On the WEAK MEASUREMENT of the ELECTRICAL THz CURRENT: a NEW SOURCE of NOISE?

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2. Novel approach to model the measurement of THz current

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1 - Open problem

How do we model the measurement of the high frequency current?

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$\Rightarrow$

Year	2015	2020	2025
Cutoff Frequency (GHz)	620	1137	2062

International Technology Roadmap for Semiconductors (2011)

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⇒

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⇒ Total Current = Particle + Displacement Current

$$I_d(t) = \int_{S_i} \epsilon(\mathbf{r}) \frac{d\mathbf{E}(\mathbf{r},t)}{dt} \cdot d\mathbf{s}$$

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$$I_d(t) = \int_{S_i} \epsilon(\mathbf{r}) \frac{d\mathbf{E}(\mathbf{r},t)}{dt} \cdot d\mathbf{s}$$

$\Rightarrow$ Continuous (or very High Frequency) Measurement

$\Downarrow$

Inclusion of the Back-Action (Disturbance of quantum system)

1 - Open problem

How is Measurement modelled in Quantum Mechanics?

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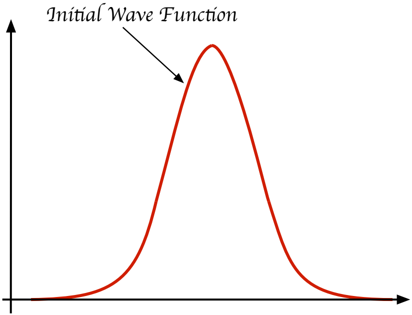
Strong



J. Von Neumann: PUP (1955)

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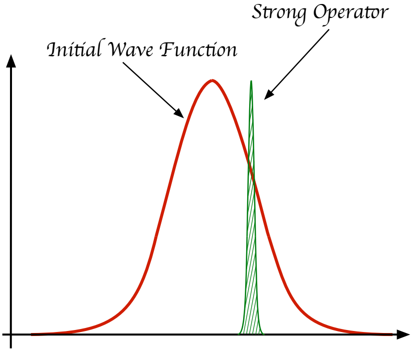

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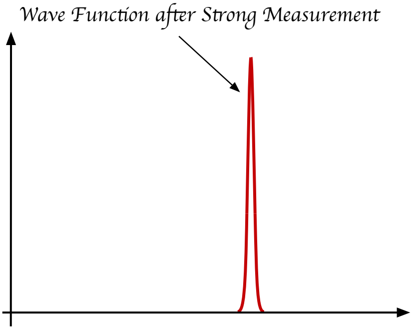

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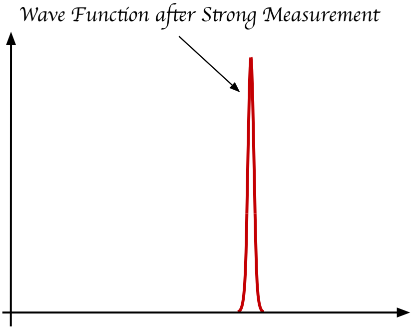

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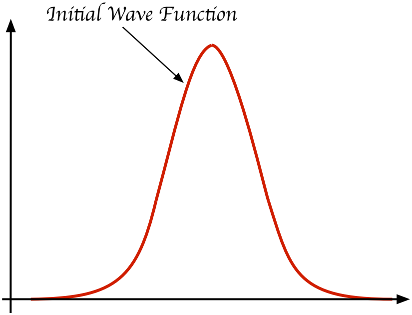

Y. Aharonov, D. Z. Albert, and L. Vaidman: PRL (1988)

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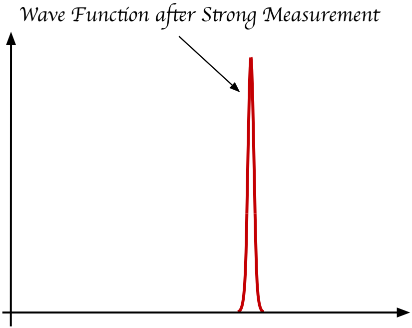

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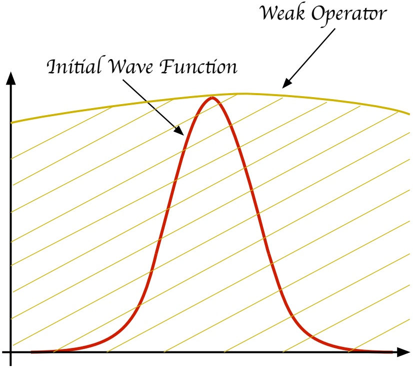

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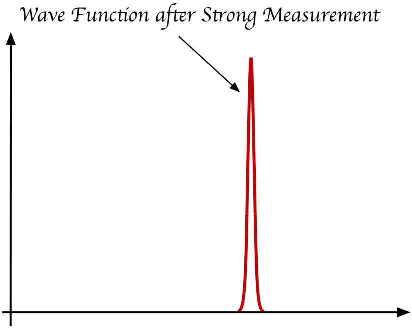

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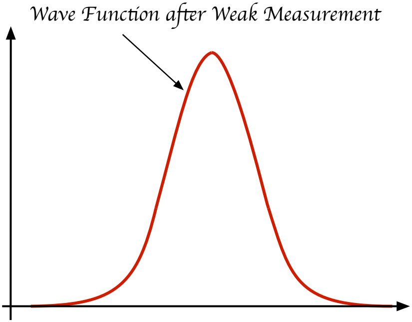

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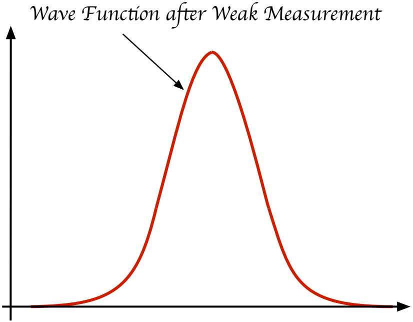
How is Measurement modelled in Quantum Mechanics?

Weak

Main features of the *Weak* Measurement

- $\langle I \rangle_{strong} = \langle I \rangle_{weak}$
- Wave Function of the system is slightly perturbed after the interaction



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Measuring the current at high frequency

Three ways:

- Taking information from the system without worrying about the apparatus $\rightarrow$ Be careful!

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- *Which is the operator that defines the (non-unitary) evolution of the wave function when measuring the total current?*
- *Is it "continuous" or "instantaneous"? Does it provide a "weak" or a "strong" perturbation on the wave function?*

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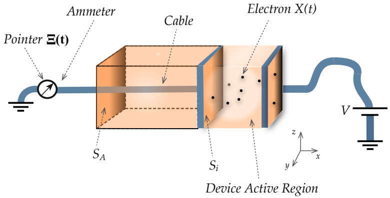

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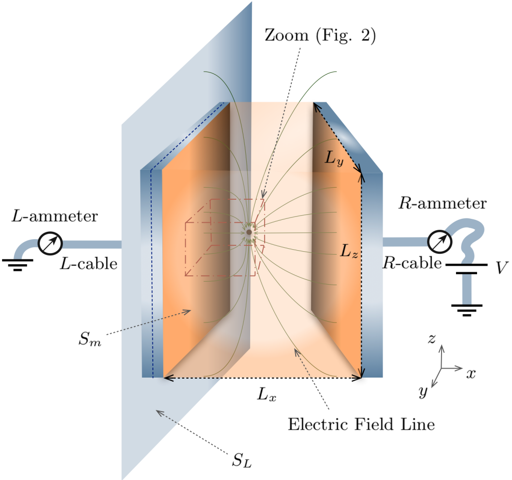
2 - Approach to model the measurement of THz current

Many-Body Problem

$$i\hbar\frac{\partial\Psi(x_1, x_2, ..., x_N, t)}{\partial t} = [H_0 + V_{Coul}]\,\Psi(x_1, x_2, ..., x_N, t)$$



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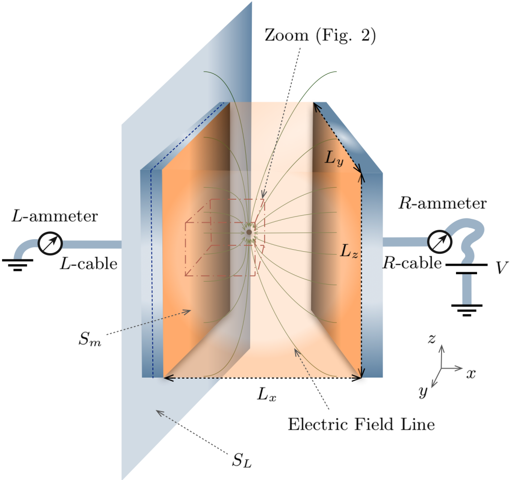

- Coulomb Interaction

$\downarrow$

$$V_{Coul} = \frac{1}{4\pi\epsilon(\mathbf{r})}\frac{1}{2}\sum_{i=1,j\neq i}^{N}\frac{q_i q_j}{\sqrt{(x_i - x_j)^2}}$$

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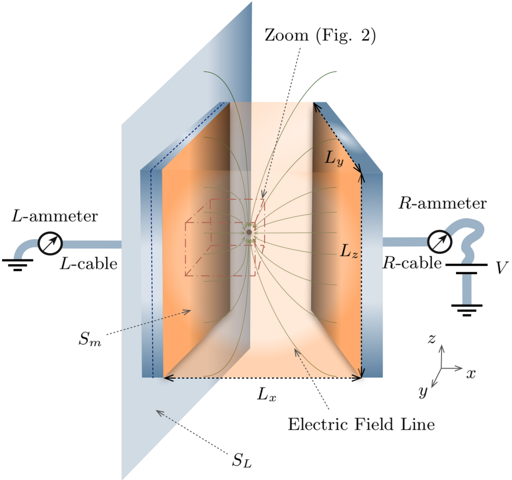
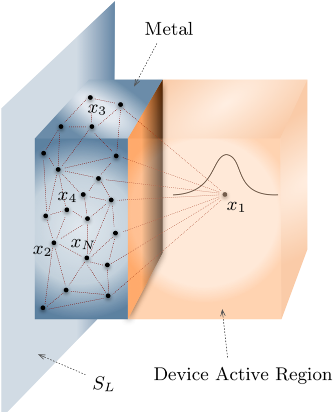


- Coulomb Interaction
  ↓
  $$V_{Coul}=\frac{1}{4\pi\epsilon(\mathbf{r})}\frac{1}{2}\sum_{i=1,j\neq i}^{N}\frac{q_iq_j}{\sqrt{(x_i-x_j)^2}}$$
- Many Particle
  ↓
  Numerically inaccessible

Conditional Wave Function

$$i\hbar\frac{\partial\psi(x_1,t)}{\partial t}=[H_0+V_{Cond}]\,\psi(x_1,t) \qquad \frac{dX_1(t)}{dt}=\frac{\hbar}{m}\text{Im}\left(\frac{\nabla\psi}{\psi}\right)$$

D. Dürr, S. Goldstein, and N. Zanghì: JSP (1992;2005)



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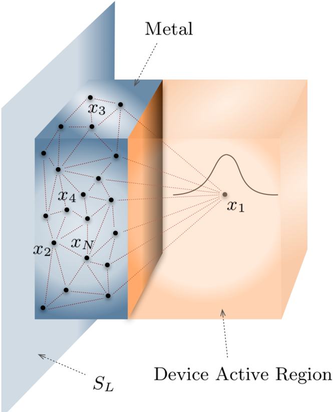

- $\psi(x_1,t)=\Psi(x_1,X_2(t),...,X_N(t))$

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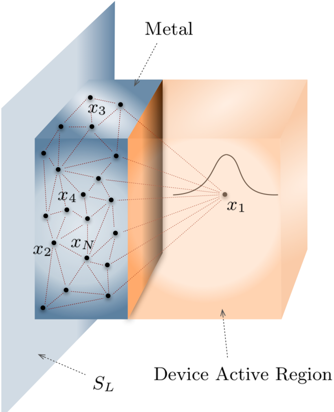

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$$V_{Cond} = \frac{1}{4\pi\epsilon}\sum_{i=2}^{N}\frac{q_1 q_i}{\sqrt{(x_1 - X_i(t))^2}}$$

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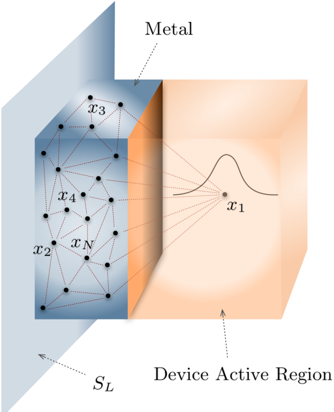

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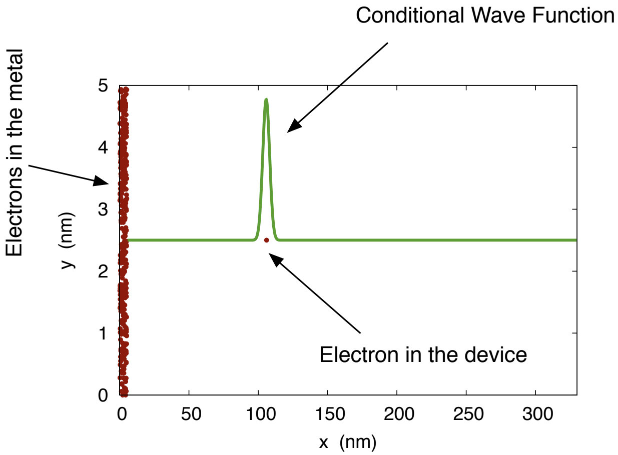
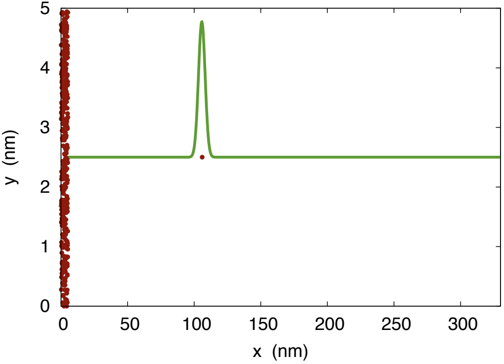
- $I_d(t) = \int_{S_L} \epsilon(r) \frac{dE(r; X_1(t), ..., X_N(t), t)}{dt} \cdot ds$

2 - Approach to model the measurement of THz current



Evolution of the system under the interaction with the apparatus



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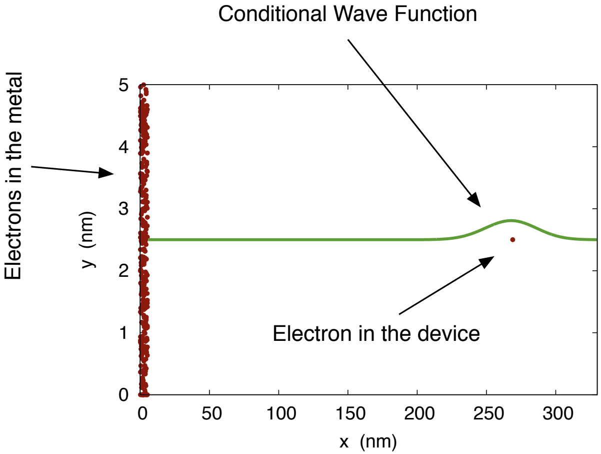

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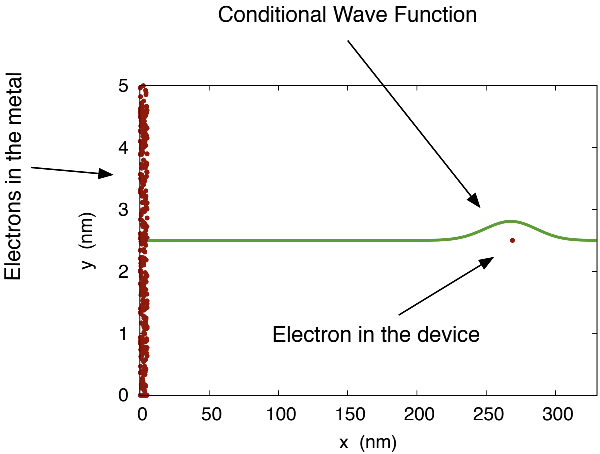

We have a numerical method to tackle the many-body problem!

$\Downarrow$

We include the Back-Action of the measuring apparatus!!!

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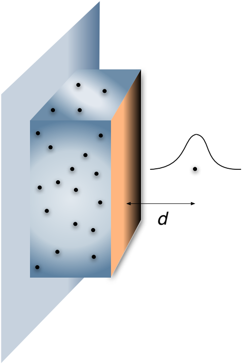
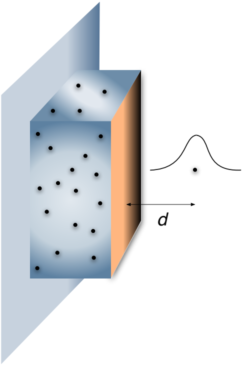
3 - New source of noise

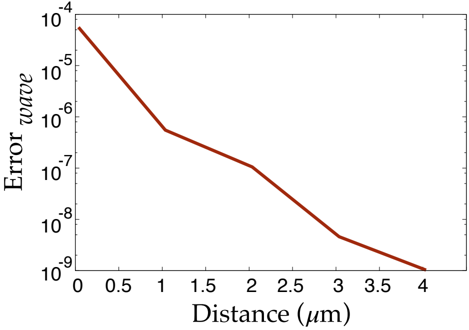
The wave function of the system is only "slightly" perturbed



Different distances from the metal surface → Parameter d

The wave function of the system is only "slightly" perturbed





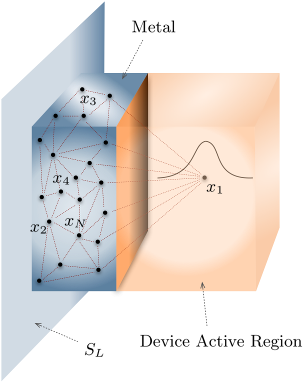
Different distances from the metal surface → Parameter d

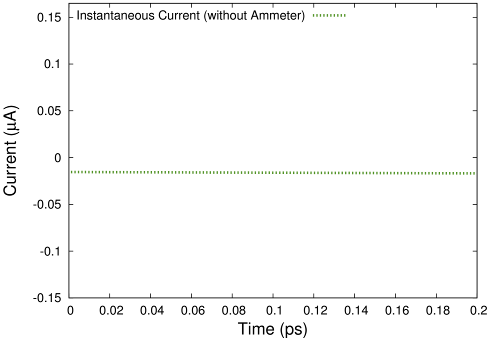
Error Wave Function

$$\mathrm{Error}_{Wave} = \int |\psi_{int} - \psi_{free}|^2 dx$$

The error in the Wave Function decreases with the distance!

Total Current Measured in the Surface S_L





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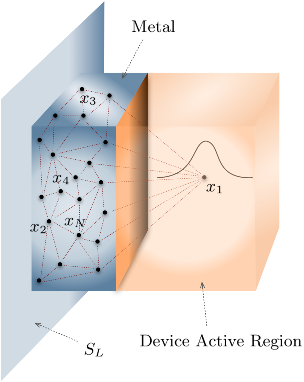

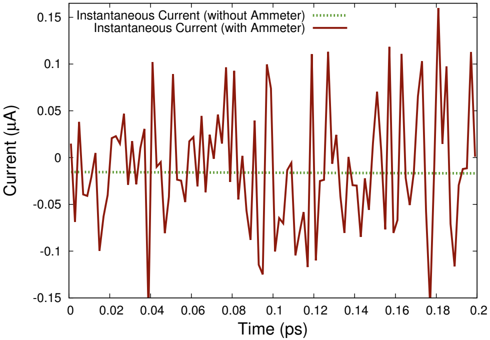

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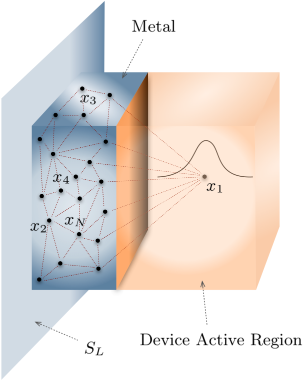
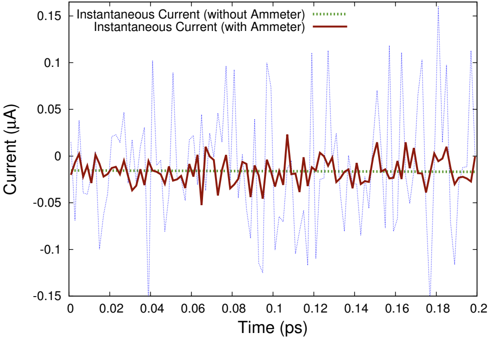


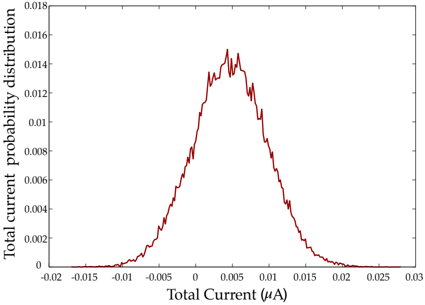


Repeating many times the experiment

we reach the mean value!!!

3 - New source of noise

Probability distribution of the total current in a large metallic surface



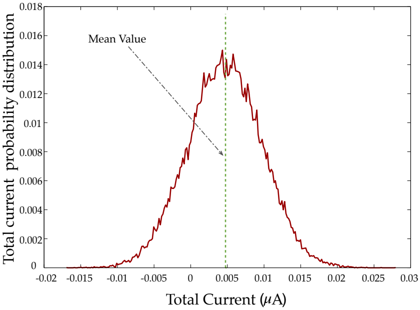
Output current very noisy!

⇓

Additional source of noise!!!

3 - New source of noise

Probability distribution of the total current in a large metallic surface



$$\langle I \rangle_{strong} = \langle I \rangle_{weak}$$

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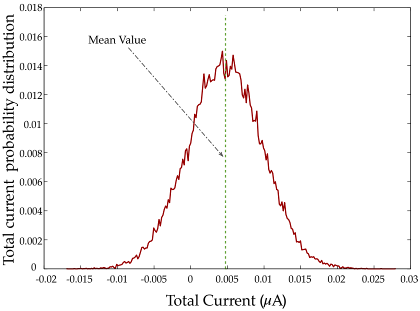

⇒ Output current very noisy!

⇒ The error in the Wave Function decreases with the distance!

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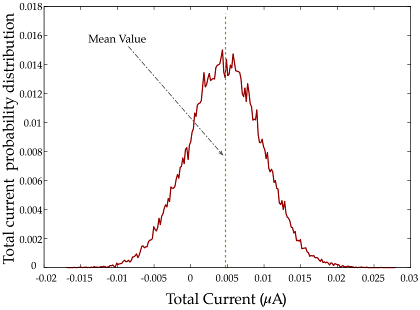

$\Rightarrow$ Output current very noisy!

$\Rightarrow$ The error in the Wave Function decreases with the distance!

$\Rightarrow \langle I \rangle_{strong} = \langle I \rangle_{weak}$

Weak Measurement!!!

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4 - Measurement of the local (Bohmian) velocities

Total Current (conduction *plus* displacement components)

$$\langle I(t)\rangle = \int_{S_i} \langle \mathbf{J}_c(\mathbf{r},t)\rangle \cdot d\mathbf{s} + \int_{S_i} \epsilon \frac{d\langle \mathbf{E}(\mathbf{r},t)\rangle}{dt} \cdot d\mathbf{s}$$

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Or from the Ramo-Schockley-Pellegrini theorem

$$\langle I(t)\rangle = -\int_{\Omega} \mathbf{F}(\mathbf{r}) \cdot \langle \mathbf{J}_c(\mathbf{r},t)\rangle \cdot dv + \int_{S} \epsilon \cdot \mathbf{F}(\mathbf{r}) \cdot \frac{d\langle V(\mathbf{r},t)\rangle}{dt} \cdot d\mathbf{s}$$

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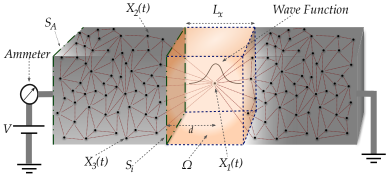
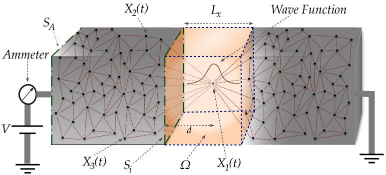


In the case of two metallic surfaces S_i at distance $S_i \gg L_x^2$:

$$\langle I(t)\rangle = \frac{q}{mL_x}\langle p(t)\rangle_{\Omega}$$



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Total Current in a large surface

$\Downarrow$

Weak Measurement of the Momentum!!!

4 - Measurement of the local (Bohmian) velocities

Weak measurement: a new way for measuring incompatible observables

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Weak measurement: a new way for measuring incompatible observables

$\Downarrow$

Measurement of position and momentum (for an ensemble of experiments)

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Weak measurement: a new way for measuring incompatible observables

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Measurement of position and momentum (for an ensemble of experiments)

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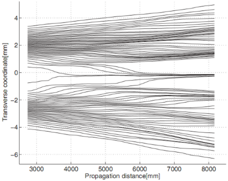
Experimental measurement of the local velocity



From: S. Kocis, *et al.*, Science, **332** (2011).

Experimental Bohmian trajectories:
Photons in a double slit set-up

4 - Measurement of the local (Bohmian) velocities

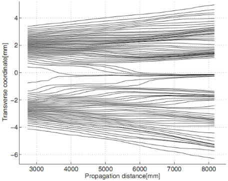
Weak measurement: a new way for measuring incompatible observables

$\Downarrow$

Measurement of position and momentum (for an ensemble of experiments)

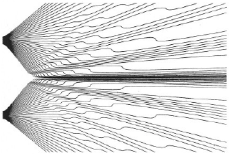
$\Downarrow$

Experimental measurement of the local velocity



From: S. Kocis, *et al.*, Science, **332** (2011).

Experimental Bohmian trajectories:
Photons in a double slit set-up



From: C. Philippidis, *et al.*, Il nuovo cimento B (1979)

Theoretical Bohmian trajectories

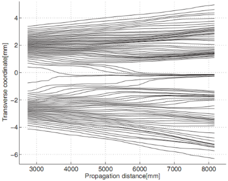
Weak measurement: a new way for measuring incompatible observables

⇓

Measurement of position and momentum (for an ensemble of experiments)

⇓

Experimental measurement of the local velocity



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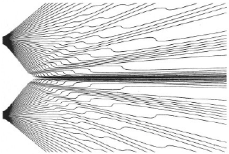

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Theoretical Bohmian trajectories

⇓

Is it possible to envisage an analogous experiment for electrons?

Experimental proposal for measuring the local (Bohmian) velocity

$$v(x) = \frac{1}{m}\mathrm{Re}\frac{\langle x|\hat{p}|\psi\rangle}{\langle x|\psi\rangle} \qquad E[p_w|x_s] = \frac{\int dp_w p_w \mathcal{P}(p_w \cap x_s)}{\mathcal{P}(x_s)} = v(x_s,\tau)$$

H M Wiseman 2007 New J. Phys. 9 165

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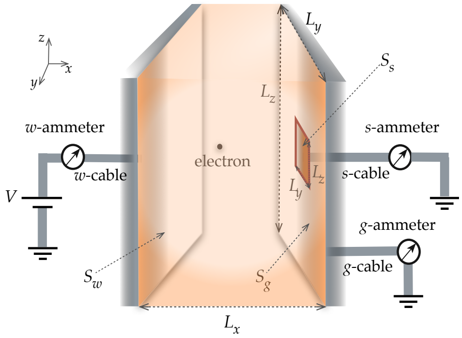

4 - Measurement of the local (Bohmian) velocities

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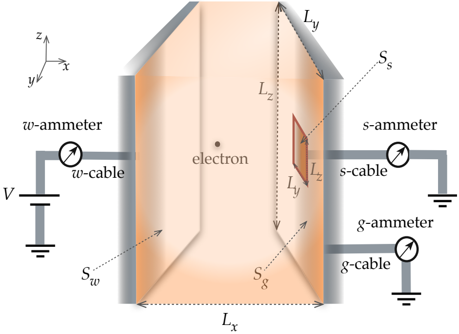

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  WM of total current = WM of momentum

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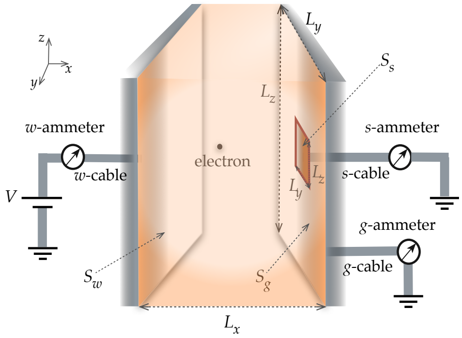

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- S_s where $L'_y, L'_z \ll L_x$
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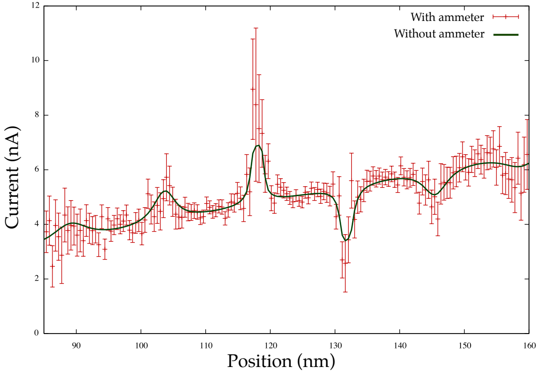
  Post-selection with position measurement

Numerical Experiments



Reconstruction of the Bohmian velocity from an ensemble of 55000 numerical experiments

$$E[p_w|x_s] = \frac{\int dp_w p_w \mathcal{P}(p_w \cap x_s)}{\mathcal{P}(x_s)} =$$
$$= \frac{J(x_s,\tau)}{|\psi(x_s,\tau)|^2} \equiv v(x_s,\tau)$$

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Strong Measurement

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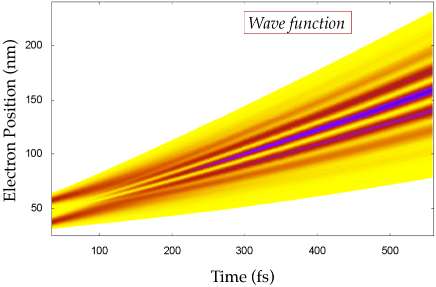
Numerical Experiments



Wave function

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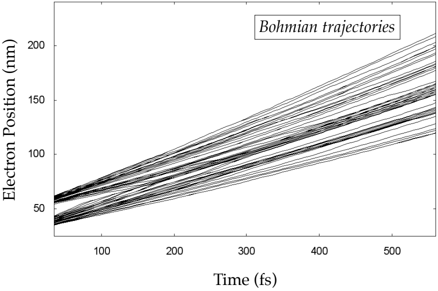

Bohmian trajectories

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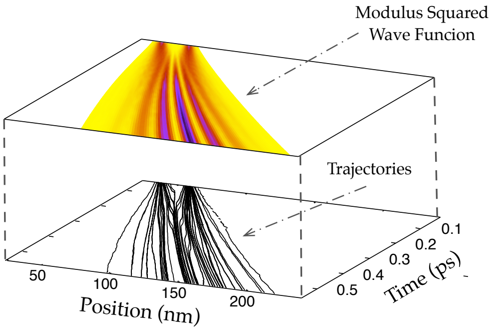

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Wave function *and* Bohmian trajectories

D. M., X. Oriols and N. Zanghì: *Weak Values from Displacement Currents in Multiterminal Electron Devices*, in preparation.
D. Dürr, S. Goldstein, and N. Zanghí, *On the Weak Measurement of Velocity in Bohmian Mechanics*, Journal of Statistical Physics 134, 1023 (2009).

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2. Novel approach to model the measurement of THz current
3. New source of noise
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5. Concluding Remarks

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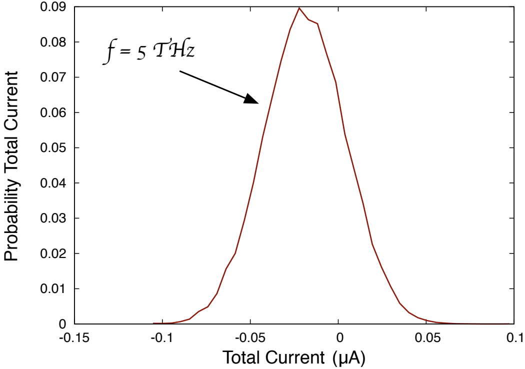
On the WEAK MEASUREMENT of the ELECTRICAL THz CURRENT: a NEW SOURCE of NOISE?

Damiano Marian, Nino Zanghì, Xavier Oriols

THANKS FOR THE ATTENTION!!!

Question: *At which frequency is this additional noise relevant?*



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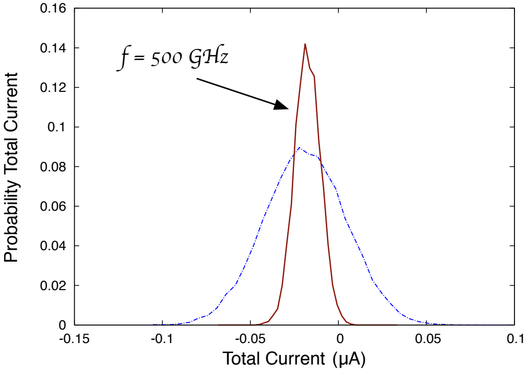

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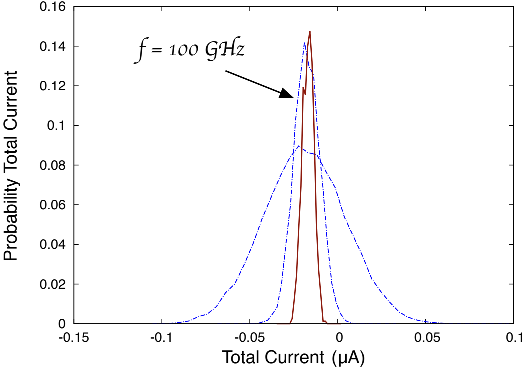

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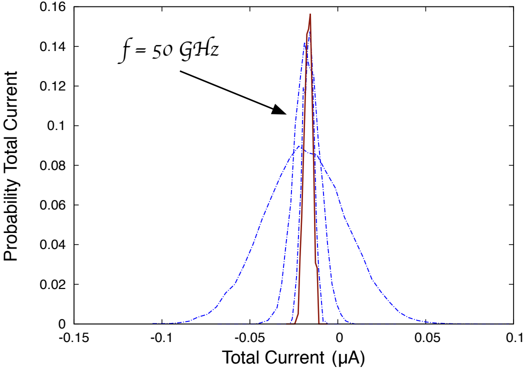

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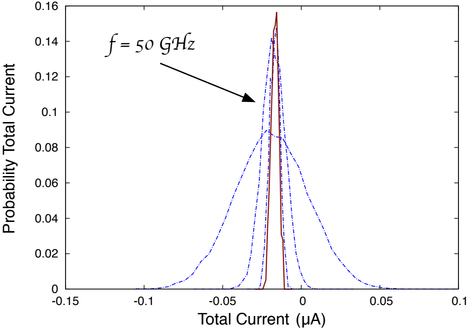

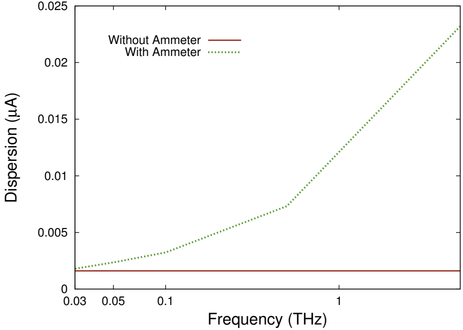

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For the configuration considered $\approx$ 50 GHz