Non-zero probability of detecting identical electrons at the same position:

How does it affect the Landauer-Büttiker noise expression at high temperatures?

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Outline

- 1.- Motivation of an unsolved problem
- 2.- Two-particle scattering with tunneling and exchange
- 3.- Many-particle scattering with tunneling and exchange
- 4.- Consequences on quantum noise
- 5.- Final remarks

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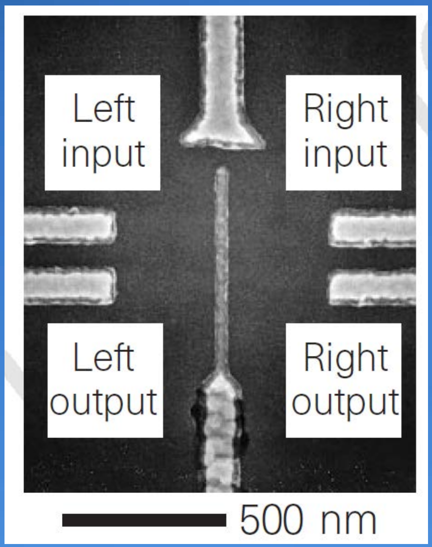
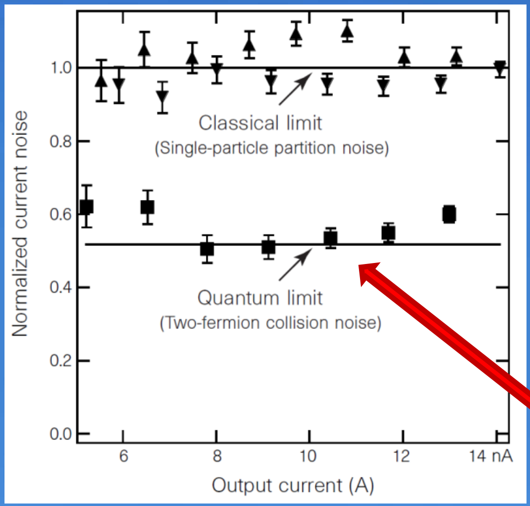
Motivation

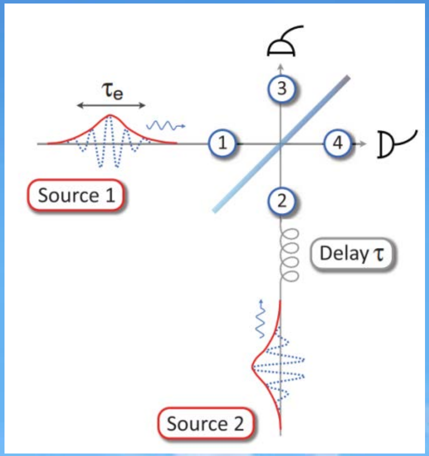


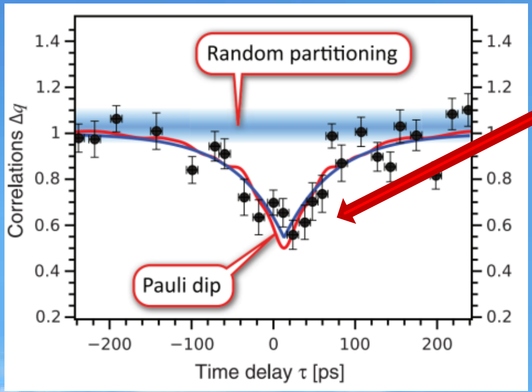


Two experiments:

R. C. Liu *et al.* *Nature 391, 1998*

We would expect **zero** fluctuations in the number of detected electrons in 3 or 4. Experiments do not provide such ideal zero.





E. Bocquillon *et al.* *Science 339, 2013*

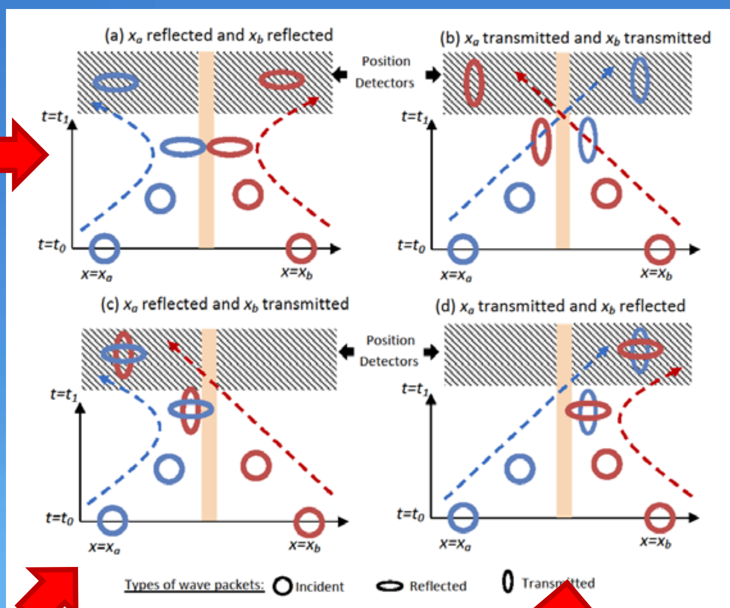
Objective

Study of the scattering probabilities of two identical electrons injected simultaneously from opposite sides of a tunneling barrier.

What is new in our approach ?

Instead of…

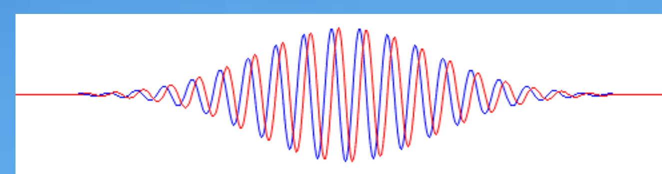
P_{LR}



P_{LL}

P_{RR}

Scattering States



Spatially-localized wave packet

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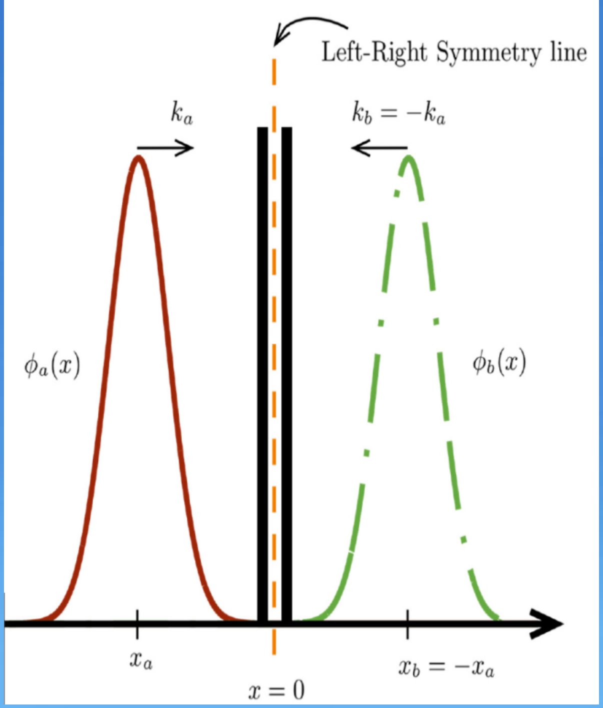
Two-particle scattering

We consider exchange and tunneling in a two-particle system



Antisymmetric wave function

$$\Phi(x_1, x_2, t_0) = \frac{\phi_a(x_1, t_0)\phi_b(x_2, t_0) - \phi_a(x_2, t_0)\phi_b(x_1, t_0)}{\sqrt{2}}$$

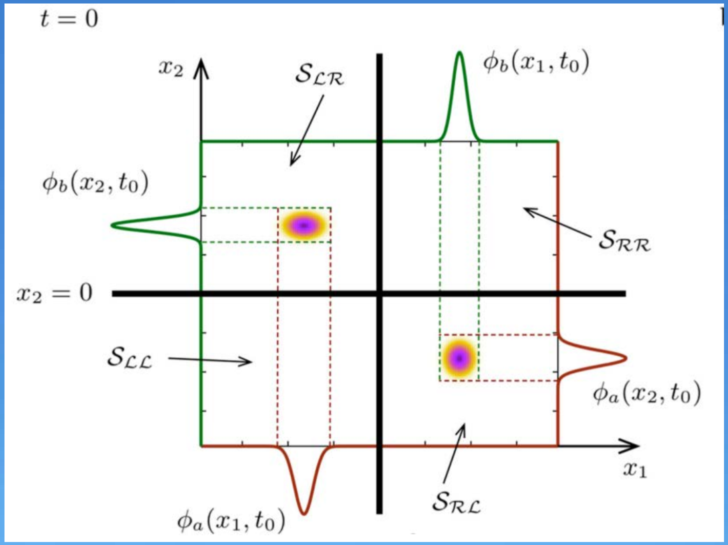
+

Time-dependent Schrödinger Equation

$$i\hbar\frac{\partial \Phi}{\partial t} = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2} + V(x_1, x_2)\right]\Phi$$

Two-particle scattering

The 4 probabilities are computed, when the scattering process with the barrier is finished, using Born rule.



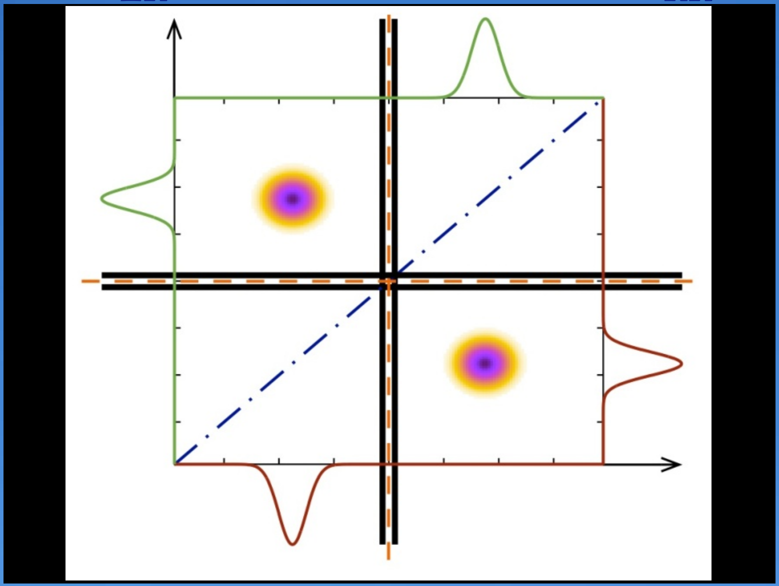
$$\mathcal{P}_{\mathcal{LR}} = \int_{\mathcal{S}_{\mathcal{LR}}} |\Phi|^2 dx_1 dx_2 + \int_{\mathcal{S}_{\mathcal{RL}}} |\Phi|^2 dx_1 dx_2 = 2 \int_{\mathcal{S}_{\mathcal{LR}}} |\Phi|^2 dx_1 dx_2$$

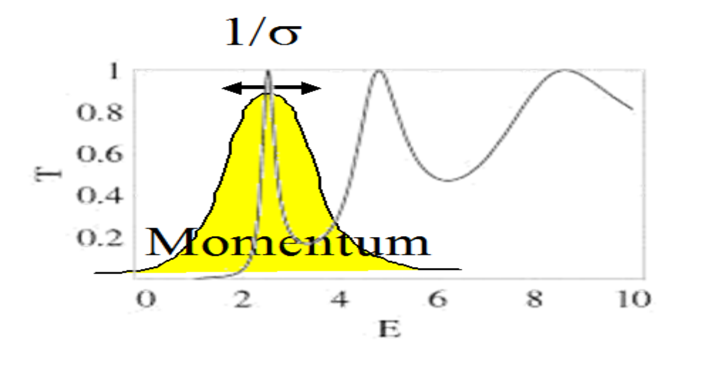
$$\mathcal{P}_{\mathcal{LL}} = \int_{\mathcal{S}_{\mathcal{LL}}} |\Phi|^2 \mathrm{d}x_1 \mathrm{d}x_2$$

$$\mathcal{P}_{\mathcal{RR}} = \int_{\mathcal{S}_{\mathcal{RR}}} |\Phi|^2 \mathrm{d}x_1 \mathrm{d}x_2$$

D. Marian, E. Colomés and X. Oriols *J. Phys. Cond. Matter 27 2015*

Two-particle scattering





Two-particle scattering probabilities

$$\mathcal{P}_{\mathcal{LL}} = \int_{-\infty}^{0} dx_1 \int_{-\infty}^{0} dx_2 \; |\Phi|^2 = R_a T_b - |I_{a,b}^{r,t}|^2$$

$$\mathcal{P}_{\mathcal{RR}} = T_a R_b - |I_{a,b}^{r,t}|^2$$

$$\mathcal{P}_{\mathcal{LR}} = R_a R_b + T_a T_b + 2|I_{a,b}^{r,t}|^2$$

Overlapping term:

$$I_{a,b}^{r,t} = \int_{-\infty}^{0} dx \; \phi_a^r(x,t_1) \; \phi_b^{*t}(x,t_1)$$

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Two-particle scattering

Overlapping term:

$$I_{a,b}^{r,t} = \int_{-\infty}^{0} dx\ \phi_a^r(x,t_1)\ \phi_b^{*t}(x,t_1)$$

Probabilities for distinguishable particles	Probabilities for scattering states
$I_{a,b}^{r,t} = 0$	$I_{a,b}^{r,t} = RT$
$\mathcal{P}_{\mathcal{LL}}^{m} = \mathcal{P}_{\mathcal{RR}}^{m} = RT$	$\mathcal{P}_{\mathcal{LL}}^{M} = \mathcal{P}_{\mathcal{RR}}^{M} = RT - RT = 0$
$\mathcal{P}_{\mathcal{LR}}^{m} = R^2 + T^2$	$\mathcal{P}_{\mathcal{LR}}^{M} = (R+T)^2 = 1$

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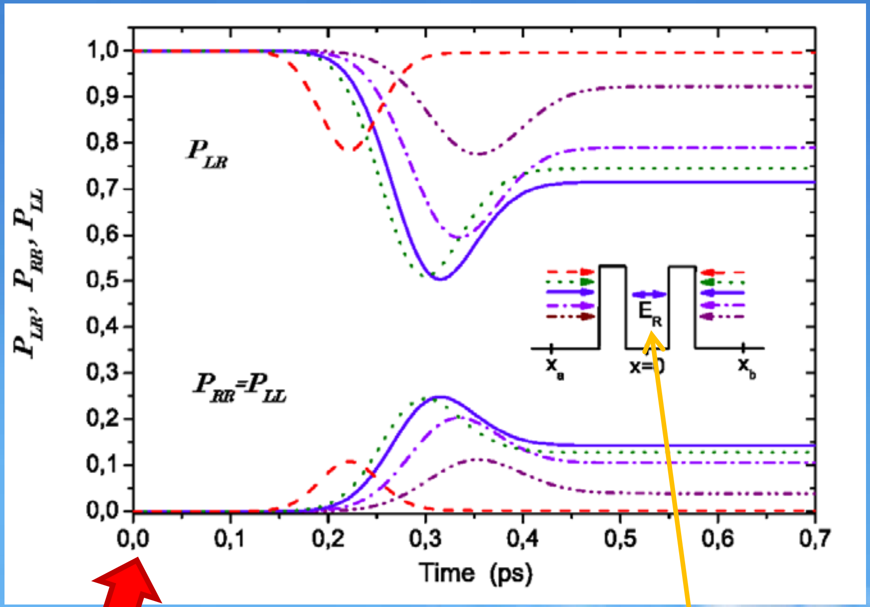
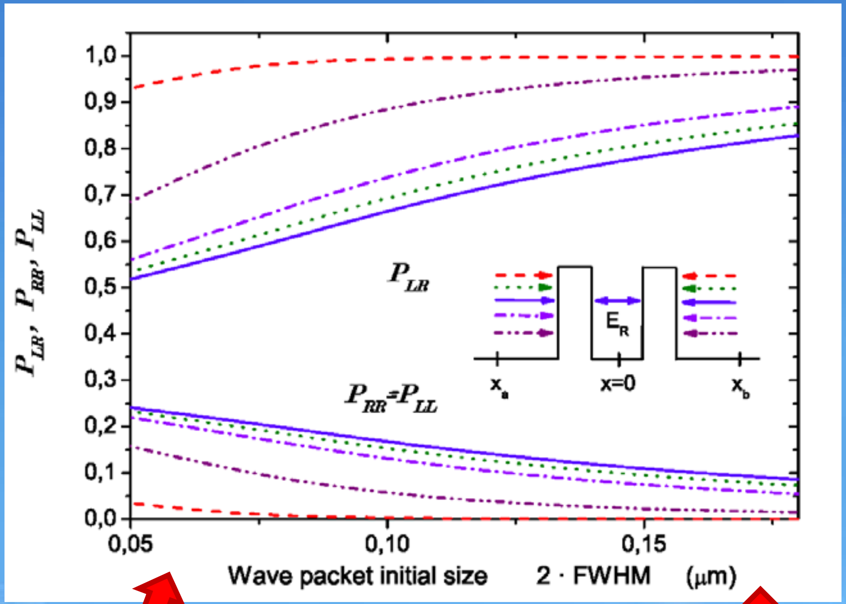
Numerical Results

Initial Gaussian state

$$\psi(x,t_0) = \left(\frac{1}{2\pi a_0^2}\right)^{\frac{1}{4}} e^{ik_0(x-x_0)} exp\left(-\frac{(x-x_0)^2}{4a_0^2}\right)$$

P_{LR}





P_{LL}=P_{RR}

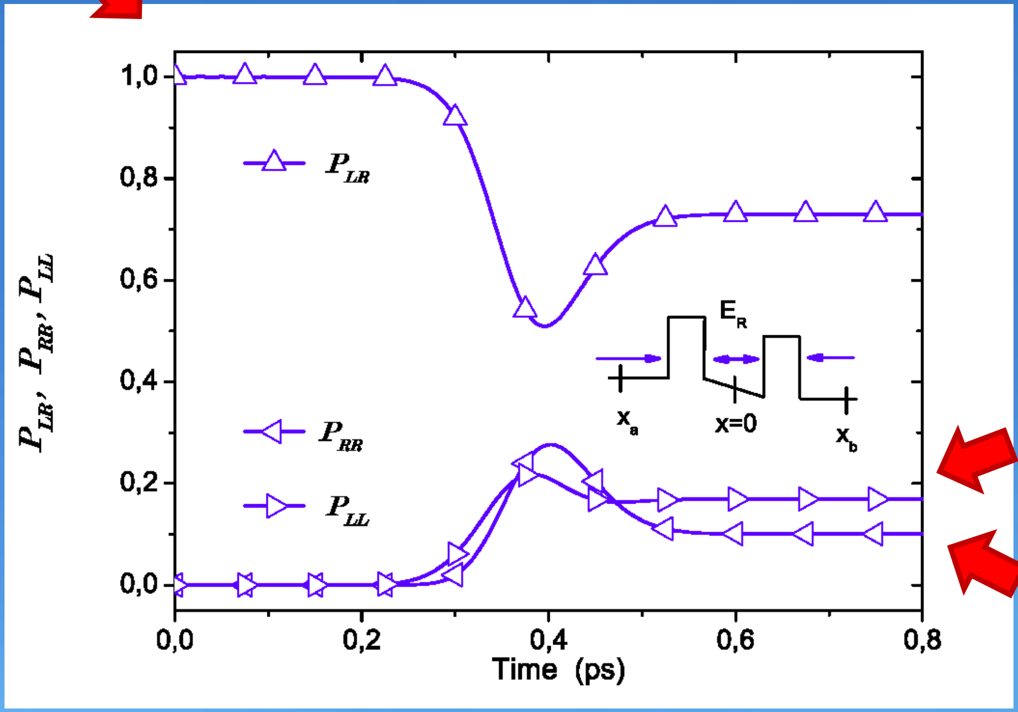
Resonant energy

Small wave packets

Large wave packets

Numerical Results

P_{LR}



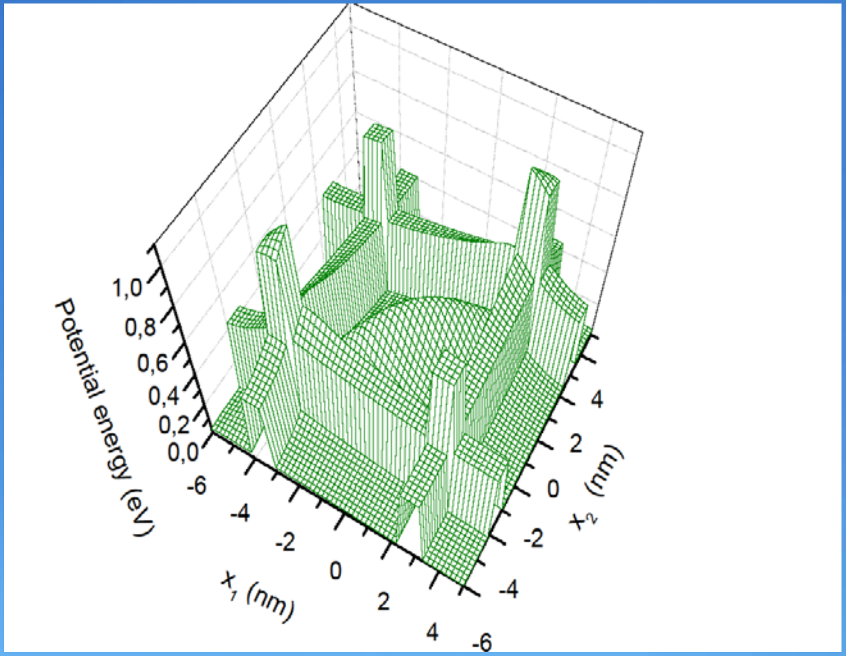
P_{LL}

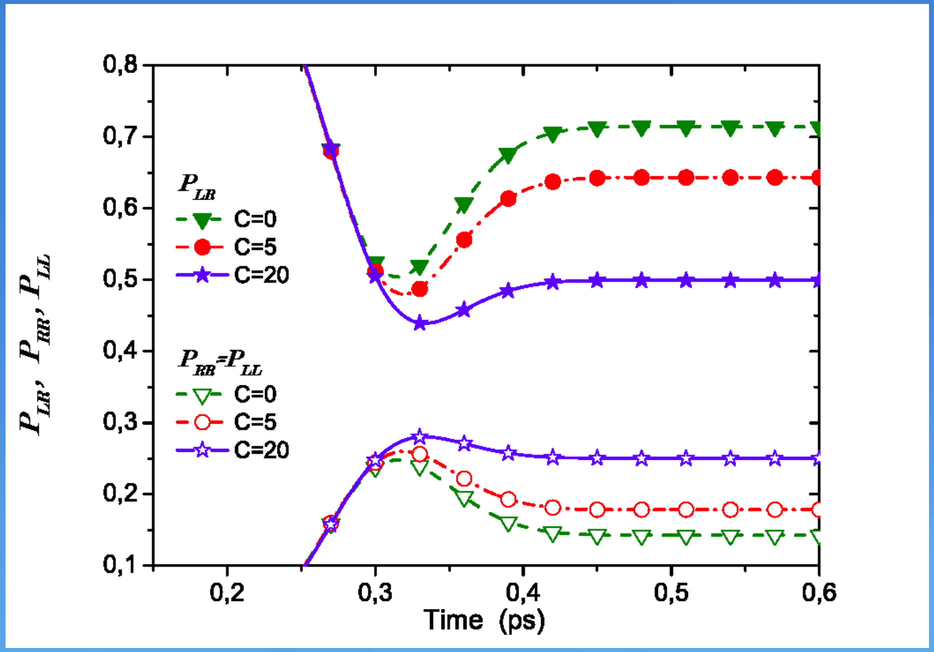
P_{RR}

Double barrier + Applied Bias

Numerical Results

Double barrier + Coulomb Potential

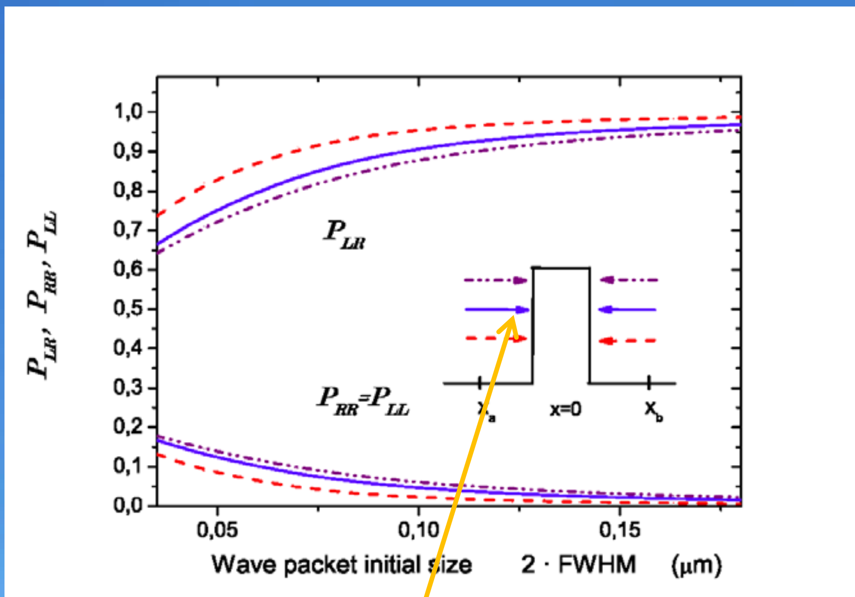




$$H = H_0 + V_0 + C \cdot V_C$$

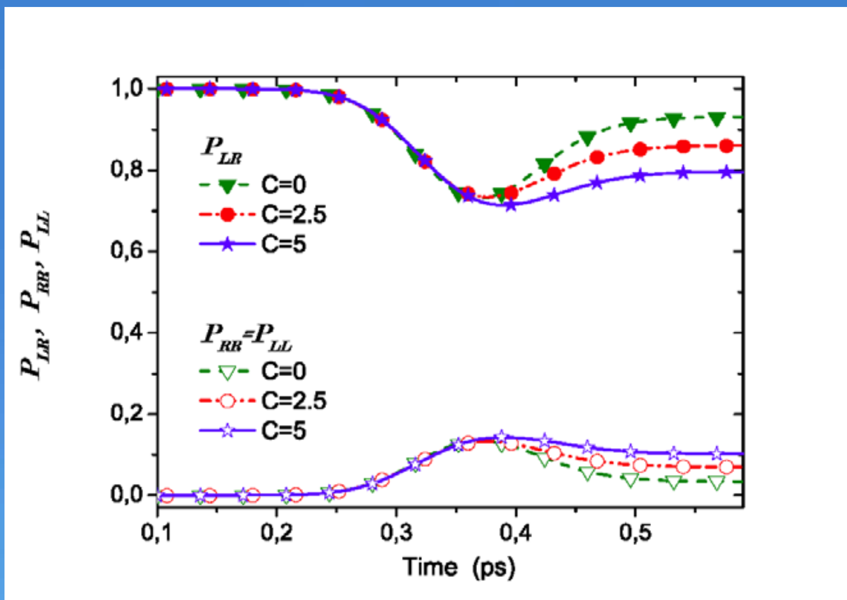
Numerical Results

Single Barrier



Energy where T=1/2

Single Barrier + Coulomb potential



$$H = H_0 + V_0 + C \cdot V_C$$

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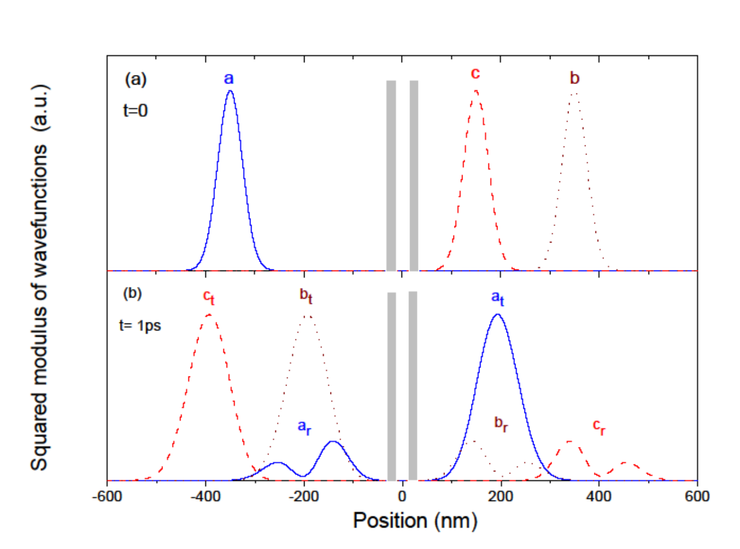
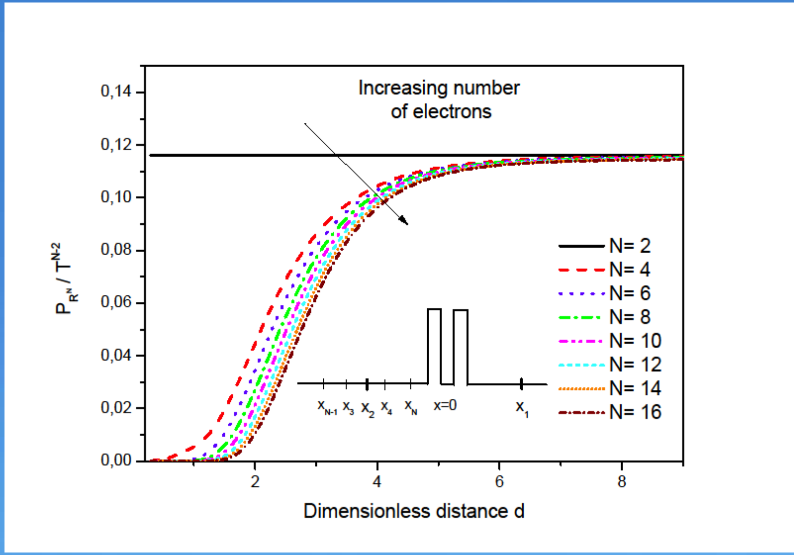
Many-particle scattering

Three-particle scattering



Overlapping is increased!!

Probability of finding N-electrons at the same place decrease



$$d = \frac{(k_i - k_j)^2}{2\sigma_k^2} + \frac{(x_i - x_j)^2}{2\sigma_x^2}$$

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Consequences on noise

One- and two-particle zero-frequency noise

$$\langle S(E)\rangle = lim_{t_d\to\infty} 2q^2 \frac{\langle N^2\rangle_{t_d} - \langle N\rangle_{t_d}^2}{t_d}$$

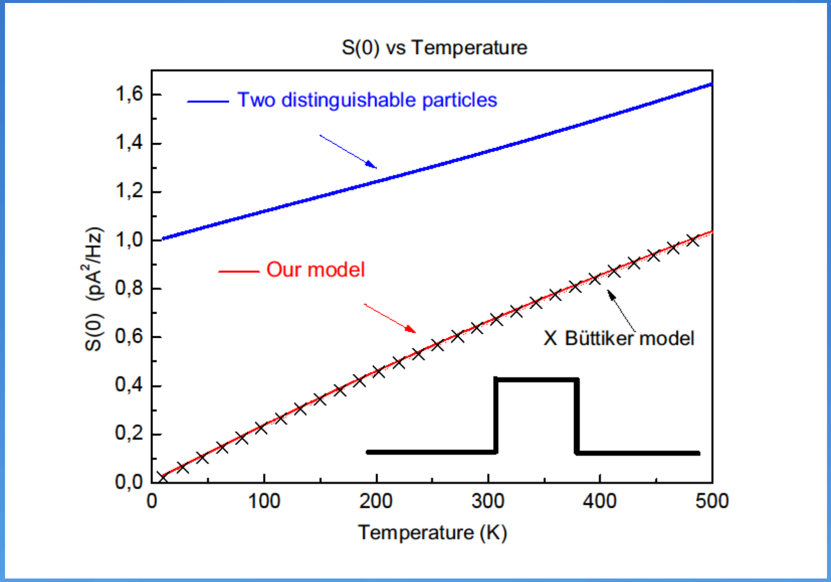
$$\langle N\rangle_{t_d} = \sum_{N=-\infty}^{N=\infty} P(N,t_d)N$$

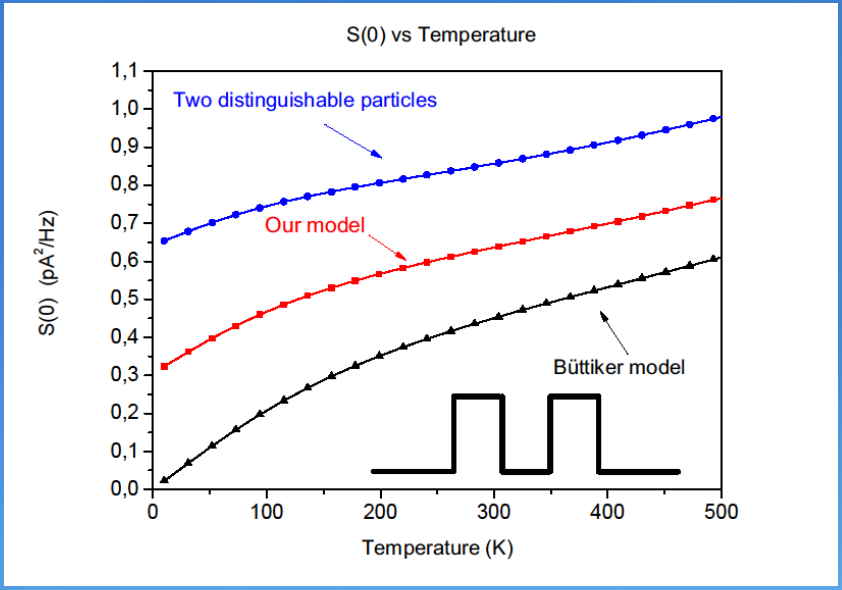
$$\langle N^2\rangle_{t_d} = \sum_{N=-\infty}^{N=\infty} P(N,t_d)N^2$$

b \ a		Injection: Transmitted	Injection: Reflected	No injection
Injection	Transmitted	$\frac{\mathcal{P}_{\mathcal{LR}}}{2} f_a f_b$	$\mathcal{P}_{\mathcal{LL}} f_a f_b$	$T(1-f_a)f_b$
		0	-1	-1
	Reflected	$\mathcal{P}_{\mathcal{RR}} f_a f_b$	$\frac{\mathcal{P}_{\mathcal{LR}}}{2} f_a f_b$	$R(1-f_a)f_b$
		1	0	0
No injection		$T f_a(1-f_b)$	$R f_a(1-f_b)$	$(1-f_a)(1-f_b)$
		1	0	0

$$\langle S\rangle = \frac{4q^2}{h}[(T[f_a(1-f_a)+f_b(1-f_b)]) + T(1-T)(f_a-f_b)^2 + 2\mathcal{P}_{\mathcal{RR}} f_a f_b]$$

Consequences on noise

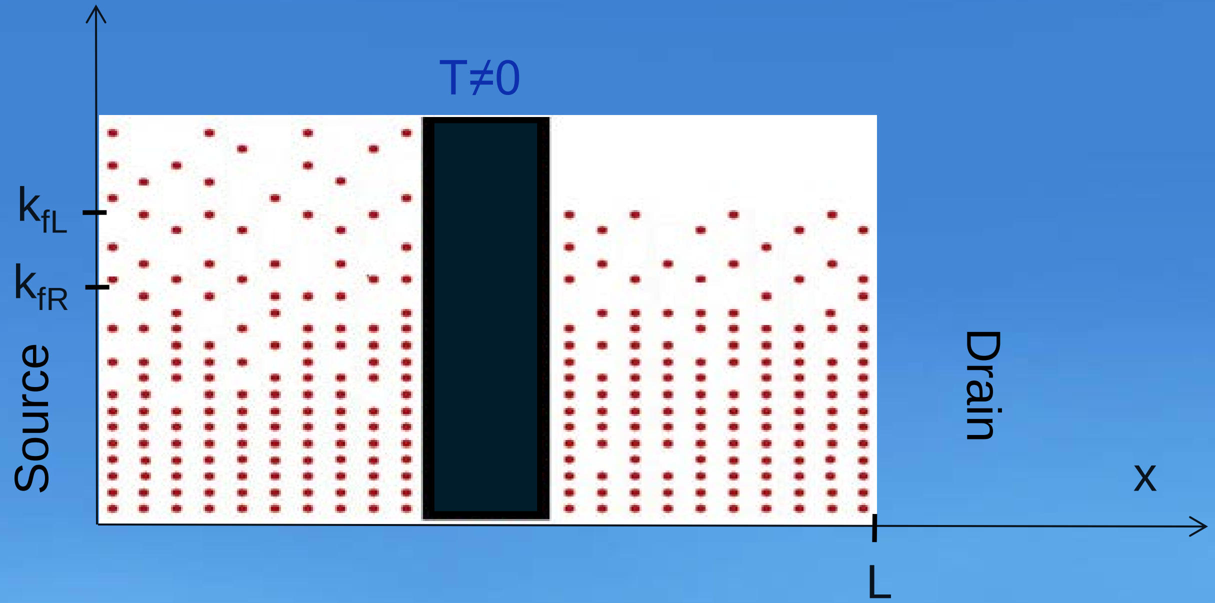




$$\langle S \rangle = \frac{4q^2}{h}[(T[f_a(1-f_a)+f_b(1-f_b)])+T(1-T)(f_a-f_b)^2+2\mathcal{P}_{\mathcal{R}\mathcal{R}}f_af_b]$$

Consequences on noise

What about the fluctuation-dissipation theorem?
Is it fulfilled?



$$\langle S\rangle = \frac{4q^2}{h}\left[(T[f_a(1-f_a)+f_b(1-f_b)]) + T(1-T)(f_a-f_b)^2 + 2\mathcal{P}_{\mathcal{R}\mathcal{R}} f_a f_b\right]$$

Consequences on noise

What about the fluctuation-dissipation theorem?
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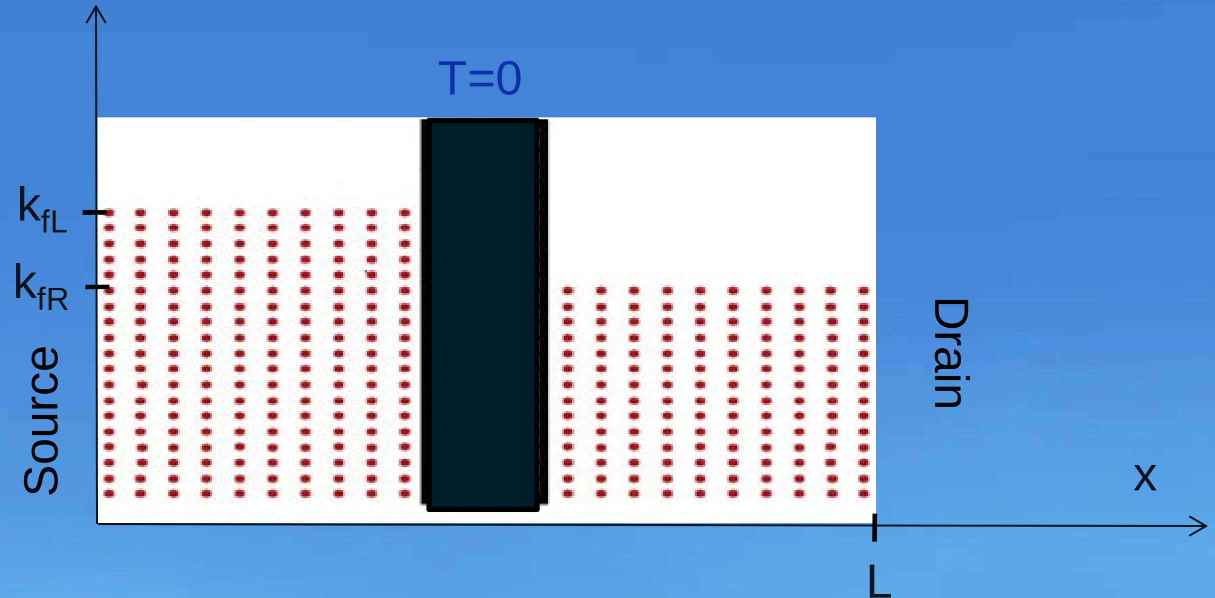

$$\langle S \rangle = \frac{4q^2}{h}[(T[f_a(1-f_a)+f_b(1-f_b)]) + T(1-T)(f_a-f_b)^2 + 2\mathcal{P}_{\mathcal{RR}} f_a f_b]$$

Consequences on noise

What about the fluctuation-dissipation theorem?
Is it fulfilled?

Yes, it is!

$$\langle S(E)\rangle = lim_{t_d\to\infty} 2q^2 \frac{\langle N^2\rangle_{t_d} - \langle N\rangle^2_{t_d}}{t_d}$$

$$< N_M >= \sum_{l_0=0}^{M/2} \sum_{i=1}^{\binom{M/2}{l_0}} \sum_{r_0=0}^{M/2} \sum_{j=1}^{\binom{M/2}{r_0}} \mathcal{P}_{L^{l_f} R^{l_0+r_0-l_f}}(\phi_{l_0+r_0,l_0,i,j}) \times f_a^{l_0}(1-f_a)^{M/2-l_0} f_b^{r_0}(1-f_b)^{M/2-r_0}(l_0-l_f)$$

$$< N_M^2 >= \sum_{l_0=0}^{M} \sum_{i=1}^{\binom{M}{l_0}} \sum_{r_0=0}^{M} \sum_{j=1}^{\binom{M}{r_0}} \mathcal{P}_{L^{l_f} R^{l_0+r_0-l_f}}(\phi_{l_0+r_0,l_0,i,j}) \times f_a^{l_0}(1-f_a)^{M-l_0} f_b^{r_0}(1-f_b)^{M-r_0}(l_0-l_f)^2$$

$$\langle S\rangle = \frac{4q^2}{h}[(T[f_a(1-f_a)+f_b(1-f_b)]) + T(1-T)(f_a-f_b)^2 + 2\mathcal{P}_{\mathcal{RR}} f_a f_b]$$

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Conclusions

✓ Non-zero probabilities of detecting two identical injected electrons at the same place.

✓There is an important overlapping term, depending on it we can recover as two particular limits: - the distinguishable particles probabilities
- the scattering states theory results

✓The results give theoretical explanation to experimental results.

✓Quantum noise in mesoscopic systems needs to be revisited in some scenarios.

Thank you for your attention!!