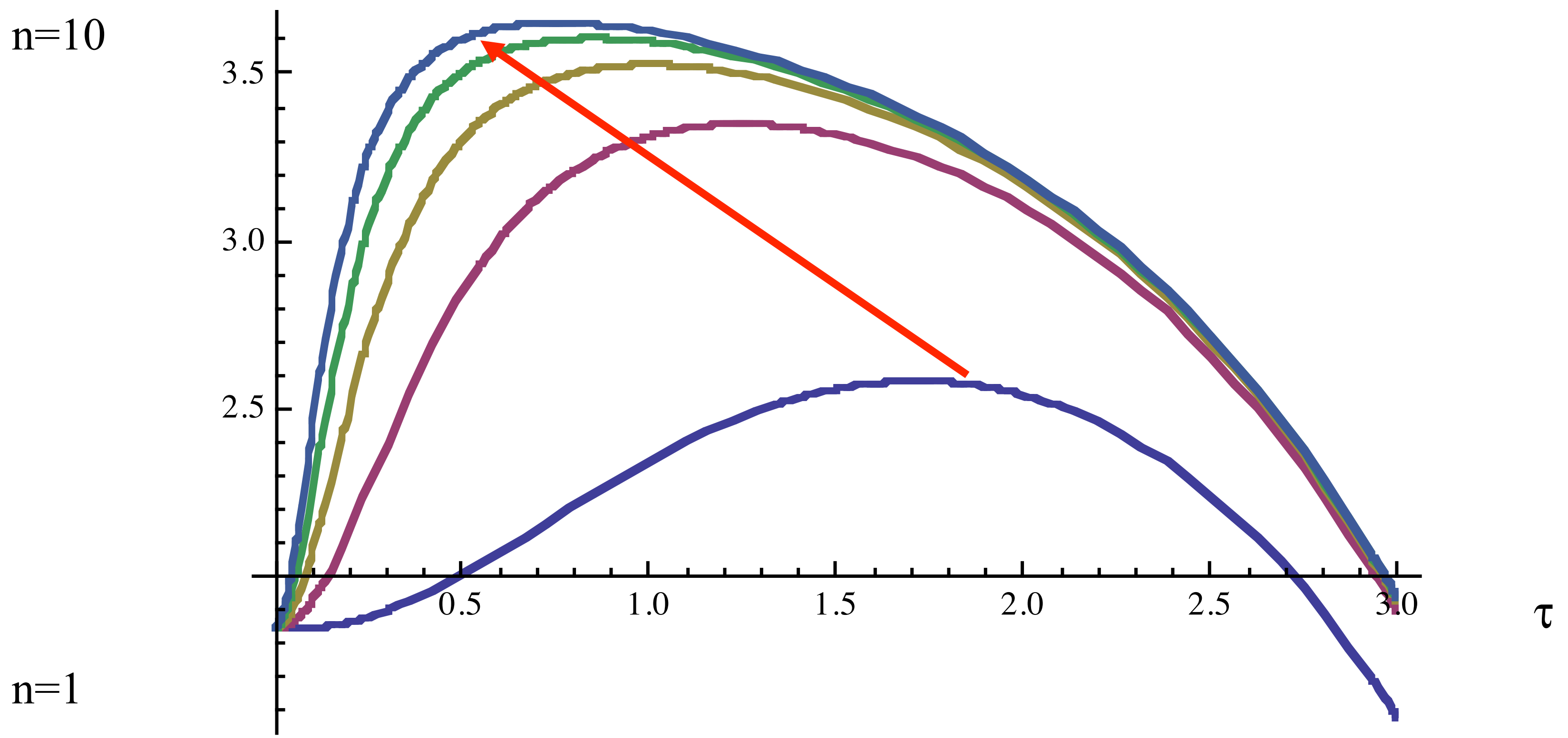


in the figure on the right, the probability p that the target is in region 2 decreases exponentially with time.

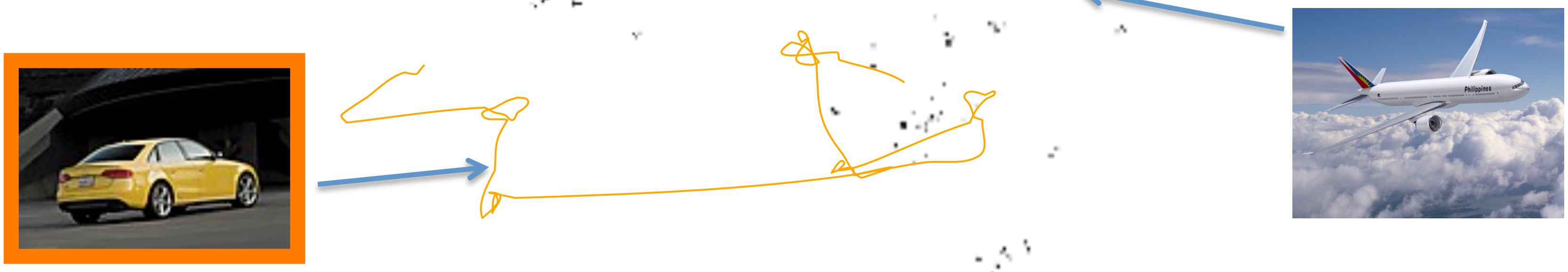
Small tau means search maximized by spending shorter time in region 2.

Slower than exponential search success rate in region 2

$$\lambda \exp(-\lambda t) \Rightarrow \frac{\tau_0}{\tau^2{}_0 + t^2 / n}$$



Levy Flights vs Drives

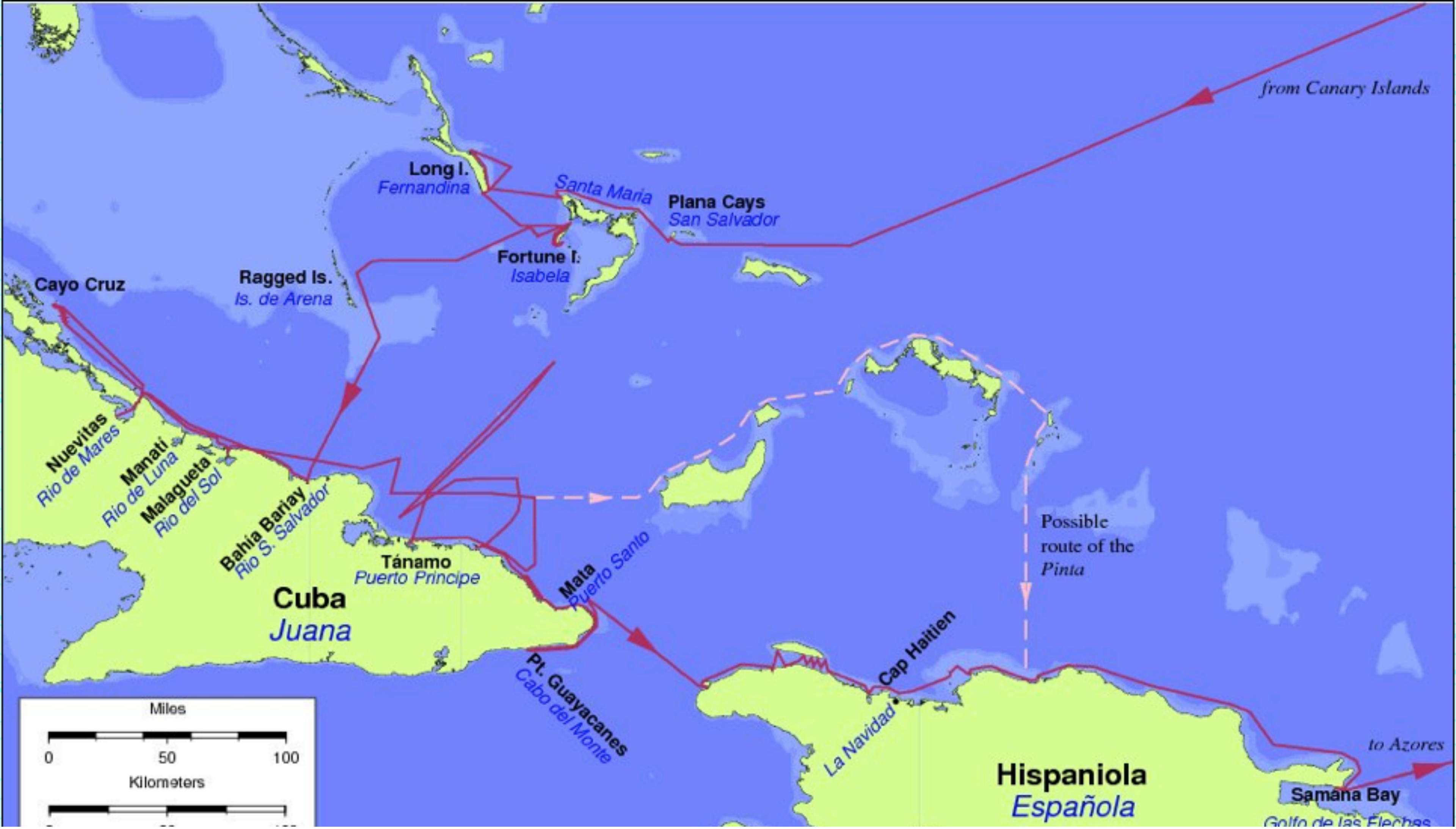


$$\Psi(\ell,t)=\psi(t|\ell)\,p(\ell)$$

$$\psi(t|\ell)=\delta\left(t-\frac{\ell}{V(\ell)}\right)$$

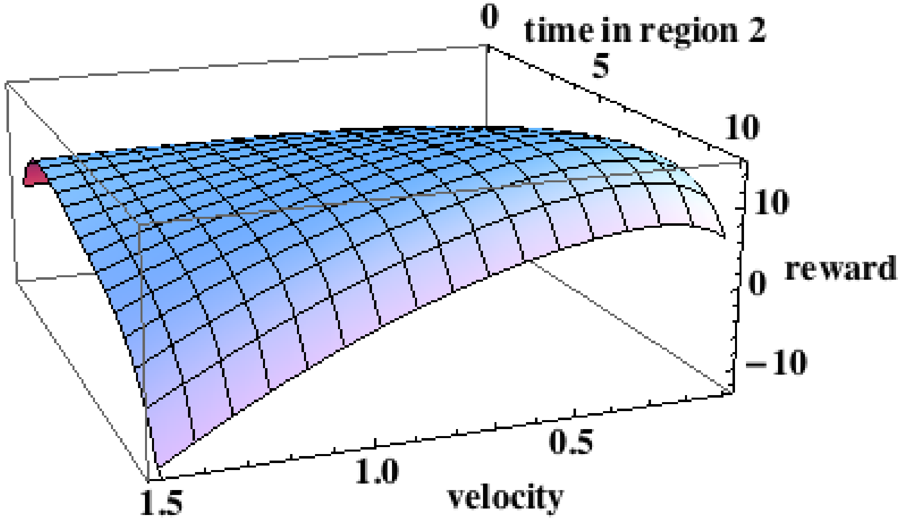
diffusion

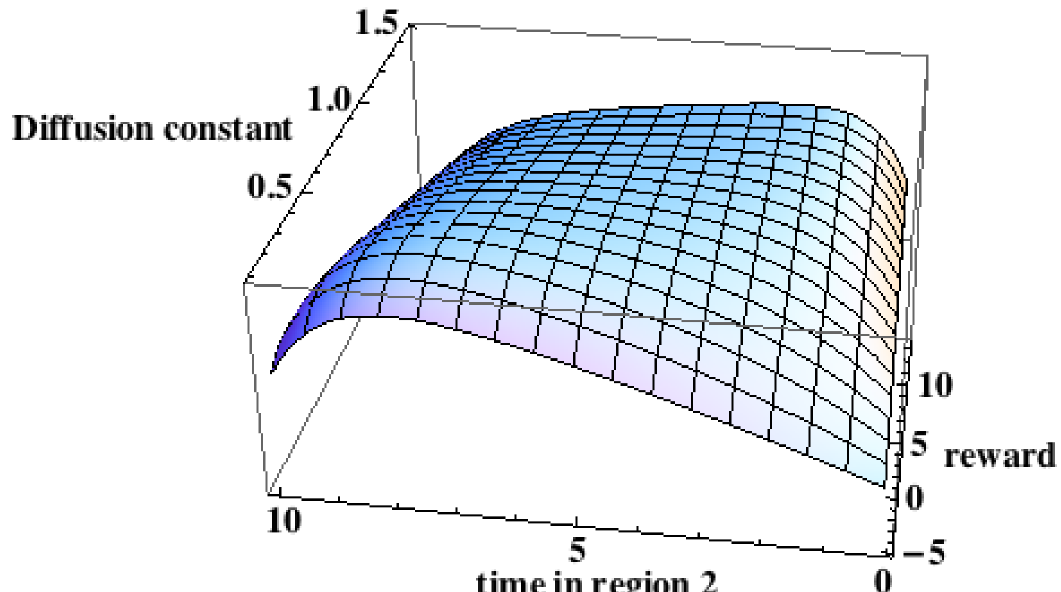
What if target sites looks like this, a fractal set?



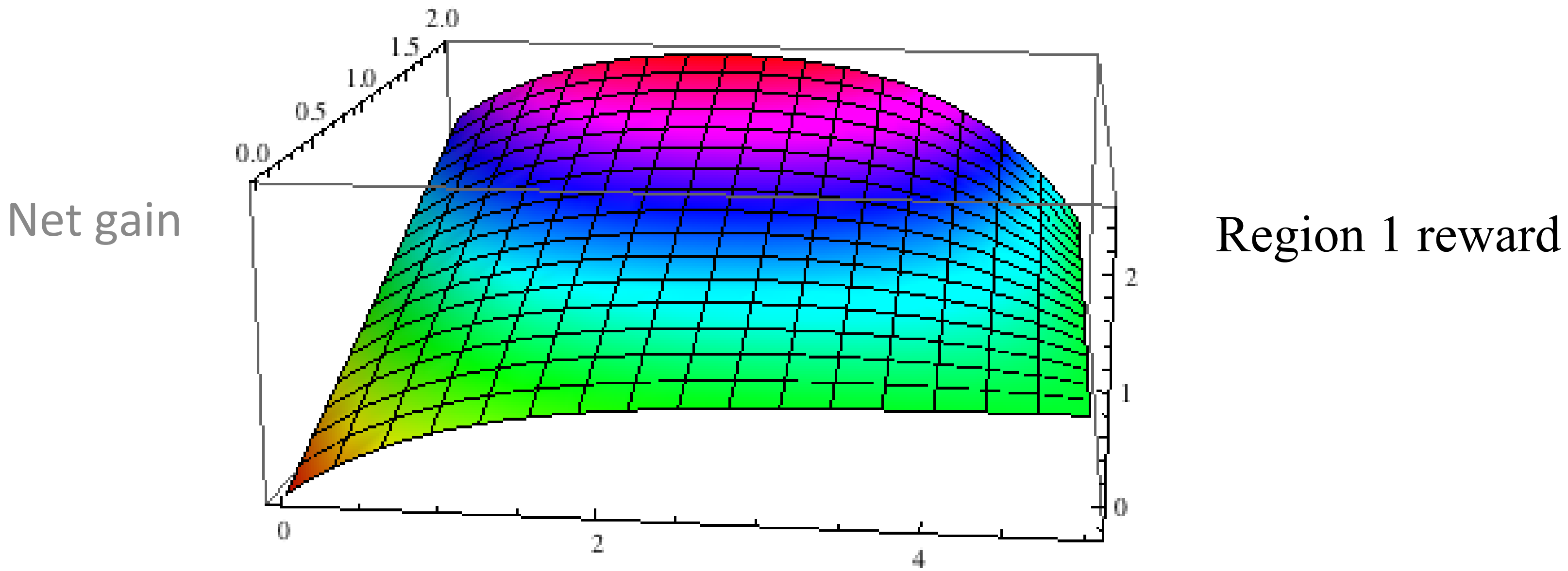
Sometimes a long flight pays off: C. Columbus (1492)

Success rate λ depending on velocity v or diffusion constant D





Gain as a function of time in region 2 and reward value in region 1

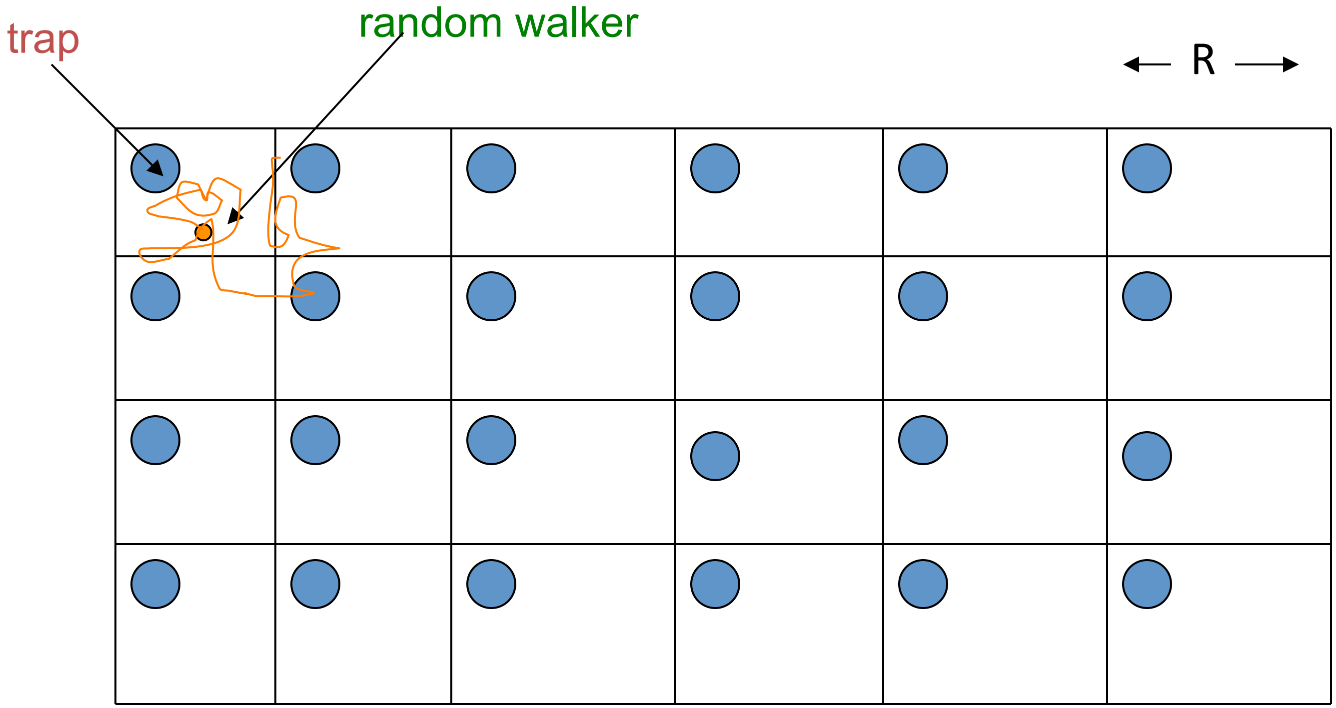


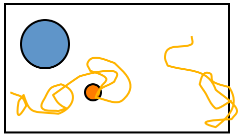
Brownian Random Walker Lifetime Probability

the trapping problem

$$\Phi(t) \propto \exp\left(-\frac{4Dt}{R^2}\right)$$

B. Ya Balagorov and V. G. Vaks, Sov. Phys. JETP **38** 968 (1974)





Equivalent to one cell with periodic boundary conditions

Mean number of contacts in time t

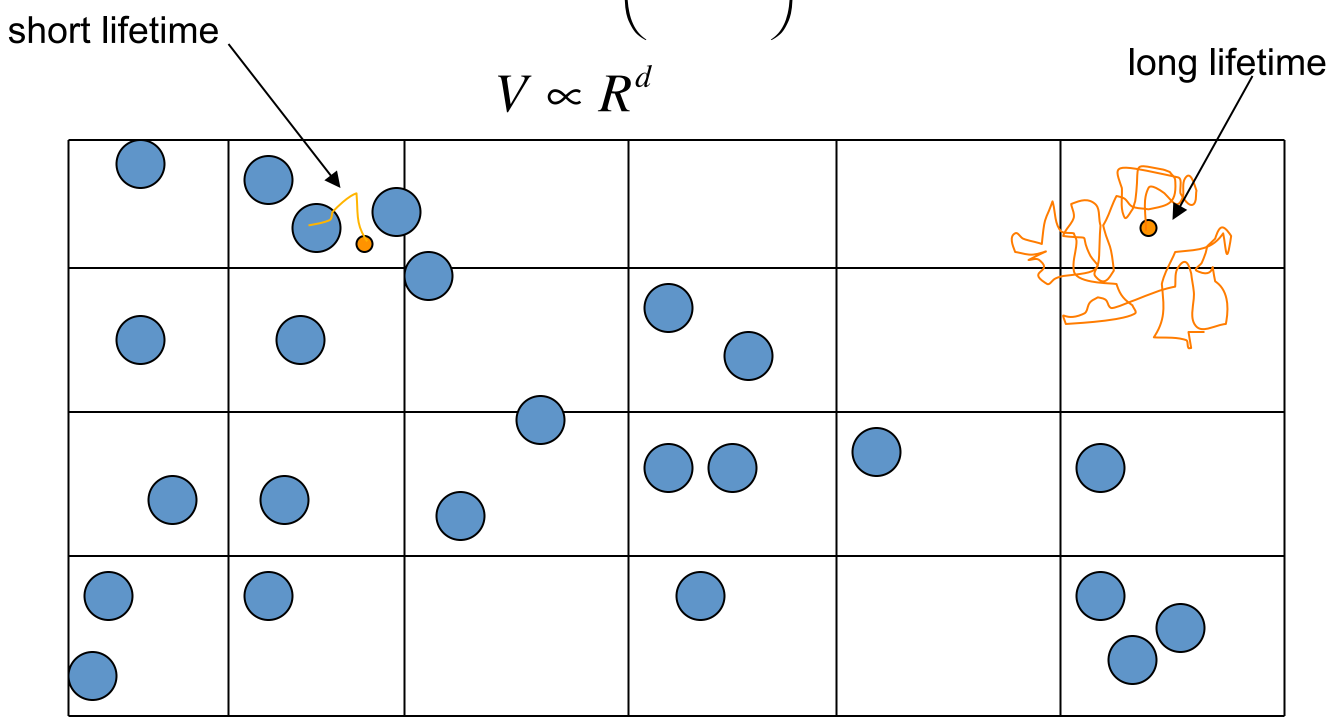
$$\langle n(t) \rangle \propto \frac{4D}{R^2} t$$

Donsker-Varadhan Brownian trapping problem

$$\lim_{t\to\infty}\Phi(t) \propto \lim_{t\to\infty}\int^{R} \exp\left(-\frac{Dt}{R^2}\right)\frac{1}{V_0}\exp\left(-\frac{V}{V_0}\right)dR$$

$$\propto \exp\left(-t^{\frac{d}{d+2}}\right), \qquad t >> 0$$

$$V \propto R^d$$

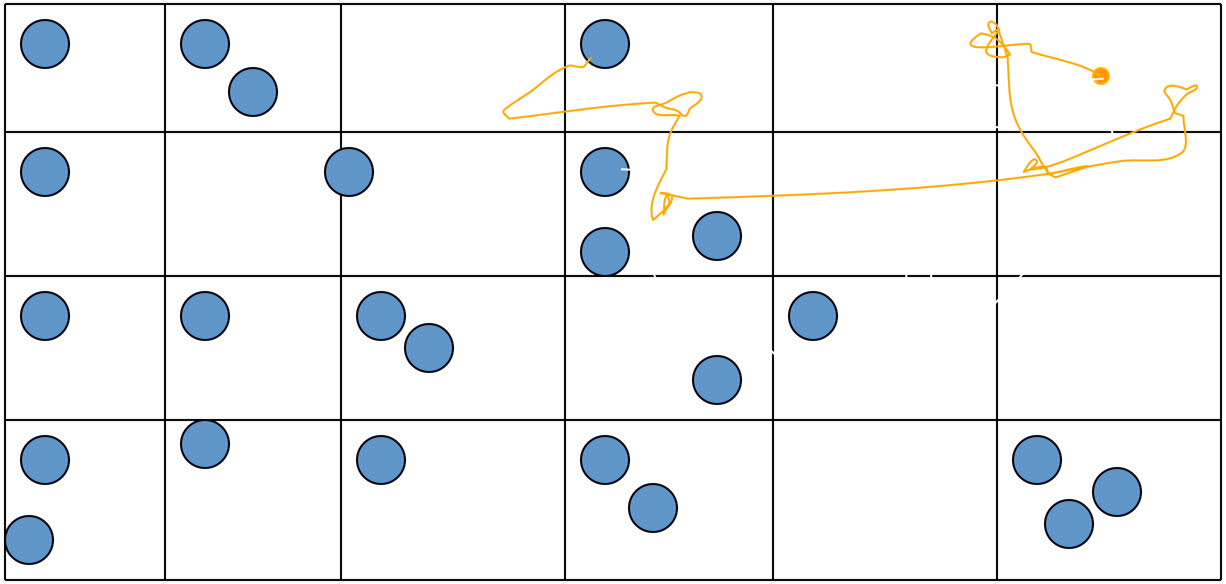


Poisson distribution of trap sites

Donsker-Varadhan Levy trapping problem

$$\lim_{t\to\infty}\Phi(t)\propto\lim_{t\to\infty}\int^{R}\exp\left(-\frac{Dt}{R}\right)\frac{1}{V_0}\exp\left(-\frac{V}{V_0}\right)dR\propto\exp\left(-t^{\frac{d}{d+1}}\right)$$

$$\text{shallower tail than }\exp\left(-t^{\frac{d}{d+2}}\right)$$



When should Brownian search give up and take a Levy flight?

Estimation of success p* decreases in time with continued failure

$$p^* = P(T\ present | search\ fails, T\ not\ found)$$

$$= \frac{P(T\ not\ found | T\ present) P(T\ present)}{P(T\ not\ found | T\ present) P(T\ present) + P(T\ not\ found | T\ not\ present) P(T\ not\ present)}$$

$$= \frac{(1-f)p}{(1-f)p + 1*(1-p)}$$

$$= \frac{p - fp}{1 - fp} < p$$

$$p*(t) = \frac{p\phi(t)}{p\phi(t) + (1-p)} = \frac{pe^{-\lambda t}}{pe^{-\lambda t} + (1-p)} < p$$