Classical and quantum non-linear dynamics in optomechanical systems

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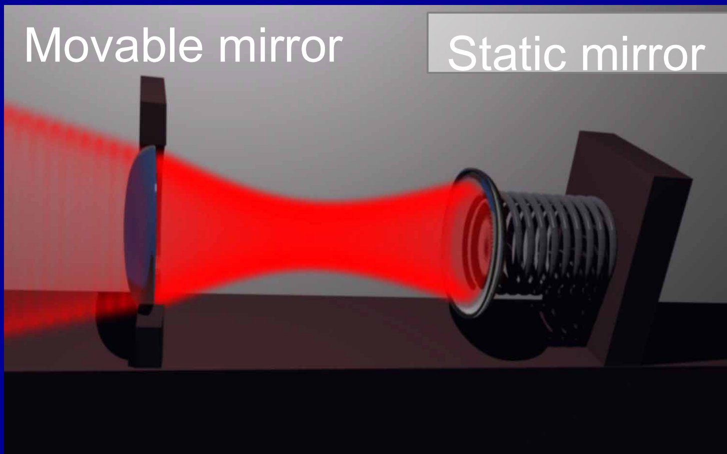
- Cavity/circuit optomechanics
- Non-linear mechanical resonators
- Non-linear cavities and self-sustained oscillations
- Unsolved problems

Cavity/circuit optomechanics



Kippenberg's Group website

Radiation pressure coupling

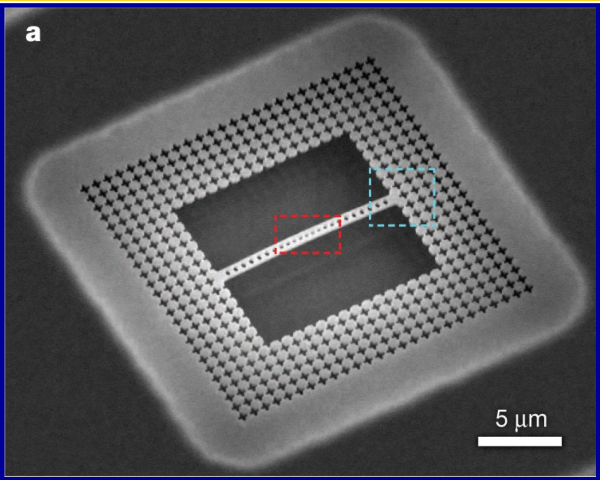
$\omega_{cav}(x)$

$$H = \hbar\omega_{cav}\hat{a}^{\dagger}\hat{a} + \hbar\omega_{m}\hat{b}^{\dagger}\hat{b} - \hbar g_0 \hat{a}^{\dagger}\hat{a}(b^{\dagger} + b)$$

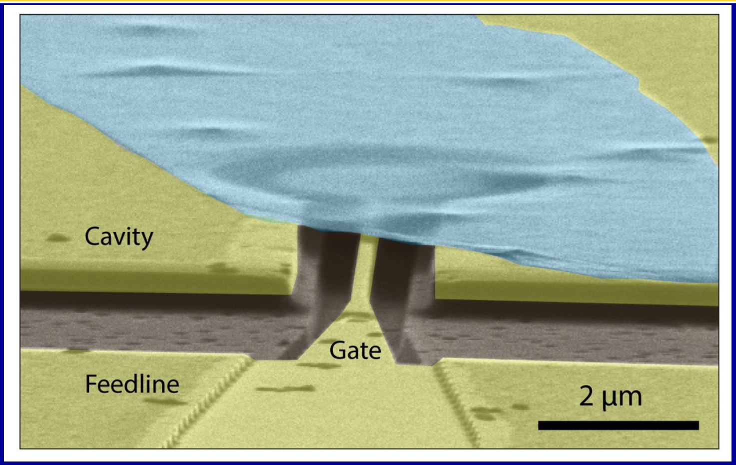
Cavity

Mechanical resonator

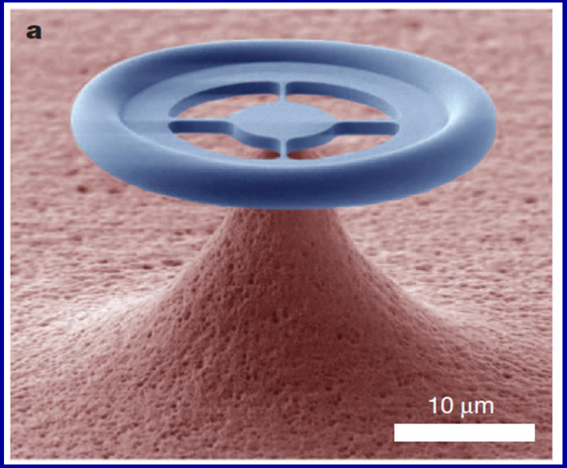
Cavity/circuit optomechanics



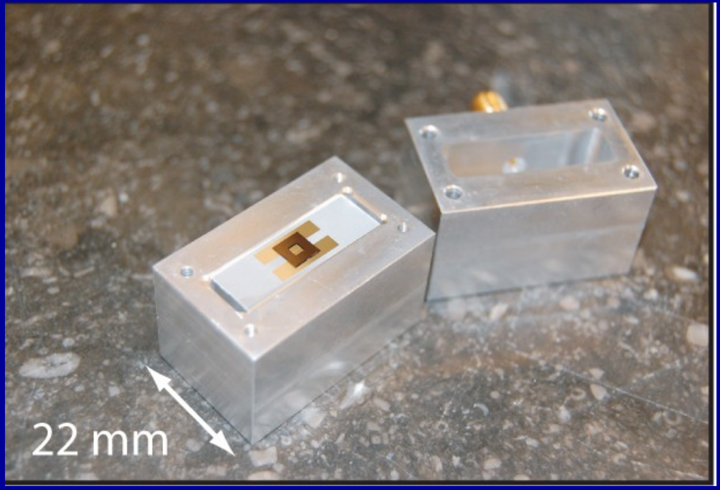
Chan et al, Nature **478**, 89 (2011)



Singh et al, Nature Nanotech. **9**, 820 (2014)



Verhagen et al, Nature **482**, 63 (2012)



Yuan et al, arXiv:1507.08898

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Coupling

$$H = \hbar\omega_{cav}\hat{a}^{\dagger}\hat{a} + \hbar\omega_m\hat{b}^{\dagger}\hat{b} - \hbar g_0\hat{a}^{\dagger}\hat{a}(b^{\dagger} + b)$$

Dissipation rate in the cavity

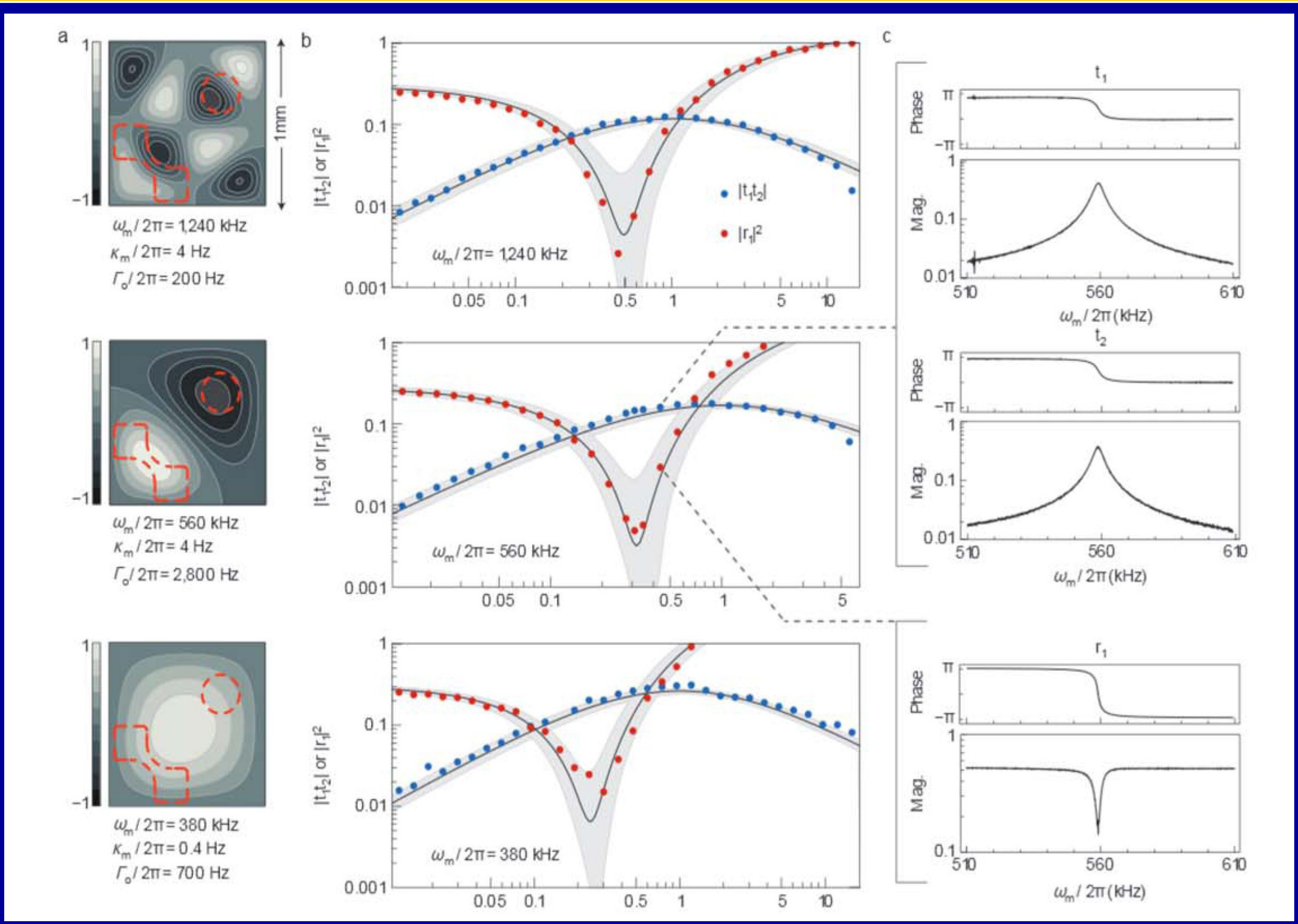
$$\kappa \ll \omega_m \ll \omega_{cav}$$

Where is g_0?

Weak coupling ← → Strong coupling

Driving and linearization: $g = g_0\sqrt{n_{cav}}$ $\quad \hbar g\left(\hat{a}^{\dagger}b + \hat{a}b^{\dagger}\right)$

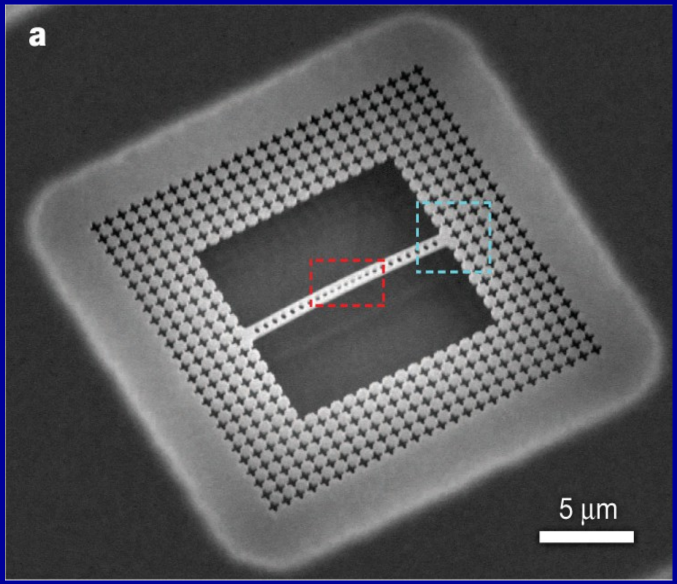
Conversion between optical and microwave light

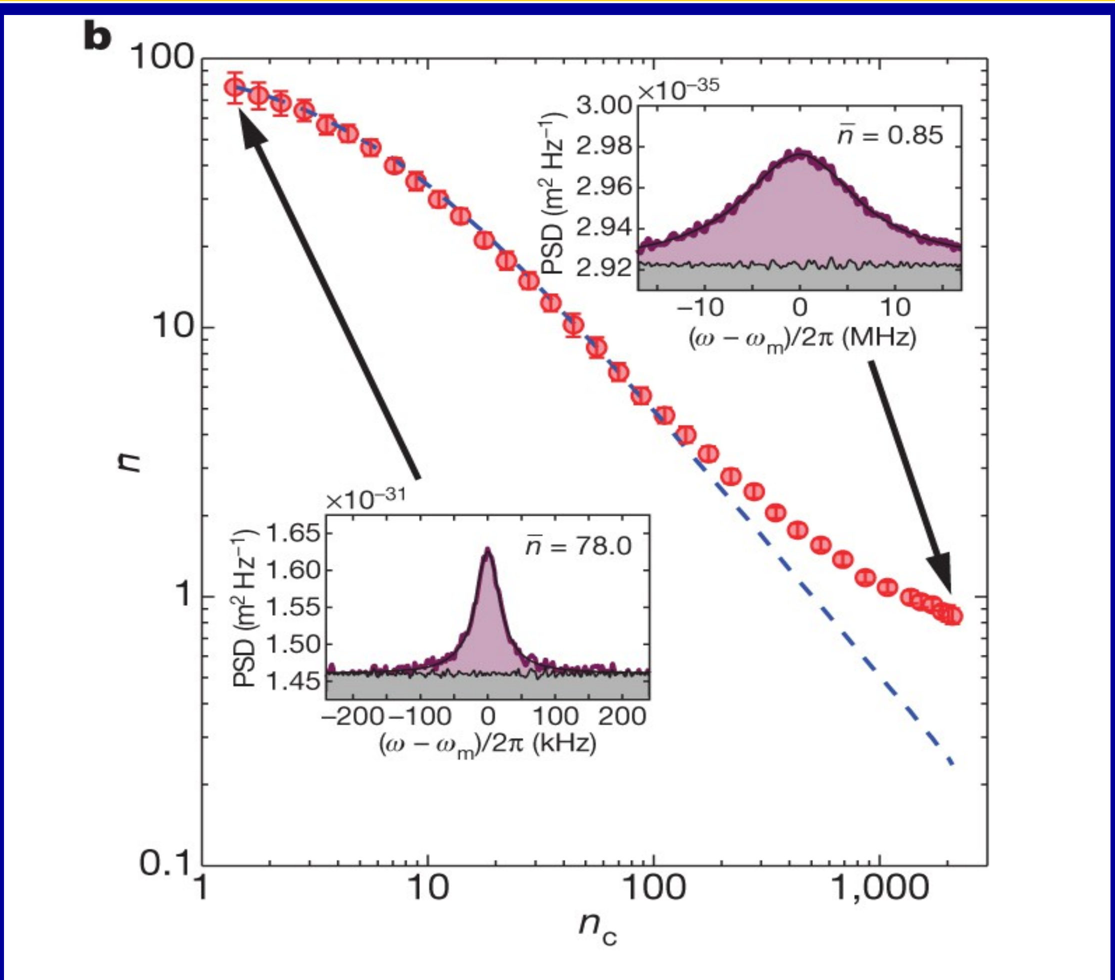


Andrews et al, Nature Physics **10**, 321 (2014)

Quantum signatures of mechanical motion

Chan et al, Nature **478**, 89 (2011)





Also: O'Connell et al, Nature **464**, 697 (2010); Teufel et al, Nature **475**, 359 (2011); Verhagen et al, Nature **482**, 63 (2012)

Non-linear optomechanics

Why is non-linearity important?

– Because it is there

– Modifies the behavior, especially at strong coupling

– Preparation of non-classical states of mechanical resonator

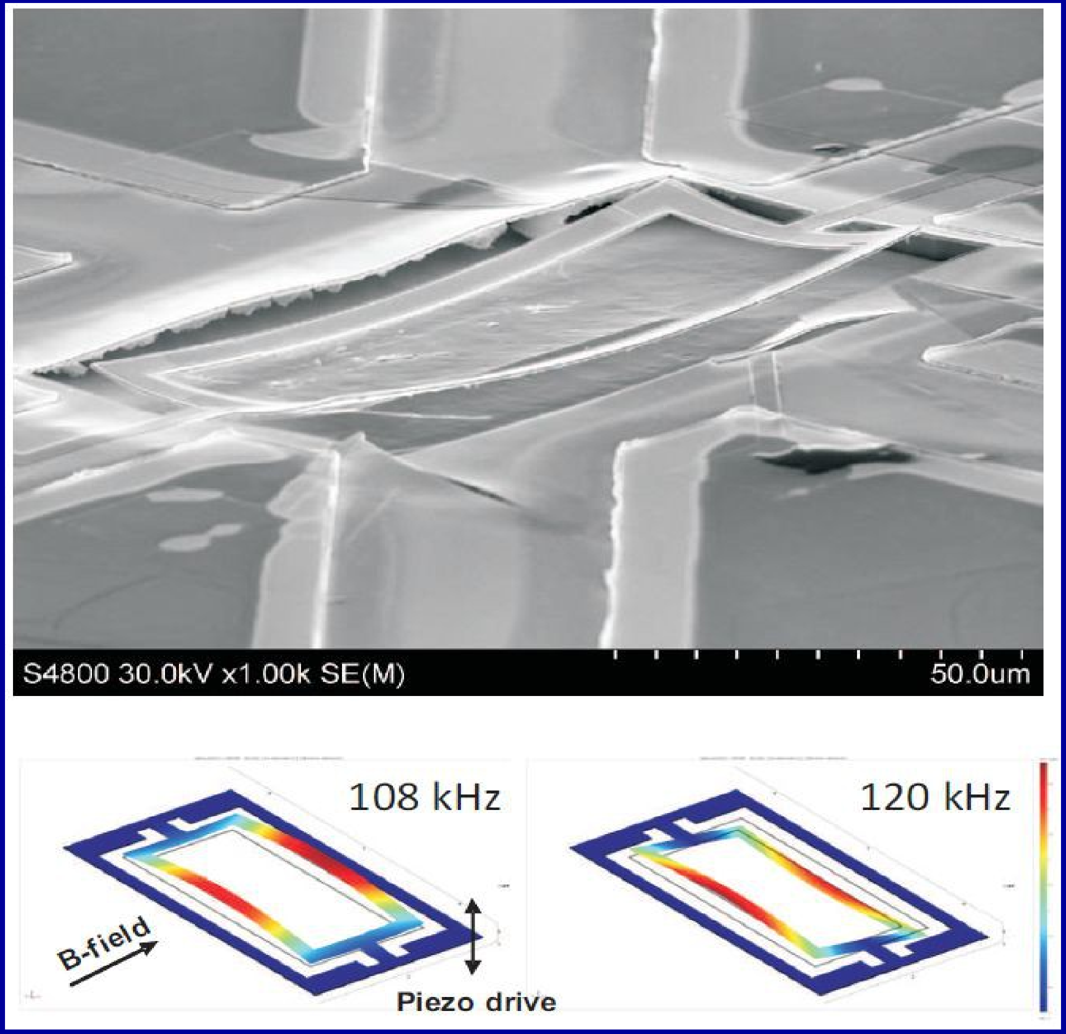
Non-linear optomechanics

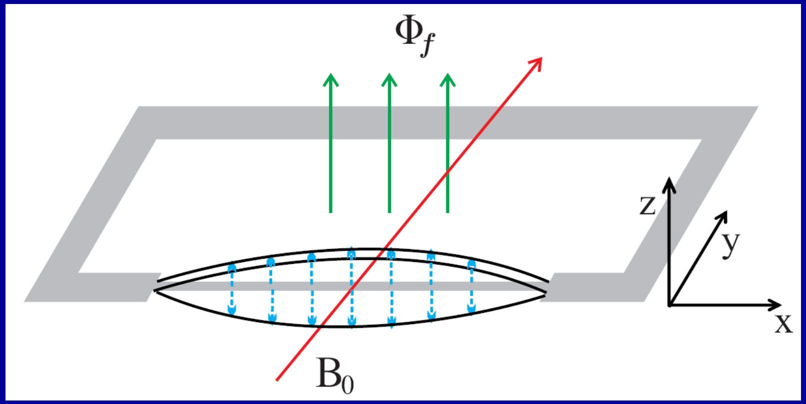
What is non-linear?

- Cavity: Microwave with a Josephson junction
- Mechanical resonator
- Radiation pressure interaction

Non-linear cavity

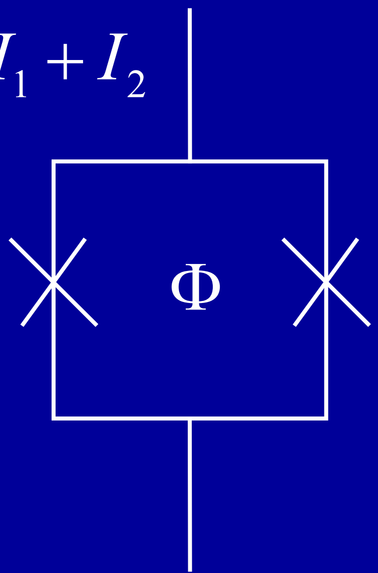
S. Etaki, F. Konschelle, H. Yamaguchi, YMB, H. S. J. van der Zant, Nature Comm. **4**, 1803 (2013)





dc SQUID

$$I = I_1 + I_2$$



$$\Phi = \Phi_0(\varphi_2 - \varphi_1)/(2\pi)$$

$$\Phi_0 \equiv \frac{\pi\hbar c}{e}$$

Josephson junctions:

$$I_{1,2} = I_0 \sin\varphi_{1,2}$$

Total current through the loop:

$$I = 2I_0 \cos\left(\frac{\pi\Phi}{\Phi_0}\right)\sin\varphi$$

Very sensitive detector of magnetic field

Coupling to mechanical motion:

$$E_c = -E_J \cos\left(\frac{\pi\Phi}{\Phi_0} + \alpha x\right) = g_1 x \sin\left(\frac{\pi\Phi}{\Phi_0}\right) + g_2 x^2 \cos\left(\frac{\pi\Phi}{\Phi_0}\right)$$

Non-linear cavity

S. Etaki, F. Konschelle, H. Yamaguchi, YMB, H. S. J. van der Zant,
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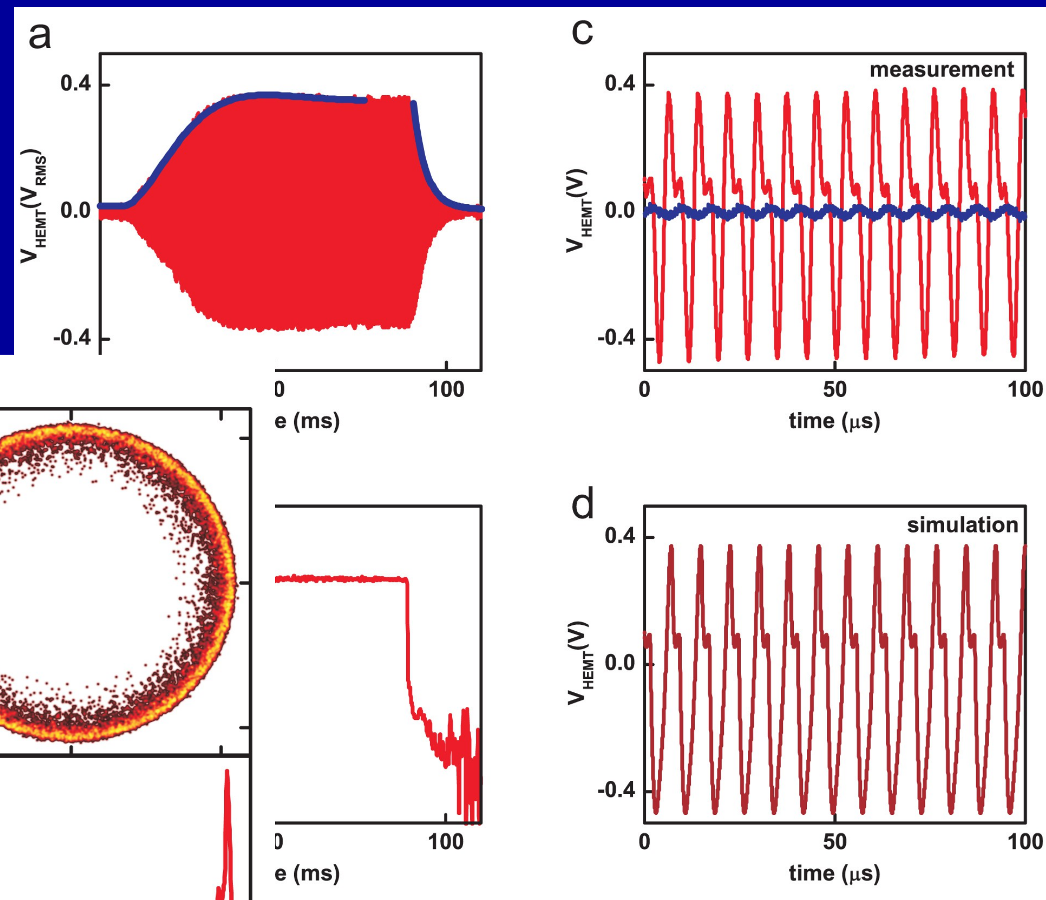
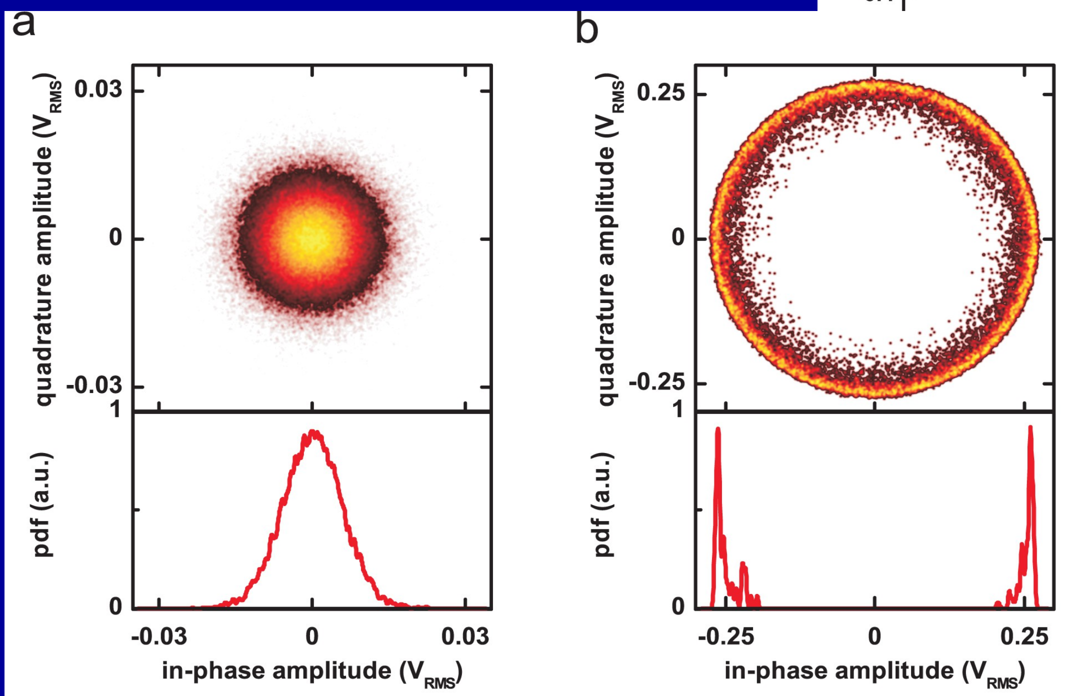
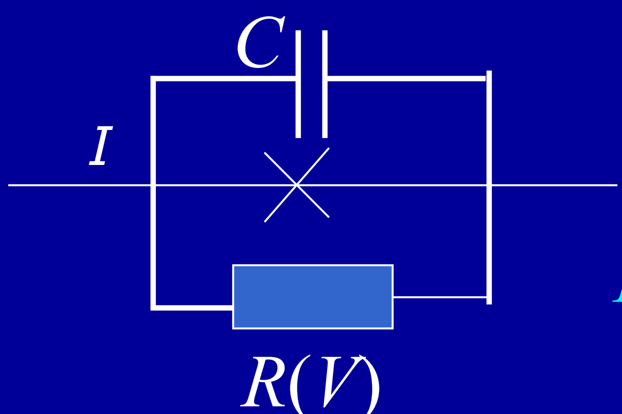




Self-sustained oscillations!

Non-linear cavity

- Josephson junctions: RSJ model
- Coupling: Lorentz force depends on the phases
- Generally: coupled non-linear (stochastic) differential equations
- Inertia term essential
- Physics: Lorentz force back-action renormalizes the quality factor of the mechanical resonator and makes it negative

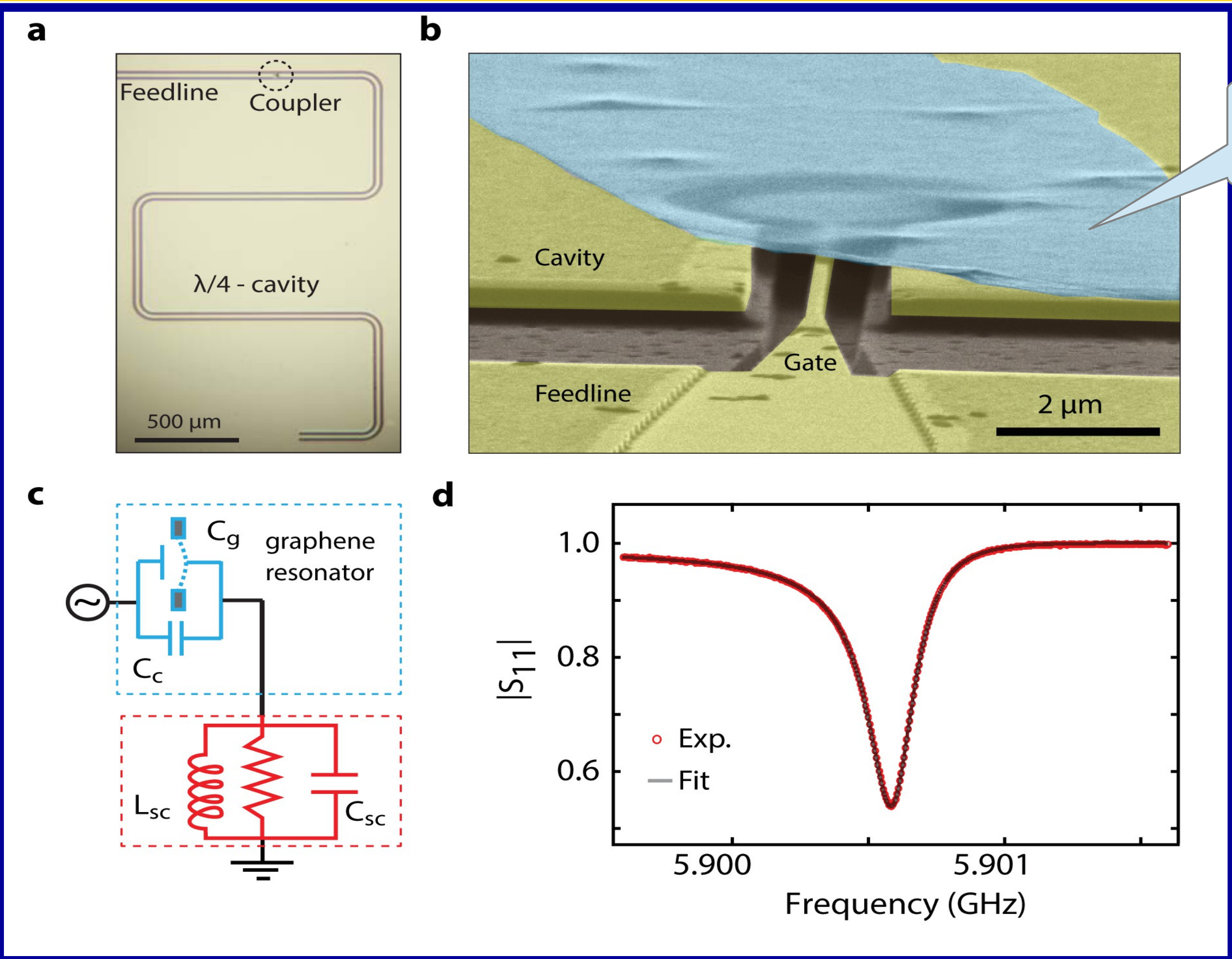


$$I_{1,2} = I_0 \sin \varphi_{1,2} + \frac{V_{1,2}}{R} + C\dot{V}_{1,2}, \; V_{1,2} = \frac{\Phi_0}{2\pi}\dot{\varphi}_{1,2}$$

$$M\ddot{x} + \frac{M\omega}{Q}\dot{x} + M\omega^2 x = F\cos\omega t + aBlI_1[x,\dot{x}]$$

S. Etaki, F. Konschelle, H. Yamaguchi, YMB, H. S. J. van der Zant, Nature Comm. **4**, 1803 (2013)

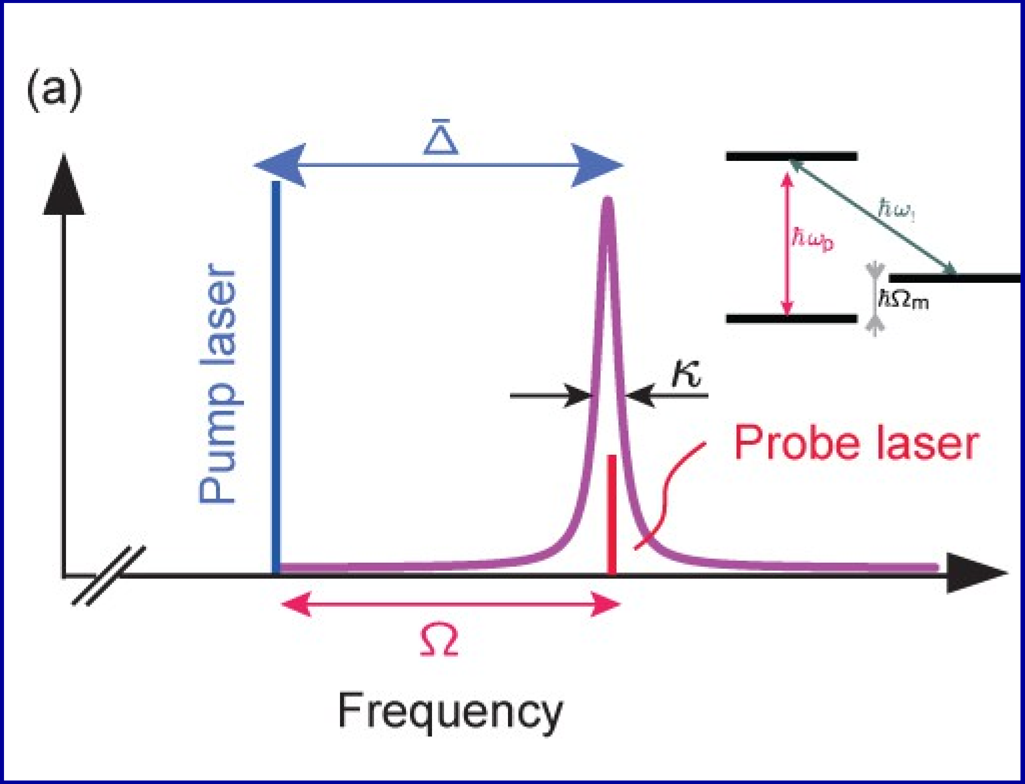
Non-linear mechanical resonator



Graphene membrane

Singh et al, Nature Nanotech. **9**, 820 (2014)

Optomechanically induced transparency

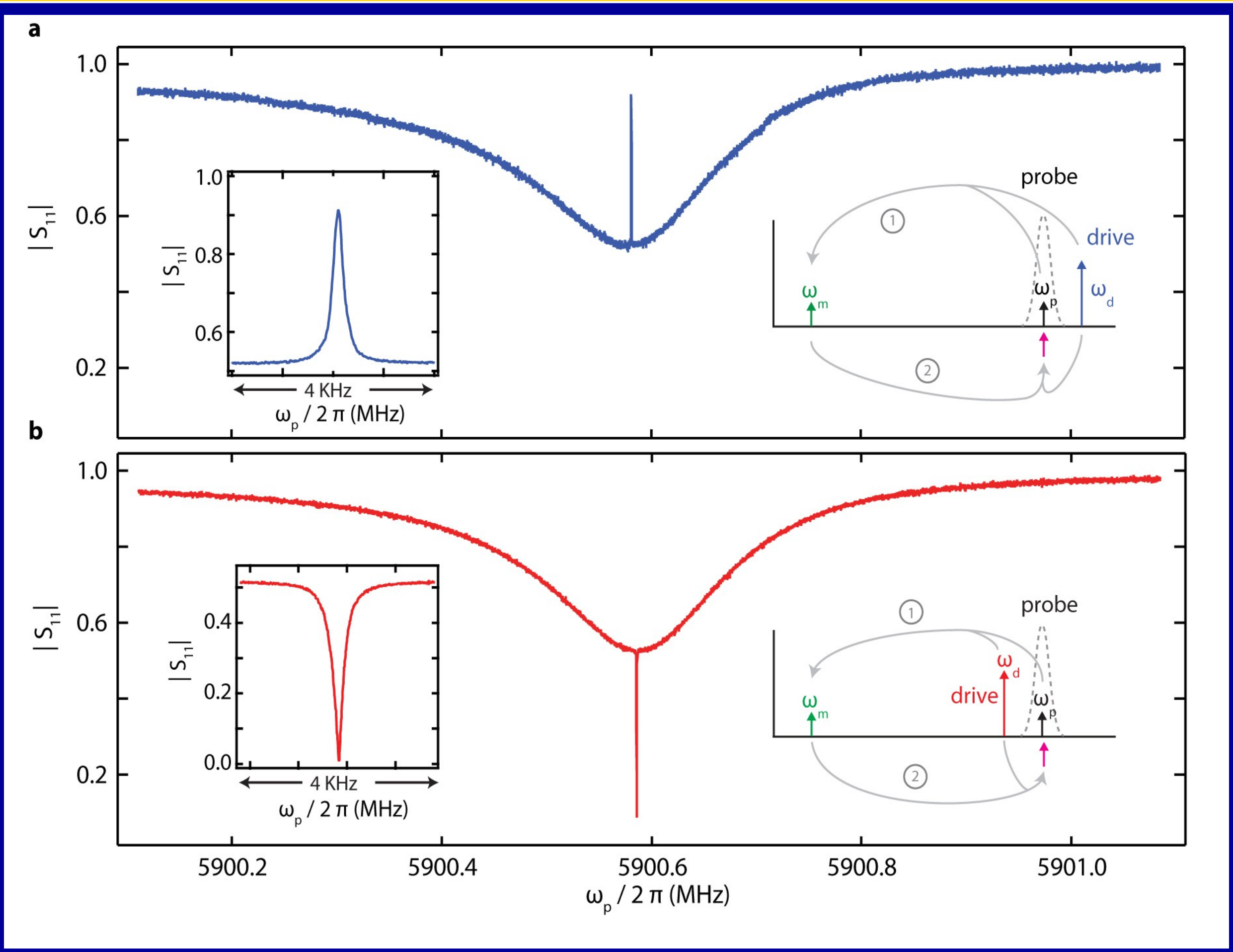


From: Aspelmeyer, Kippenberg, and Marquardt Rev. Mod. Phys. **86**, 1391 (2014)

Cavity is strongly red-driven at $\omega_{cav} - \omega_m$ (red-detuned)

Probe laser measures the transmission around the cavity resonance

Optomechanically induced transparency



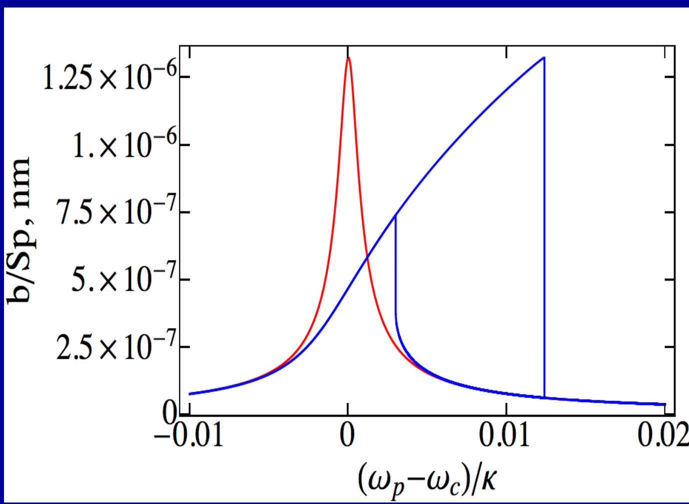
Singh et al, Nature Nanotech. **9**, 820 (2014)

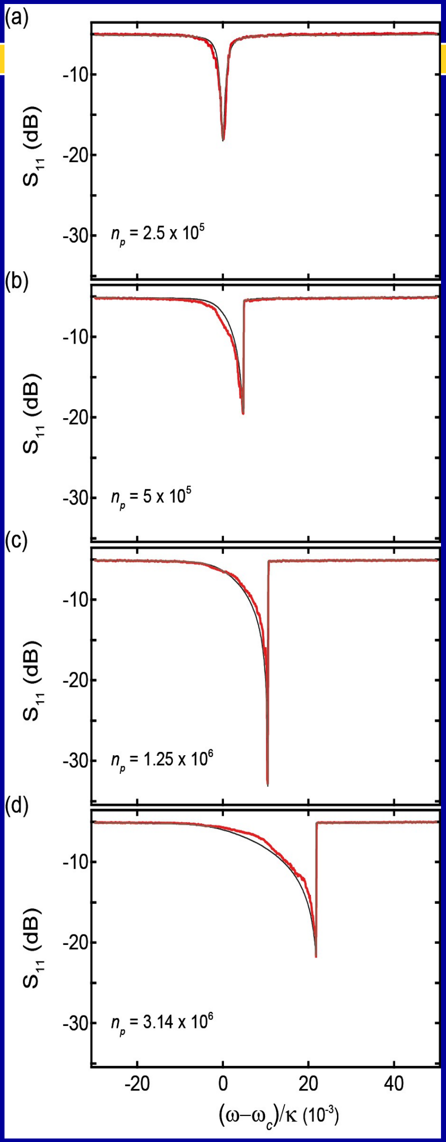
First observation:
S. Weis et al, Science 330, 1520 (2010)

Constructive interference between the two probes results in OMIT

Non-linear OMIT

Duffing oscillator:





Does the shape of the transmission maximum repeat the response of the driven Duffing ocsillator?

Not always, the phase dynamics is important.

V. Singh et al, arXiv:1508.04298

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Input-output relations

Langevin equations for the creation/annihilation operators:

$$\frac{d\hat{a}}{dt} = \left(i\Delta - \frac{\kappa}{2} \right)\hat{a} - ig\hat{x}\hat{a} + \sqrt{\kappa_{ext}}\, s_{in} + \sqrt{(1-\eta_c)\kappa}\,\delta\hat{s}_{vac}(t)$$

Detuning and dissipation in the cavity

Input signal

Quantum noise

$$\frac{d\hat{x}}{dt} = \frac{\hat{p}}{m}$$

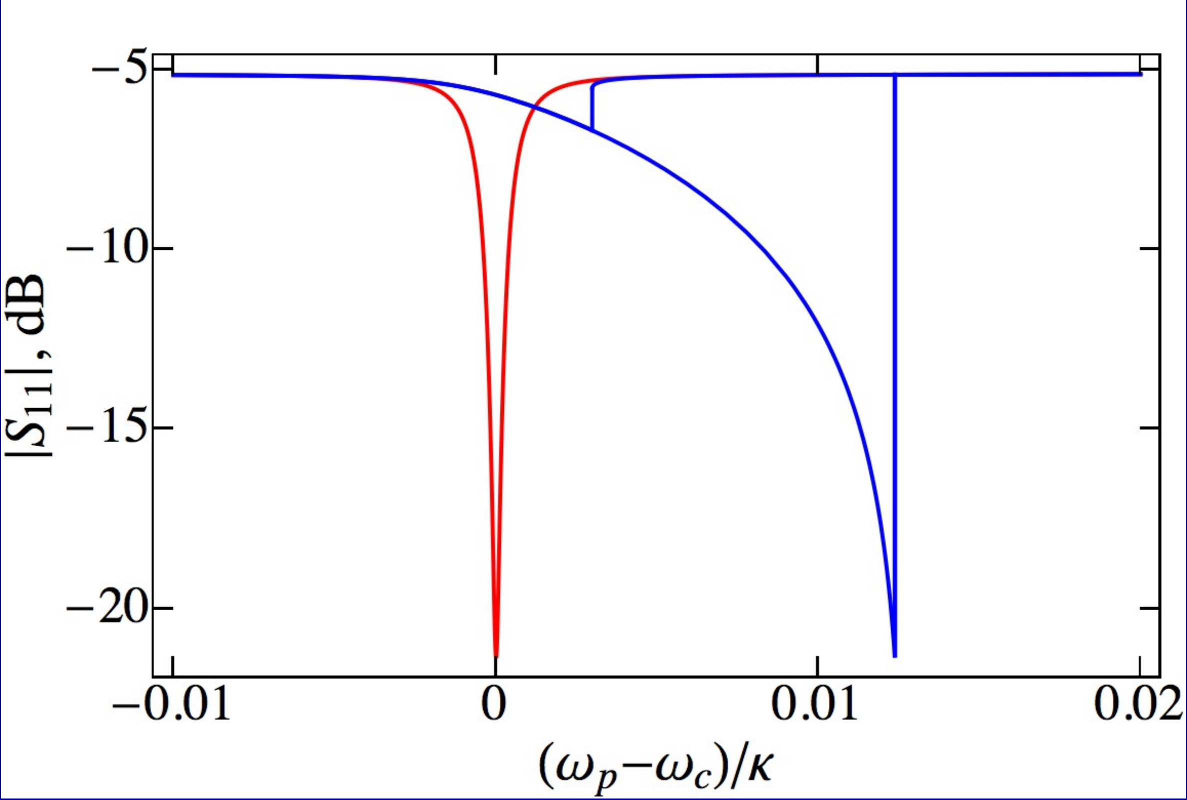
$$\frac{d\hat{p}}{dt} = -m\omega_m^{\,2}\hat{x} - \alpha\hat{x}^3 + \hbar g\hat{a}^\dagger\hat{a} - \Gamma_m\hat{p} + \delta F_{th}(t)$$

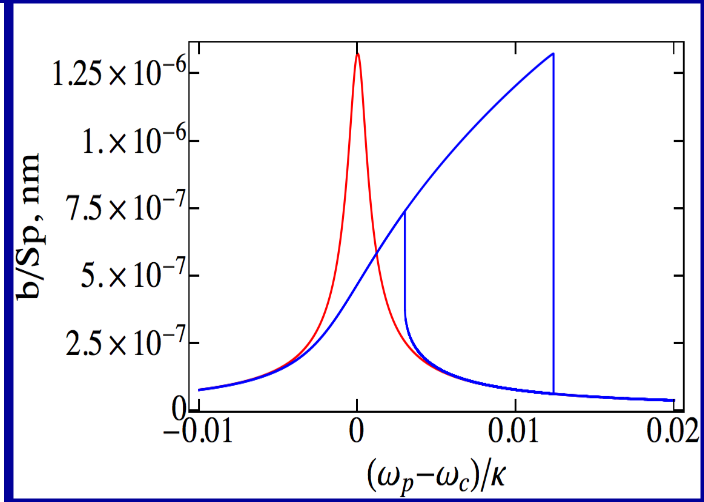
Coupling

Mechanical dissipation

Thermal noise

Non-linear OMIA





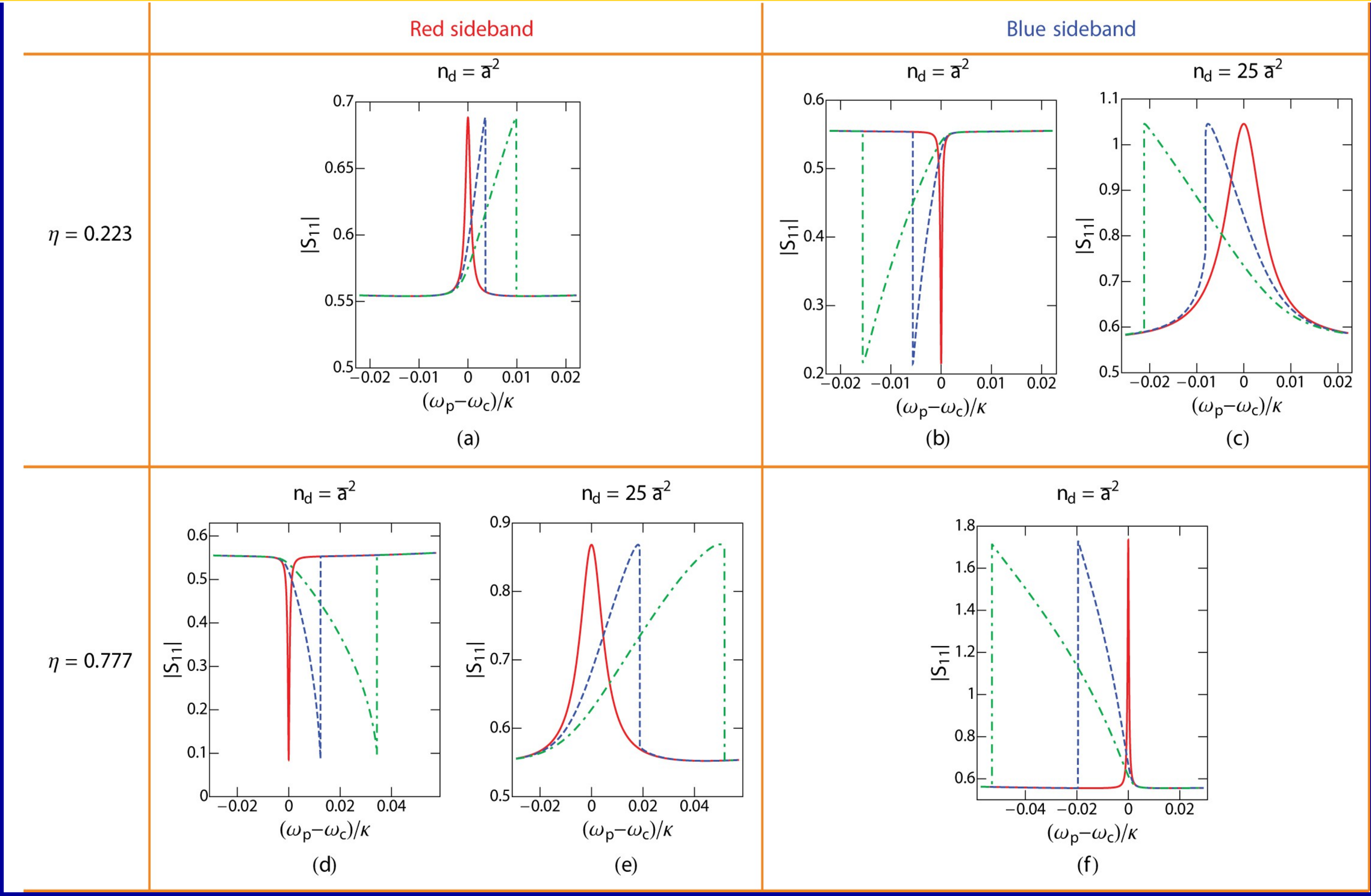
Red-detuned drive

Overcoupled cavity

$$\eta = \frac{\kappa}{\kappa + \kappa_{ext}} > \frac{1}{2}$$

O. Shevchuk et al, arXiv:1507.06851

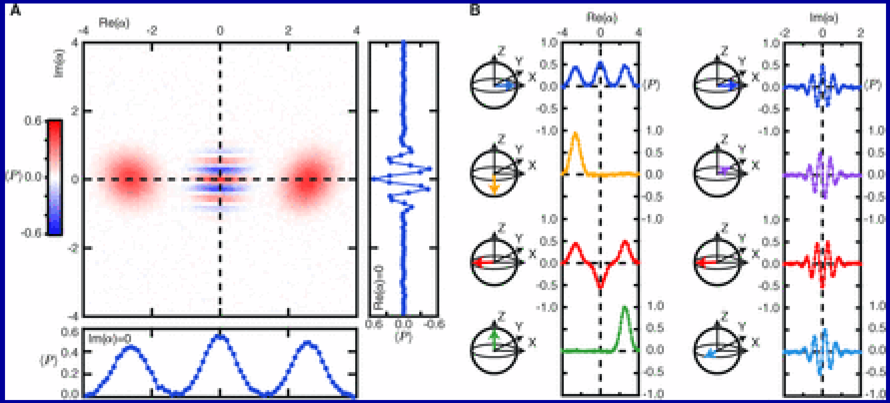
Non-linear OMIA



O. Shevchuk et al, arXiv:1507.06851

Unsolved problems: Creation of non-classical mechanical states

Schoelkopf group, Yale: created cat states in a cavity



Vlastakis et al, Science **342**, 607 (2013)

How can we make non-classical mechanical states in the cavity architecture?

- Transfer from the cavity
- Make the cavity or the resonator non-linear (Yurke, Stoler)

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Unsolved problems: Strong coupling

$$H = \hbar\omega_{cav}\hat{a}^{\dagger}\hat{a} + \hbar\omega_{m}\hat{b}^{\dagger}\hat{b} - \hbar g_{0}\hat{a}^{\dagger}\hat{a}(b^{\dagger} + b) \quad / \hbar g\left(\hat{a}^{\dagger}b + \hat{a}b^{\dagger}\right)$$

- Use non-linearized interaction pressure: need single-photon strong coupling (Nunnenkamp, Borkje, Girvin) $g_o \gg \kappa$
- For cooling the resonator to the ground state: Cooperativity $C = \frac{g_0^2}{\kappa\Gamma}$
- Photon blockade: a non-linear resonator; entangles phonons and photons (Didier, Pugnetti, YMB, Fazio)

What else?

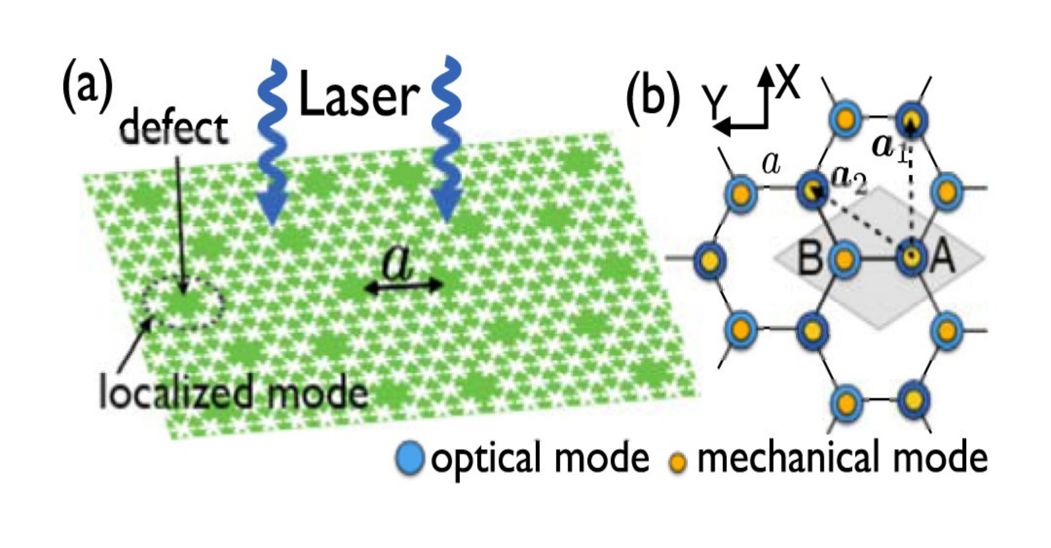
Unsolved problems: Strong coupling

- What happens at very strong coupling? $g_o \gg \omega_m$
- Role of non-linearity at strong coupling? Beyond Kerr $K\hat{a}^{\dagger}\hat{a}^{\dagger}\hat{a}\hat{a}$
- Role of quantum noise?
- Novel physical phenomena?

Unsolved problems: Optomechanical arrays

Next complexity level?

Example: M. Schmidt, V. Peano, F. Marquardt, arXiv:1410.8483: optically tunable Dirac-type structire



- What happens to dissipation?
- Non-linear effects? Chaos?
- Quantum effects?

Conclusions

- Non-linear effects are important
- The are even more important if coupling is strong
- There are many things ahead of us

Classical and quantum non-linear dynamics in optomechanical systems

Delft-Theory

Olga Shevchuk
João Machado
François Konschelle

SNS Pisa

Nicolas Didier
Stefano Pugnetti
Rosario Fazio

NTT

Imran Mahboob
Koji Onomitsu
Hiroshi Yamaguchi

Delft-Experiment

Vibhor Singh
Sal Bosman
Ben Schneider
Mingyun Yuan
Gary Steele

Samir Etaki
Menno Poot
Herre van der Zant

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