

# UAB: Visualizing light-matter interactions through simulations

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<https://europe.uab.es/qcslim/>



**UAB**  
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de Barcelona

There is a lot of mathematics in a physical theory, but a **physical theory** is not (only) a **mathematical theory**. The elements that constitute a mathematical theory can be abstract, but the elements that constitute a physical theory must be connected to the **real world**.

The connection between **mathematical formalism** and the **real world** is especially difficult when dealing with quantum phenomena:

**Shut up and calculate !**

**QC Slim** is a software tool that **helps visualize** how the mathematical formalism of quantum electrodynamics relates to the physical concepts of **matter** and **light**.

1.- Introduction: What are we talking about?

2.- Classical Electrodynamics

3.- Quantum Electrodynamics with wave functions

4.- Quantum Electrodynamics with  $a^\dagger$  and  $a$  operators

5.- Quantum Electrodynamics with quantum trajectories

6.- Examples of the program QC slim

7.- Conclusions

## 1.- Introduction: What are we talking about?

**Teacher:** Quantum electrodynamics describes the interaction between **electrons** and **photons**.



Is a **photon** a wave or a particle?

**Teacher:** No, a **photon** is an excitation of the electromagnetic **quantum field**.



But what is a **quantum field**?

**Teacher:** A **quantum field** is an operator acting on quantum states in Fock space.



But what is the **Fock space**?

## 1.- Introduction: What are we talking about?

**Teacher:** A quantum system of  $N$  particles “lives” in the Hilbert space  $H^N$  with  $N$  fixed. A **Fock space** is  $F=H^0 \oplus H^1 \oplus H^2 \oplus H^3 \oplus \dots$



.. And what is a **quantum state** in this **Fock space**?

**Teacher:** A **quantum state** in this Fock space is a superposition of ordinary quantum states with 0, 1, 2, ... particles.



So, are **photons** particles that can be counted: 0, 1, 2, ...?

**Teacher:** No. A **photon** is not a particle if you have in mind a trajectory in physical space, as happens in a classical particle. As I said, a **photon** is an **excitation** of the quantum field.



But what is an **excitation** of the quantum field?

## 1.- Introduction: What are we talking about?

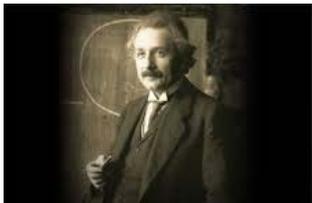
**Teacher:** An excitation (a **photon**) is a quantum state in **Fock space** that arises when a **creation operator** acts on another quantum state (for example, the **vacuum state**).



But what is a **creation operator**? What is the **vacuum state** ?....



Can you explain **quantum electrodynamics** using the language of **wave functions**? Is it possible to understand it starting from classical electrodynamics? Is it possible to **visualize** them?



If you can't explain simply, you don't understand it well enough

## 1.- Introduction: What are we talking about?

**Teacher:** Yes, **quantum electrodynamics** can be explained using the language of **wave functions**, starting from **classical electrodynamics**.  
Let's now explain the ideas behind the QCslim software.



<https://europe.uab.es/qcslim/>

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## 2.- Classical Electrodynamics

2.1.- Classical Electrodynamics: What is a electron?

2.2.- Classical Electrodynamics: What is a field?

2.2.1.- What is a classical field? Mode-decomposition

2.4.- Classical Electrodynamics: Light-matter interaction

## 2.1.- Classical Electrodynamics: What is an electron?

**Teacher:** Classical electrodynamics describes how **electrons** interact with **electromagnetic fields**.



.... and I know what a **classical electron** is.

**Teacher:** Yes. A **classical electron** is a point particle with a well-defined position  $x$  at all times.

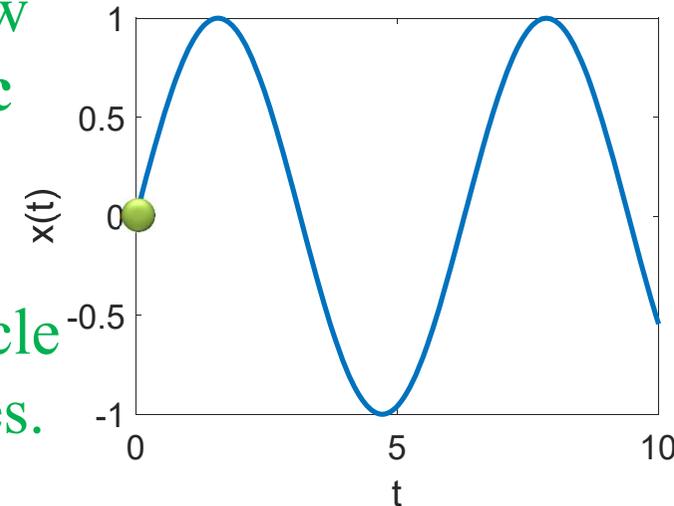


... we can **visualize** electron trajectories in **physical space**.

**Teacher:** Yes. The total energy of an electron is the sum of its kinetic and potential energies.

$$H_M = -\frac{p^2}{2m_e} + V(x)$$

...and the electron trajectory is given by the solution of **Newton's** (or **Hamilton's**) equations.

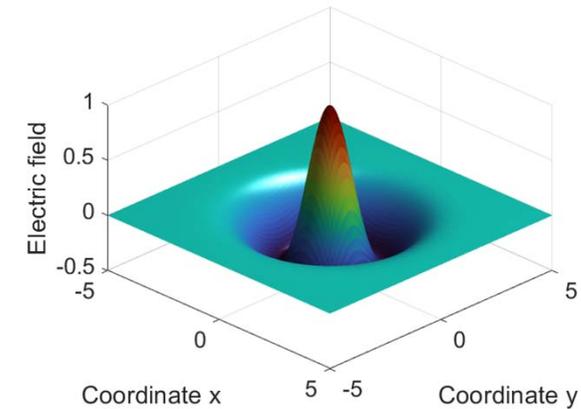


## 2.2.- Classical Electrodynamics: What is a field?

**Teacher:** The **electromagnetic field** comprises both an electric field  $E$  and a magnetic  $B$  field.



...we can represent an electric (and magnetic) field as a function defined at every point in **physical space**.



**Teacher:** The **energy** of the electromagnetic field can also be computed.

$$H_R = \frac{\epsilon_0}{2} \int d^3r (E^2 + c^2 B^2)$$

The time evolution of the electric and magnetic fields can be determined from **Maxwell's** (or **Hamilton's**) equations.



...But the amount of information required for the electromagnetic field is enormous (**millions of points in physical space**)! Is there a simpler way to define the electromagnetic fields?

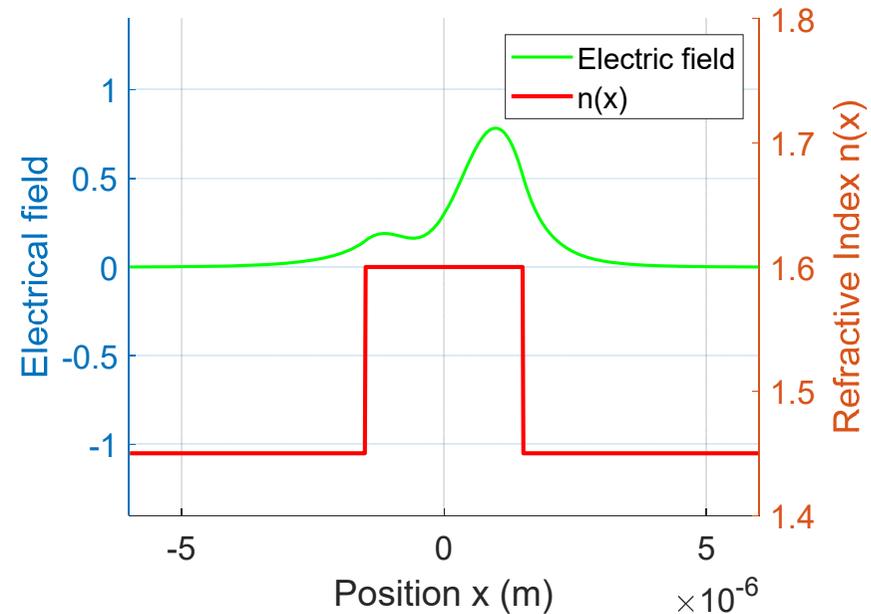
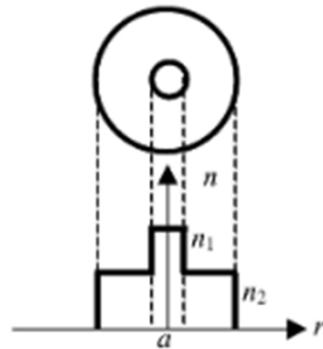
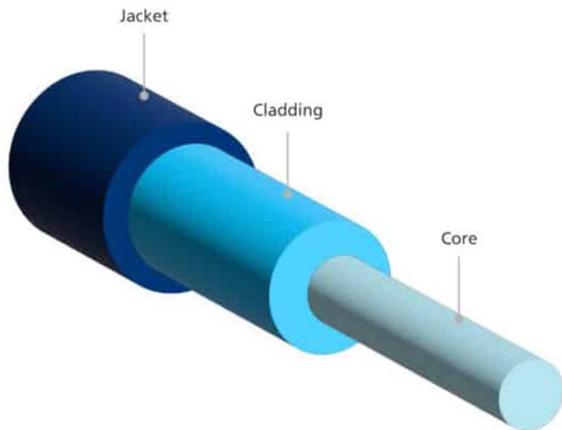
## 2.2.1.- What is a classical field? Mode-decomposition

**Teacher:** Yes, the electromagnetic fields can be represented using the so-called **mode decomposition**.



Can you illustrate this with a concrete example?

**Teacher:** Let us consider the **electromagnetic field** inside an **optical fiber**.



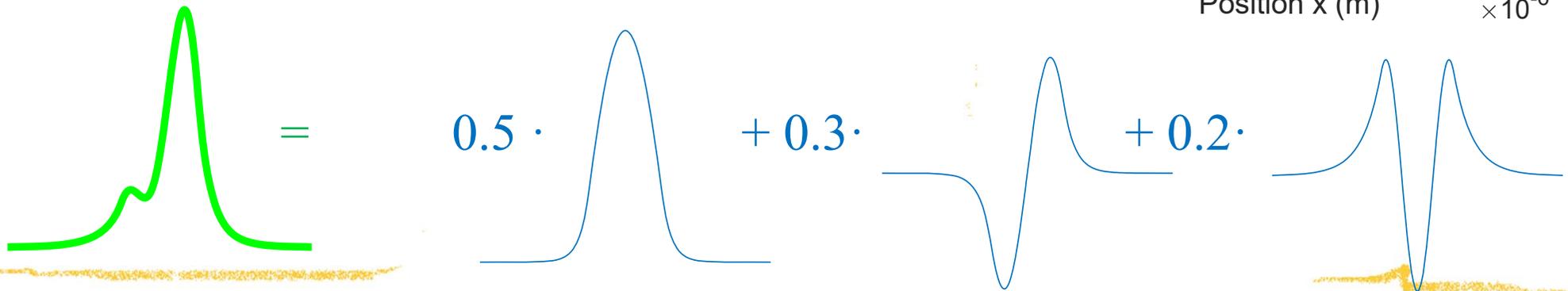
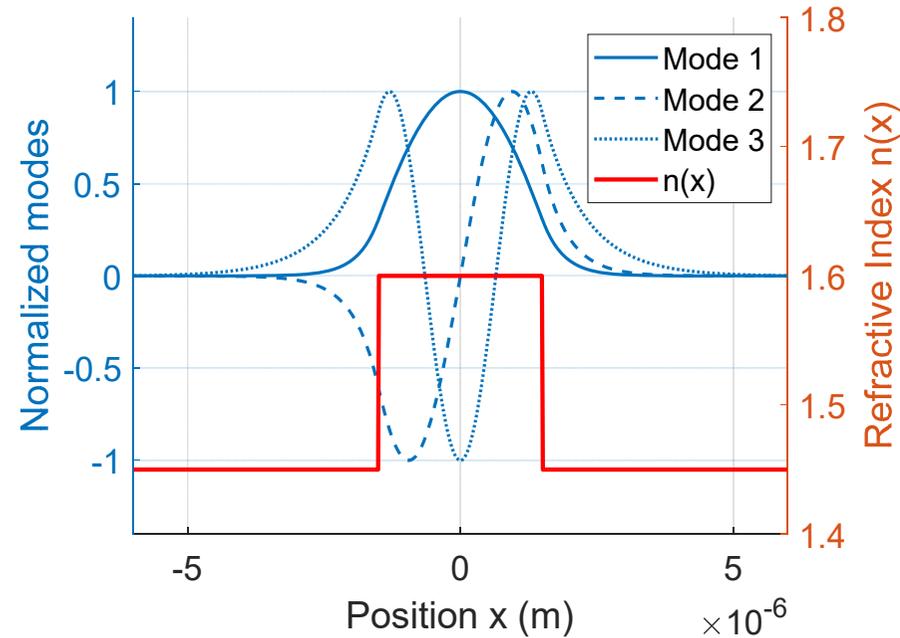
## 2.2.1.- What is a classical field? Mode-decomposition

**Teacher:** Let us calculate the modes of the electromagnetic field of the optical fiber.



Why are these **modes** necessary?

**Teacher:** Now, a single **electric field** can be completely specified by just **three numbers**:  $q_1=0.5$ ,  $q_2=0.3$  and  $q_3=0.2$ !

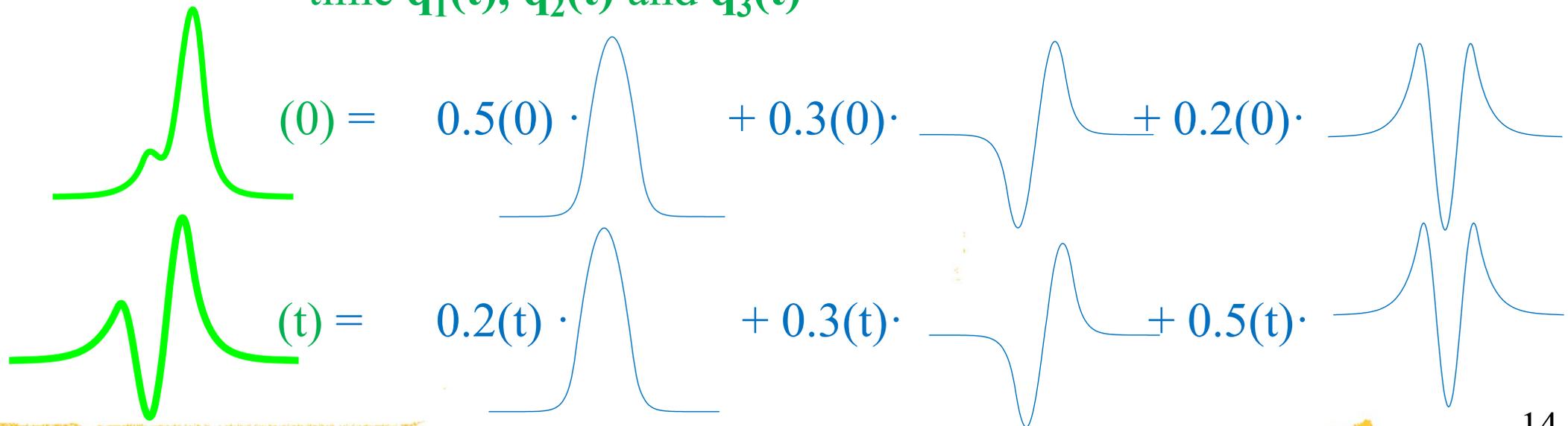


## 2.2.1.- What is a classical field? Mode-decomposition



But... the shape of the electric field changes with time. What happens to the coefficients  $q_1$ ,  $q_2$  and  $q_3$  of the mode decomposition at another time?

**Teacher:** An excellent question! The coefficients must now be time-dependent; they are no longer fixed numbers but functions of time  $q_1(t)$ ,  $q_2(t)$  and  $q_3(t)$



## 2.2.1.- What is a classical field? Mode-decomposition

**Teacher:** Thus, the **time-dependent field** is described by the **time-dependent coefficients**  $q(t)$ , one for each mode



Is  $q(t)$  a position in physical space?

No!  $q(t)$  is the time-dependent coefficient corresponding to a particular mode of the field.

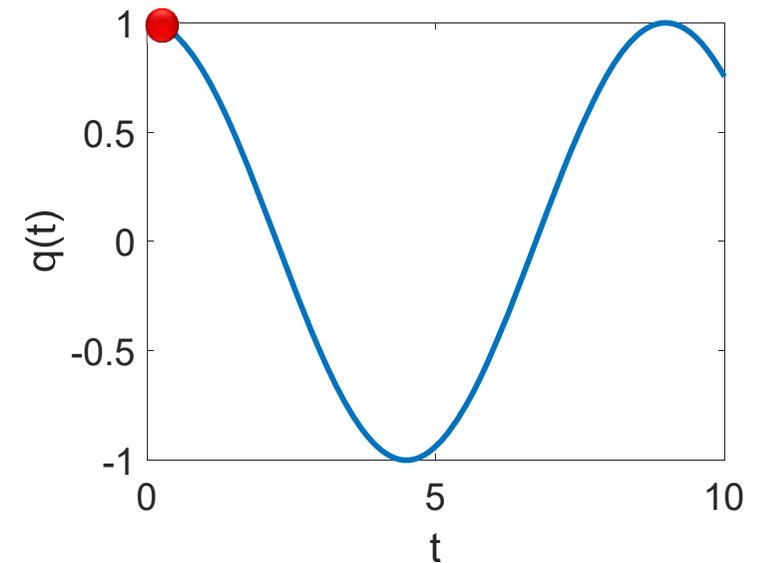
**Teacher:**



In which space does  $q(t)$  “live”?

The coefficient  $q(t)$  exists in  **$q$ -space**. The structure of this space depends on the mode decomposition: for instance, using plane waves as modes leads to a  $q$ -space that is the **Fourier space**.

**Teacher:**



## 2.2.1.- What is a classical field? Mode-decomposition



But are there Maxwell's (or Hamilton's) equations for describing these  $q(t)$ ?

**Teacher:** Yes, they are mathematically different equations, but conceptually they are the same. For example, you can write the **energy of the electromagnetic field** using  $q(t)$  as:

$$H_R = \frac{\epsilon_0}{2} \int d^3r (E^2 + c^2 B^2) = \sum_i \frac{\hbar \omega_i}{2} (q_i^2 + s_i^2)$$



What is  $s(t)$ ? What is  $\omega_i$ ?

**Teacher:** The variable  $s(t)$  is related to the time derivative of  $q(t)$  (in the same way that  $p(t)$  is related to  $x(t)$  for an electron). The frequency  $\omega_i$  is related to the mode decomposition.

## 2.3.- Classical Electrodynamics: Light-matter interaction



How do the electron and the electromagnetic field interact?

Teacher:

As you know, electrons and the electromagnetic field interact via **Maxwell's equations**, which depend on the electron's charge and current densities, and via the **Lorentz force**, which depends on  $E$  and  $B$ .



But can you give a concrete example of how an **electron**, represented by  $x$  and  $p$ , interacts with the **electromagnetic field**, represented by  $q$  and  $s$ ?

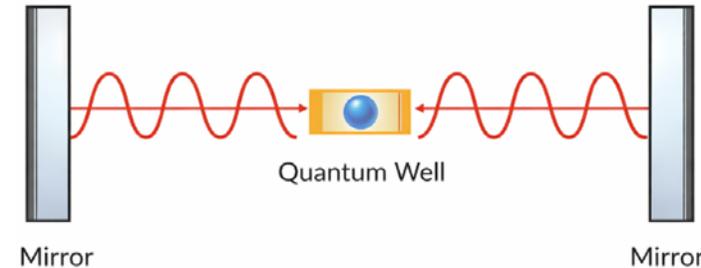
Teacher:

Ok, let me explain a concrete example, valid under some approximations.

## 2.3.- Classical Electrodynamics: Light-matter interaction

**Teacher:** We adopt the following approximations:

- (1).- A **single 1D electron** is considered.
- (2).- Only a **single mode** of the field.
- (3).- The **long-wavelength approximation**.



With these approximations, the total energy of the system, including the interaction energy, is expressed as:

$$H = -\frac{p^2}{2m_e} + V(x) + \frac{\hbar\omega}{2}(q^2 + s^2) + \alpha \cdot x \cdot q$$

Electron in the well + light in a cavity + **interaction**

Solving **Hamilton's equations** yields the light-matter interacting solutions:



What is  $\alpha$  ?

$$\begin{aligned} x &\rightarrow x(t) & q &\rightarrow q(t) \\ p &\rightarrow p(t) & s &\rightarrow s(t) \end{aligned}$$

**Teacher:**

The parameter  $\alpha$  controls the interaction. The value  $\alpha = 0$  means no interaction between light and matter.

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## 3.- Quantum Electrodynamics with wave functions

3.1.- Quantum Electrodynamics: What is an electron?

3.2.- Quantum Electrodynamics: What is a field?

3.2.1.- What is a quantum field? Mode-decomposition

3.3.- Quantum Electrodynamics: What is a photon?

3.4.- Quantum Electrodynamics: Light-matter interaction

## 3.1.- Quantum Electrodynamics: What is an electron?



What is a **quantum electron**?

A **quantum electron** can be viewed as a superposition of electrons in well-defined positions, ● each occurring with a certain probability.

Teacher:



But what is the position of a quantum electron?

Teacher:

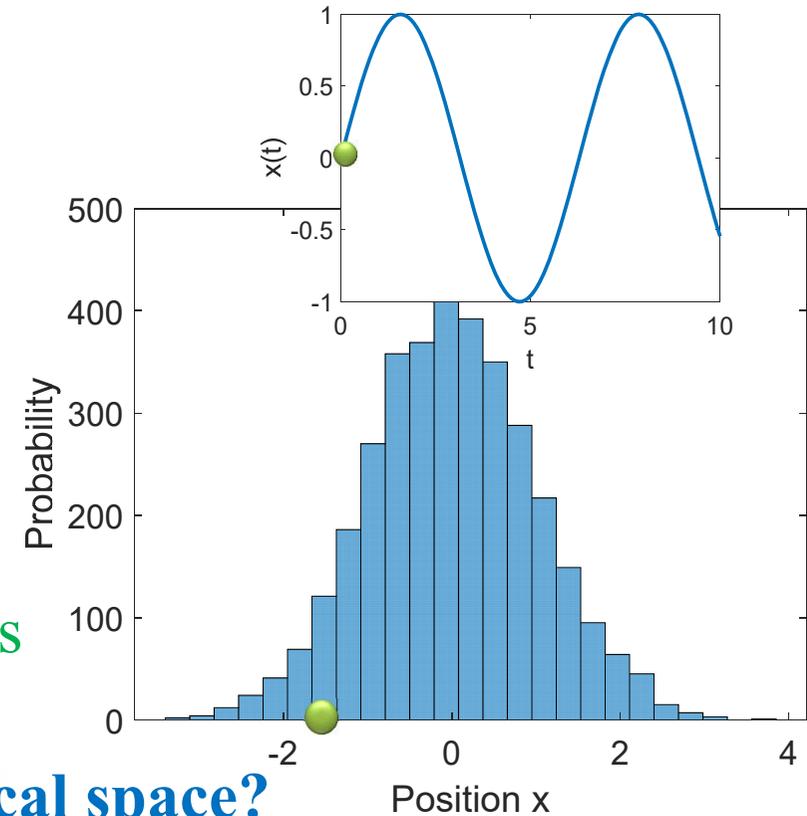
The **position** of a quantum electron is **uncertain** unless it is measured.



Can I **visualize** the electron in **physical space**?

Teacher:

Yes! You can **visualize** the probability of its position. Each point in physical space has a probability of being found when measured.

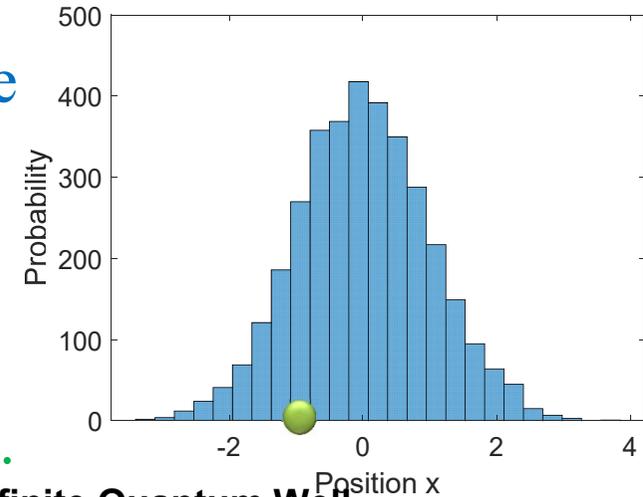


# 3.1.- Quantum Electrodynamics: What is an electron?



How I know the **probability distribution** of the electron at **different times?**

$$|\Psi(x, t)|^2 =$$



**Teacher:** The probability distribution of  $x$  is the square modulus of the wave function,  $\Psi(x,t)$ , solution of the Schrödinger equation.

1. Canonical quantization of the classical Hamiltonian:

$$[x,p]=i\hbar\mathbf{1} \quad p \rightarrow -i\hbar \frac{\partial}{\partial x} \quad x \rightarrow x$$

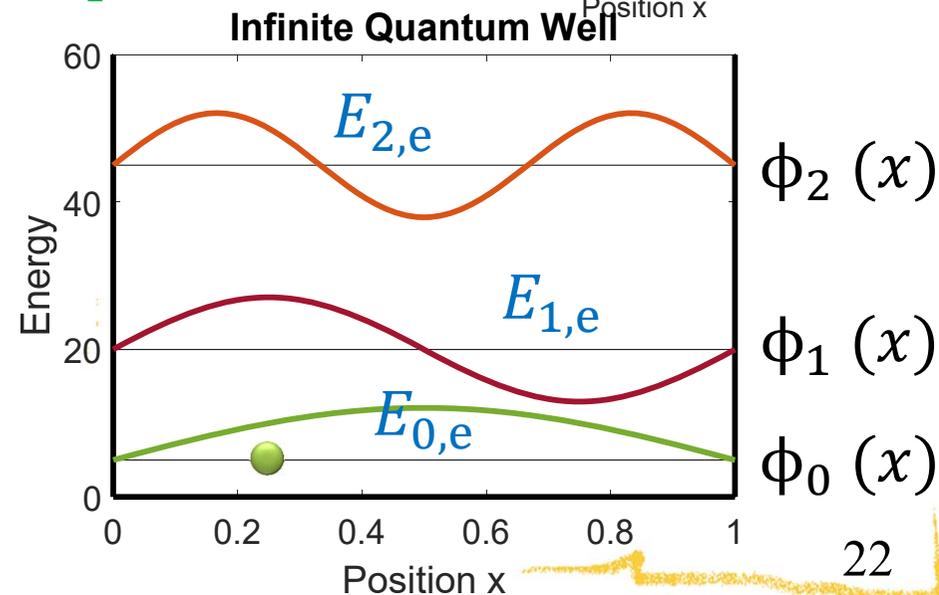
$$H_M = -\frac{p^2}{2m_e} + V(x)$$

2. Time-dependent Schrodinger equation:

$$i\hbar \frac{\partial \Psi(x, t)}{\partial t} = -\frac{\hbar^2}{2m_e} \frac{\partial^2 \Psi(x, t)}{\partial x^2} + V(x)\Psi(x, t)$$

3. Time-independent Schrodinger equation:

$$H_M \phi_i(x) = E_{i,e} \phi(x)$$



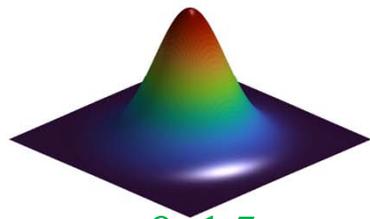
## 3.2.- Quantum Electrodynamics: What is a quantum field?



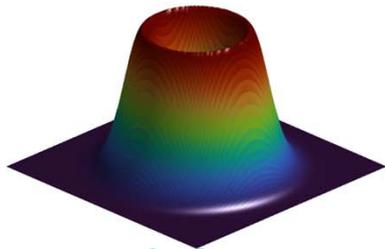
Teacher:

What is a **quantum field**?

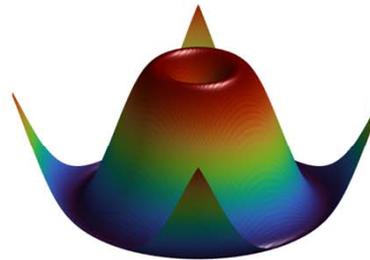
A **quantum field** can be viewed as a **superposition** of well-defined fields, each occurring with a certain probability



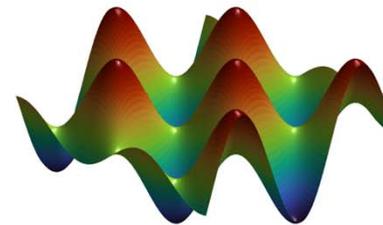
0.15



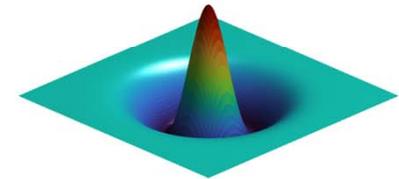
0.25



0.2



0.3



0.1

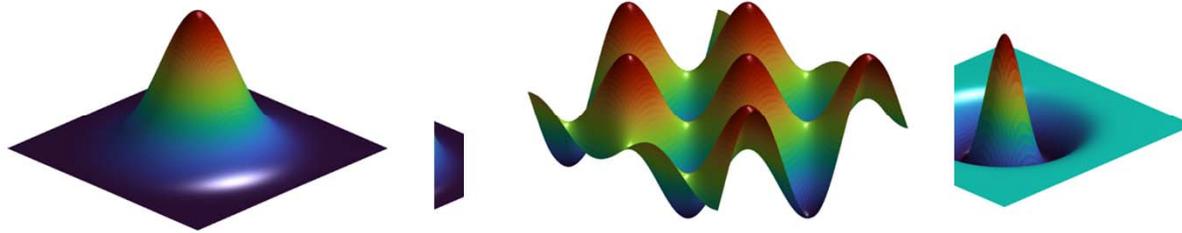


Teacher:

Which are the **properties** of the quantum field?

As with the quantum electron, the **properties** of a quantum field are **uncertain** unless measured.

## 3.2.1.- What is a quantum field? Mode-decomposition



Teacher:

Is it possible to **visualize** a quantum field in physical space?

It is not easy to visualized in physical space because each well-defined field is already defined there. You must assign a probability to each **field configuration** (not to each point in physical space).



Is it possible to use classical **mode decomposition** to aid in visualizing a quantum field?

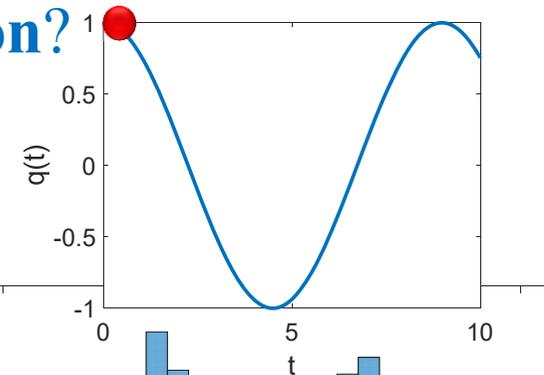
Teacher: **Wonderful** question. Yes! Let's do it.

## 3.2.1.- What is a quantum field? Mode-decomposition



What is a **quantum field** with **mode decomposition**?

A quantum field can be viewed as a superposition of fields, each characterized by a well-defined  $q$ , with each mode configuration having a certain probability.



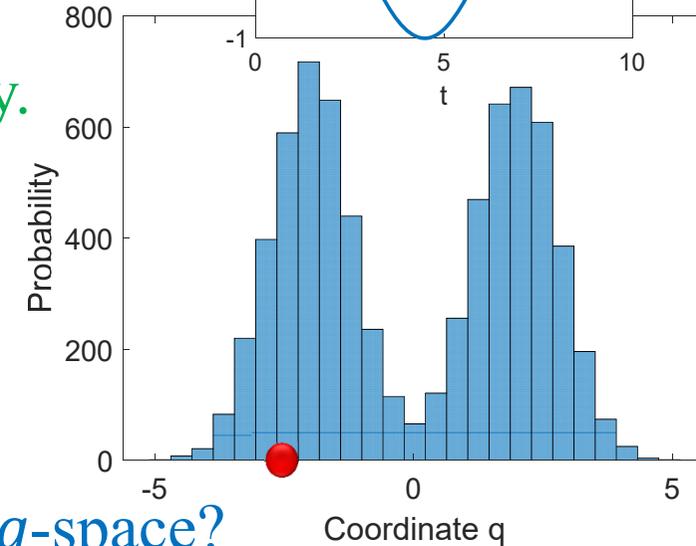
Teacher:



But which is the value  $q$  of the **quantum field**?

Teacher:

The **value of  $q$**  of the quantum field is **uncertain** unless measured.



Is it possible to **visualize** a quantum field in  $q$ -space?

Yes! As with the electron, you can **visualize** its probability in  $q$ -space. Each **point in  $q$ -space** has a probability.

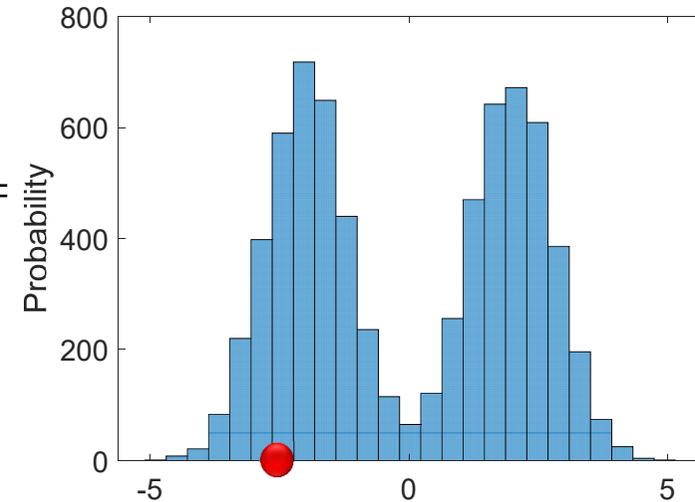
Teacher:

## 3.2.1.- What is a quantum field? Mode-decomposition



How can I **know** the probability distribution of the **quantum field** at different times?

$$|\Psi(q, t)|^2 =$$



**Teacher:** The probability distribution is the square modulus of the wave function  $\Psi(q, t)$ , solution of the Schrödinger equation.

1. Canonical quantization of the classical Hamiltonian:

$$[q, s] = i\mathbf{1} \quad q \rightarrow q \quad s \rightarrow -i \frac{\partial}{\partial q}$$

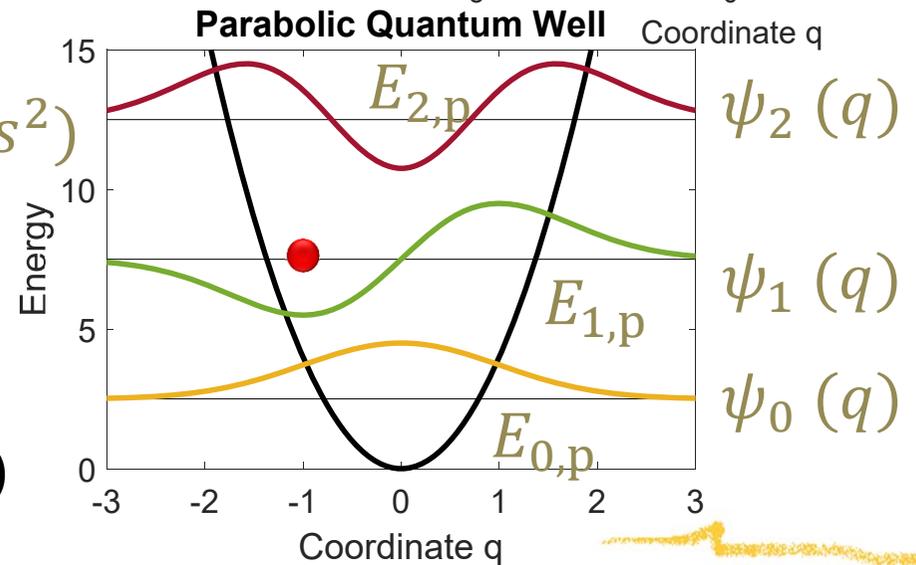
$$H_R = \frac{\hbar\omega}{2} (q^2 + s^2)$$

2. Time-dependent Schrodinger equation:

$$i\hbar \frac{\partial \Psi(q, t)}{\partial t} = -\frac{\hbar\omega}{2} \frac{\partial^2 \Psi(q, t)}{\partial q^2} + \frac{\hbar\omega}{2} q^2 \Psi(q, t)$$

3. Time-independent Schrodinger equation:

$$H\psi_i(q) = E_{i,p}\psi_i(q)$$



## 3.3.- Quantum Electrodynamics: What is a photon?



What is a **photon**: A wave or a particle?

**Teacher:**

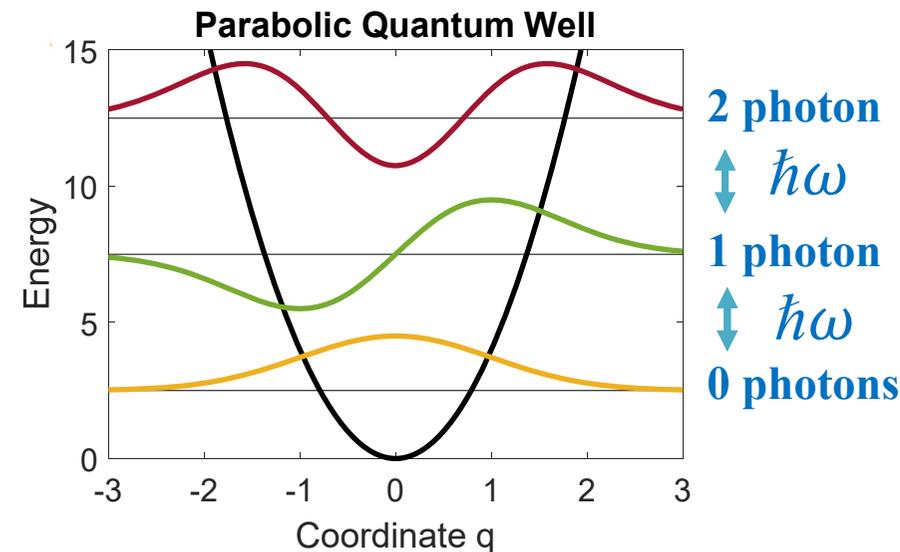
A photon is a **quantum of energy** of a single-mode electromagnetic field,  $\hbar\omega$ . The energy of the field eigenstates increases in integer multiples of this photon energy.



But the name “photon” really seems like a name for a particle?

**Teacher:**

Well, if you name the amount of energy  $\hbar\omega$  a particle, then when the electromagnetic field exchanges energy with the electron, you will have to say that photons are created and annihilated. You can use this language, but it is not mandatory!



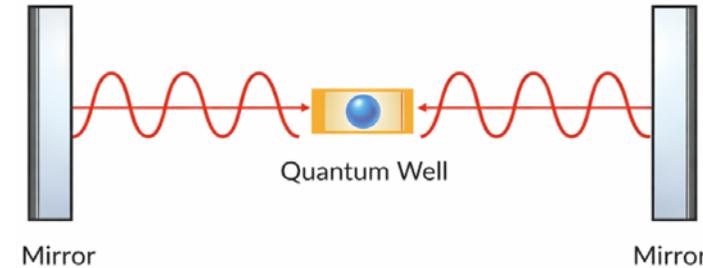
## 3.4.- Classical Electrodynamics: light-matter interaction



Can you give a concrete example of how a **quantum electron** interacts with a **quantum electromagnetic field** ?

**Teacher:** We adopt the classic example:

- (1).- A **single 1D electron** is considered.
- (2).- Only a **single mode** of the field.
- (3).- The **long-wavelength** approximation.



With these approximations, the total energy of the system, including the interaction energy, is expressed as:

1. Canonical quantization of the classical Hamiltonian:  $[x,p]=i\hbar 1$   
 $[q,s]=i 1$

$$H = -\frac{p^2}{2m_e} + V(x) + \frac{\hbar\omega}{2} (q^2 + s^2) + \alpha \cdot x \cdot q$$

2. Time-dependent Schrodinger equation:

$$p \rightarrow -i\hbar \frac{\partial}{\partial x} \quad x \rightarrow x \quad q \rightarrow q \quad s \rightarrow -i \frac{\partial}{\partial q}$$

$$i\hbar \frac{\partial \Psi(x, q, t)}{\partial t} = -\frac{\hbar^2}{2m_e} \frac{\partial^2 \Psi(x, q, t)}{\partial x^2} + V(x) \Psi(x, q, t) - \frac{\hbar\omega}{2} \frac{\partial^2 \Psi(x, q, t)}{\partial q^2} + \frac{\hbar\omega}{2} q^2 \Psi(x, q, t) + \alpha \cdot x \cdot q \cdot \Psi(x, q, t)$$

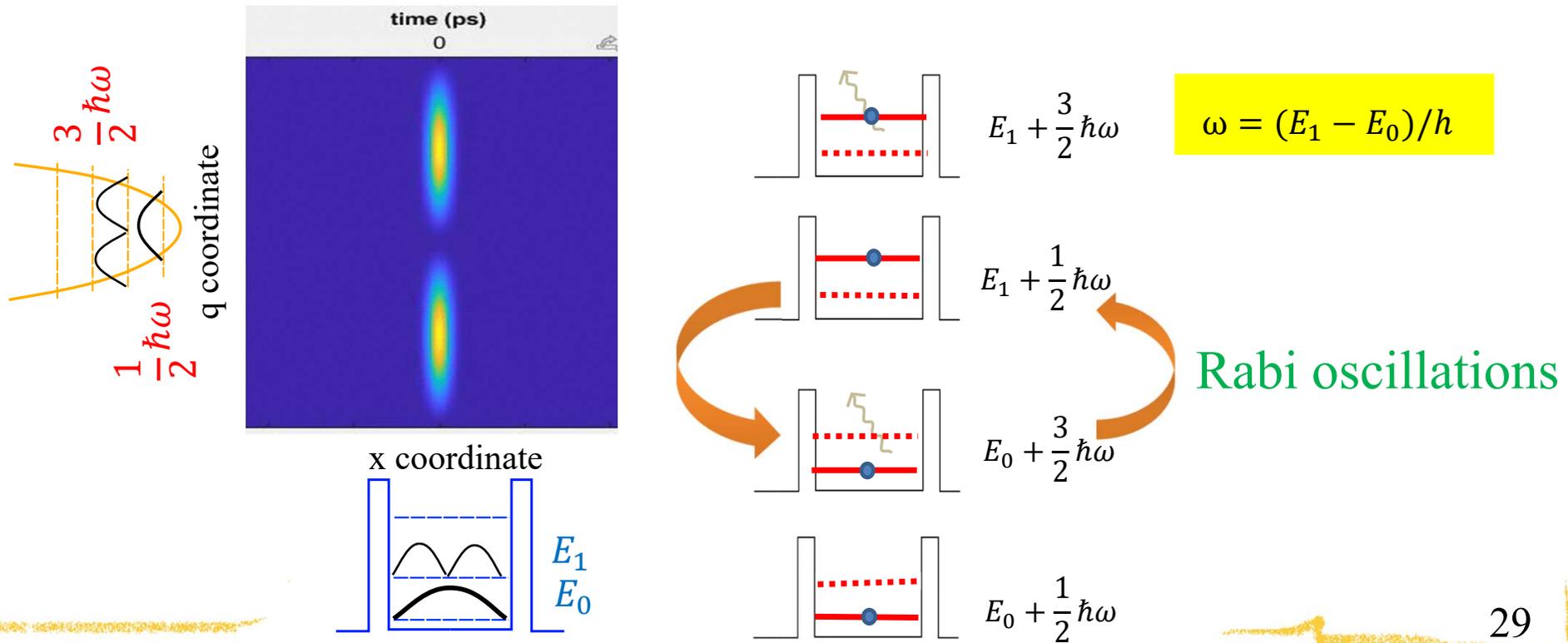
## 3.4.- Classical Electrodynamics: light-matter interaction



Can we **visualize** the wave function  $\Psi(x, q, t)$  ?

We consider an example of an electron inside an optical cavity,

**Teacher:** under resonant conditions.



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## 4.- Quantum Electrodynamics with $a^\dagger$ and $a$ operators

4.1.- Quantum Electrodynamics: What is a field?

4.2.- Quantum Electrodynamics: What is a photon?

4.3.- Quantum Electrodynamics: What is an electron?

## 4.1.- Quantum Electrodynamics: What is a field?



Can you explain how the **wave function** description of the electromagnetic field relates to the formulation using **creation** and **annihilation** operators?

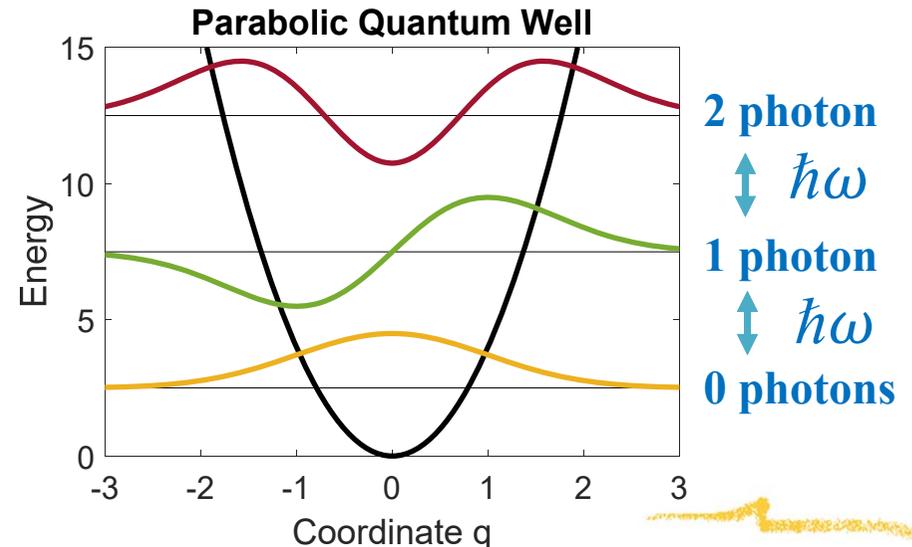
**Teacher:**

Using Dirac's **bra-ket notation**, the wave function  $\psi_i(q)$  is expressed as  $\psi_i(q) = \langle q | \psi_i \rangle = \langle q | i \rangle$ . For a harmonic oscillator, the same state  $\psi_i(q)$  can equivalently be described in the operator formalism using **creation** ( $a^\dagger$ ) and **annihilation** ( $a$ ) operators.

$$\psi_2(q) = \langle q | \psi_2 \rangle = \langle q | 2 \rangle = \langle q | a^\dagger a^\dagger | 0 \rangle$$

$$\psi_1(q) = \langle q | \psi_1 \rangle = \langle q | 1 \rangle = \langle q | a^\dagger | 0 \rangle$$

$$\psi_0(q) = \langle q | \psi_0 \rangle = \langle q | 0 \rangle$$

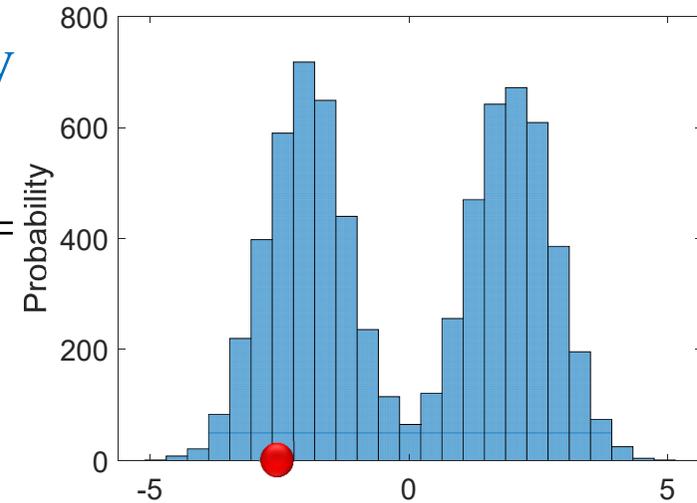


# 4.1.- Quantum Electrodynamics: What is a field?



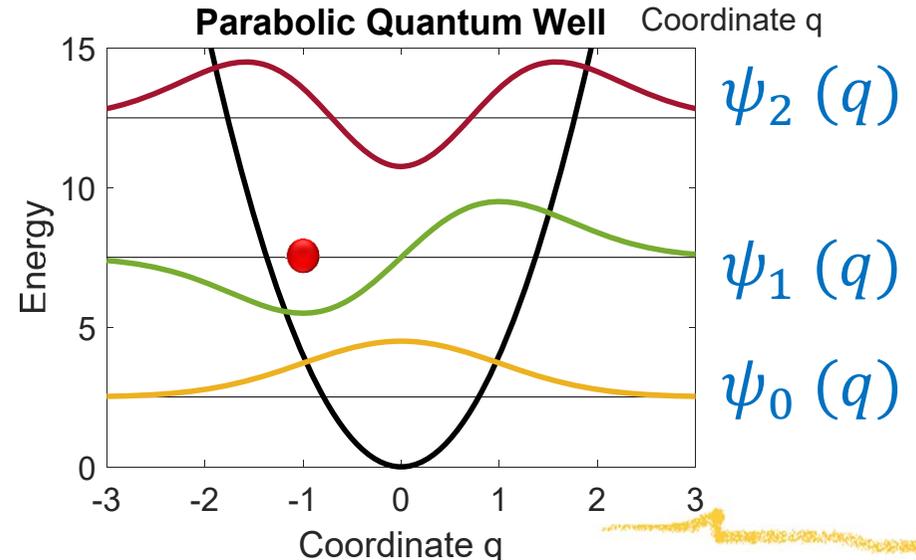
How is the state  $\Psi(q, t)$  giving the probability distribution of the **quantum field** at different times represented?

$$|\Psi(q, t)|^2 =$$



**Teacher:** The state  $\Psi(q, t)$  can be expressed as a superposition of energy eigenstates

$$\begin{aligned} \Psi(q, t) &= \langle q | \Psi \rangle = \sum_i c_i(t) \psi_i(q) \\ &= \sum_i c_i(t) \langle q | i \rangle = \sum_i c_i(t) \langle q | (a^\dagger)^i | 0 \rangle \end{aligned}$$



## 4.1.- Quantum Electrodynamics: What is a field?



Can I say that the quantum state is  $|\Psi\rangle = \sum_i c_i(t) (a^\dagger)^i |0\rangle$  ?

Teacher:

Yes. It is a **superposition** of ordinary quantum states, each with a different number of particles.



This state (with a different number of particles) lives in **Fock space** defined as  $F = H^0 \oplus H^1 \oplus H^2 \oplus H^3 \oplus \dots$

Teacher:

Yes! This language is correct, but it is not mandatory! With this language, it may seem that the light–matter interaction creates and annihilates photons (e.g.  $|1\rangle = a^\dagger |0\rangle$ ) but such interaction can also be understood as an exchange of energy between light and matter.

Moreover, if you **eliminate** the coordinate  $\langle q|$  in your description, you will have difficulties **visualizing a quantum field** in q-space.

## 4.3.- Quantum Electrodynamics: What is an electron?



What is an electron ?

Within this formalism, one can use the language of creation and annihilation operators to “create” an electron,  $|1\rangle = a^\dagger |0\rangle$ , in the Fock space of electrons.

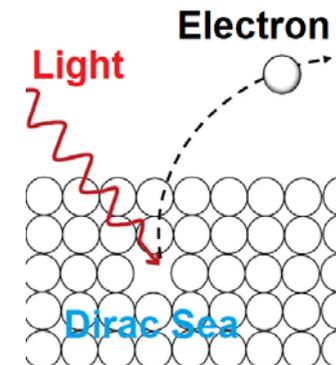
**Teacher:**



But are electrons really created?

**Teacher:** Well, in the language of wave functions, you do not create electrons.

Nevertheless, certain interactions involving electrons and positrons may be interpreted as electron creation processes. But also, as an excitation of an electron from a state below the Dirac sea to a state above it, without “creating” anything.



1.- Introduction: What are we talking about?

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## 5.- Quantum Electrodynamics with quantum trajectories

5.1.- Quantum Electrodynamics: What is an electron?

5.2.- Quantum Electrodynamics: What is a field?

5.3.- Quantum Electrodynamics: What is a photon?

5.4.- Quantum Electrodynamics: Light-matter interaction

## 5.- Quantum Electrodynamics with quantum trajectories



Dear teacher, we still think we have not properly understood what a quantum electron and a quantum field are.

Teacher:

I have provided two mathematically equivalent formulations. What aspect remains unclear?



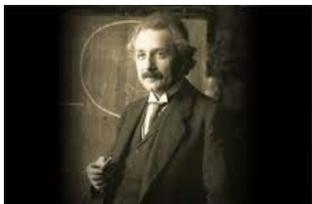
You say that a quantum electron and a quantum field have **no well-defined properties** until they are measured.

Teacher:

Yes, the quantum theories I explained worked this way.



But if the electron is not measured, where is a non-measured electron? Nowhere?



Is the moon there when nobody looks?

## 5.- Quantum Electrodynamics with quantum trajectories

**Teacher:** Ok, I will introduce a **third formulation**, empirically equivalent to the previous ones, which allows a **realistic description** of all quantum electrodynamic phenomena (retaining well-defined electron positions and electromagnetic fields, even without measuring them).

1. Canonical quantization of the classical Hamiltonian:

$$\begin{aligned} [x,p] &= i\hbar \mathbf{1} \\ [q,s] &= i\mathbf{1} \end{aligned}$$

$$H = -\frac{p^2}{2m_e} + V(x) + \frac{\hbar\omega}{2}(q^2 + s^2) + \alpha \cdot x \cdot q$$

2. Time-dependent Schrodinger equation:

$$p \rightarrow -i\hbar \frac{\partial}{\partial x} \quad x \rightarrow x \quad q \rightarrow q \quad s \rightarrow -i \frac{\partial}{\partial q}$$

$$i\hbar \frac{\partial \Psi(x, q, t)}{\partial t} = -\frac{\hbar^2}{2m_e} \frac{\partial^2 \Psi(x, q, t)}{\partial x^2} + V(x) \Psi(x, q, t) - \frac{\hbar\omega}{2} \frac{\partial^2 \Psi(x, q, t)}{\partial q^2} + \frac{\hbar\omega}{2} q^2 \Psi(x, q, t) + \alpha \cdot x \cdot q \cdot \Psi(x, q, t)$$

3. Continuity equation from the Schrödinger equation and extract “local” velocities for x and q:

$$v^{(e)}(x, q, t) = \frac{J^{(e)}(x, q, t)}{|\Psi(x, q, t)|^2} \quad v^{(f)}(x, q, t) = \frac{J^{(f)}(x, q, t)}{|\Psi(x, q, t)|^2}$$

4. From these velocities, you can extract the trajectories of the electron  $x(t)$  and the mode  $q(t)$ .

## 5.1.- Quantum Electrodynamics: What is an electron?



What is a **quantum electron** with this **realistic visualization**?

**Teacher:**

A quantum electron, in a single experiment, is given by  $x(t)$ .



Can we visualize it as a trajectory in **physical space**?

**Teacher:**

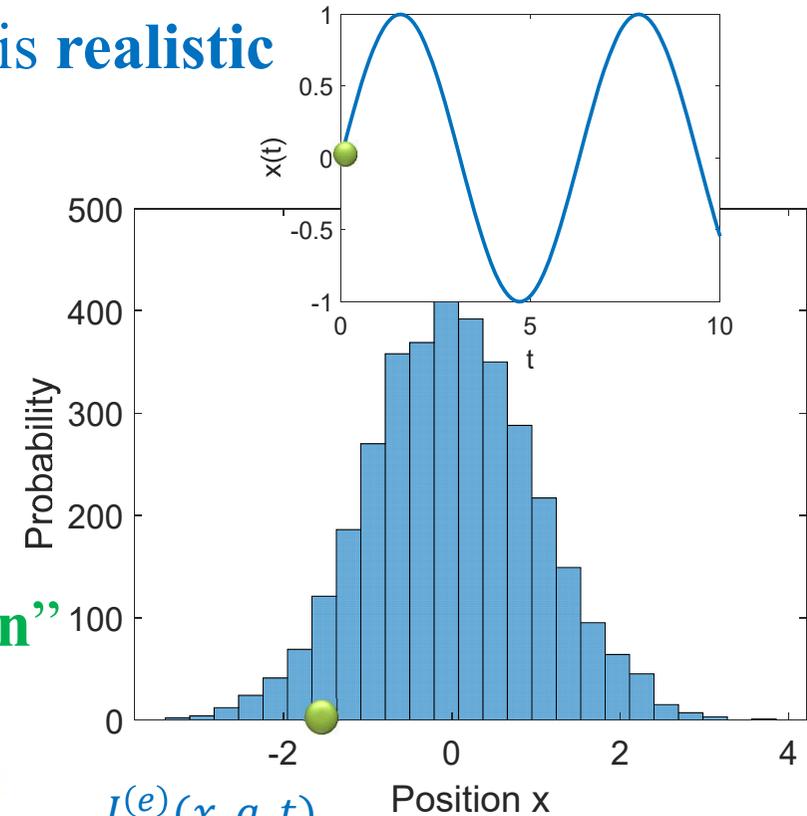
Yes! But this quantum electron  $x(t)$  moves influenced by a “**superposition**” of others  $x$ 's(t).



It moves by the wavefunction:  $v^{(e)}(x, q, t) = \frac{J^{(e)}(x, q, t)}{|\Psi(x, q, t)|^2}$

**Teacher:**

Yes! The wave function guides this electron!



## 5.2.- Quantum Electrodynamics: What is a field?



What is a **quantum field** with this **realistic visualization**?

**Teacher:** A quantum field, in a single experiment, is given by  $q(t)$ .



Can we visualize it as a function, defined everywhere, in **q-space**?

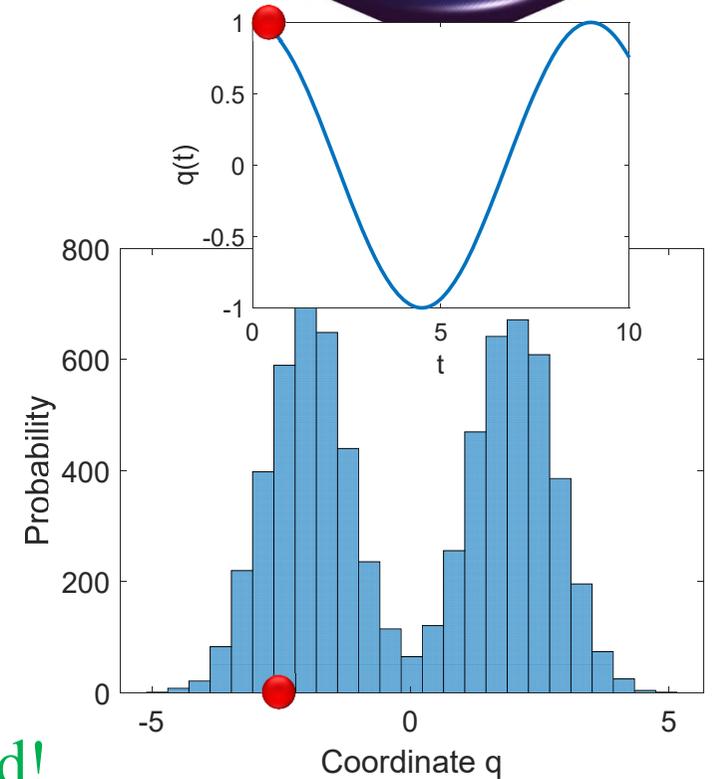
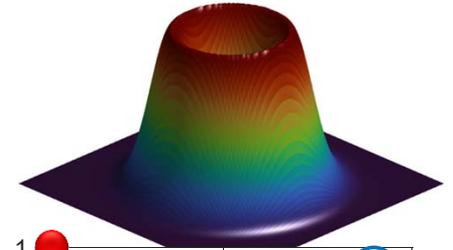
**Teacher:** Yes! But this  $q(t)$  moves influenced by a “**superposition**” of other’s  $q(t)$ .



It moves by the wavefunction

$$v^{(f)}(x, q, t) = \frac{J^{(f)}(x, q, t)}{|\Psi(x, q, t)|^2}$$

**Teacher:** Yes! The wave function guides this field!



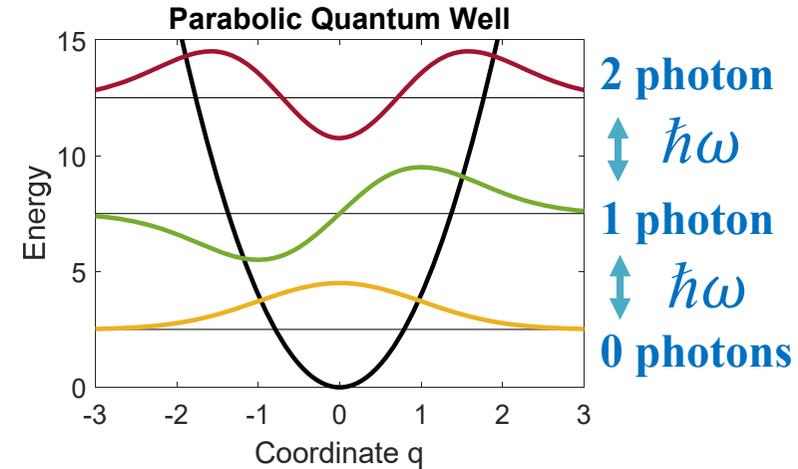
## 5.3.- Quantum Electrodynamics: What is a photon?



What is a **photon** ?

The **photon** in this picture is the same as in the picture of quantum electrodynamics with wave functions.

Teacher:



A **photon** is a **quantum of energy** of a single-mode electromagnetic field,  $\hbar\omega$ . The energy of the field eigenstates increases in integer multiples of  $\hbar\omega$ .



Is  $q(t)$  the trajectory of a photon?

No! The coordinate  $q(t)$  describes the time evolution of (one mode of) the electromagnetic field. Sometimes the field energy corresponds to one photon, and at other times it corresponds to two photons.

Teacher:

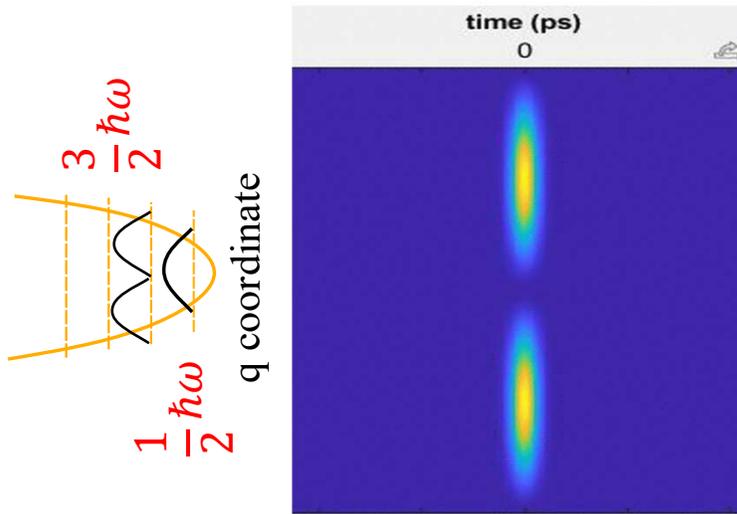
# 5.4.- Quantum Electrodynamics: Light-matter interaction



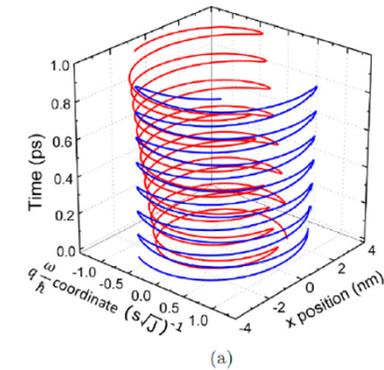
Can we visualize the quantum trajectories  $x(t)$  and  $q(t)$  ?

We consider the same example of an electron inside an optical cavity, under resonant conditions.

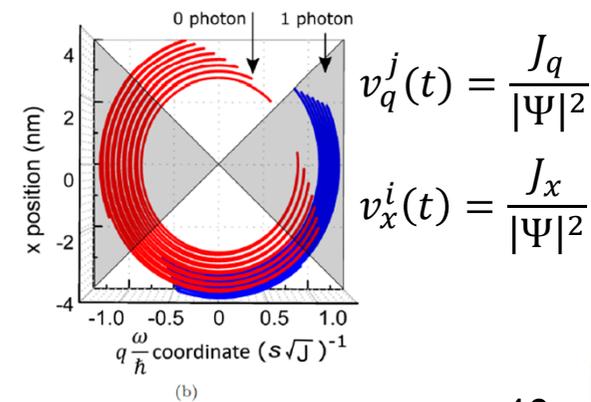
Teacher:



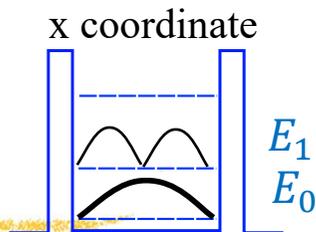
The wave function (left) provides information about the ensemble of experiments.



The red and blue trajectories (right) each correspond to the well-defined properties of a single experiment.



$$\omega = (E_1 - E_0)/\hbar$$



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## 6.- Examples of the program QC slim

6.1.- The QC slim software

6.2.- Classical Electrodynamics

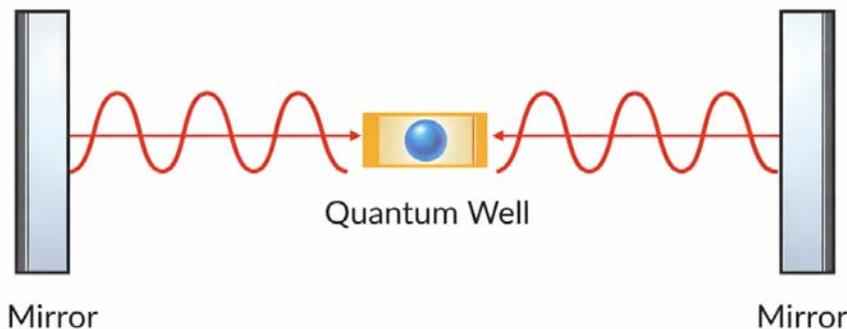
6.3.- Semiclassical Electrodynamics

6.3.- Quantum Electrodynamics with wave-functions

6.4.- Quantum Electrodynamics with trajectories

## 6.1.- The QC slim software

**Closed system:** Electron in an infinite quantum well inside an optical cavity defined by the perfect mirrors



We adopt the following approximations:

- (1).- A **single 1D electron** is considered.
- (2).- Only a **single mode** of the field.
- (3).- The **long-wavelength approximation**.

Solutions of the classical, semiclassical, and quantum equations of motion derived from the Hamiltonian.

Light-matter Hamiltonian in canonical variables:

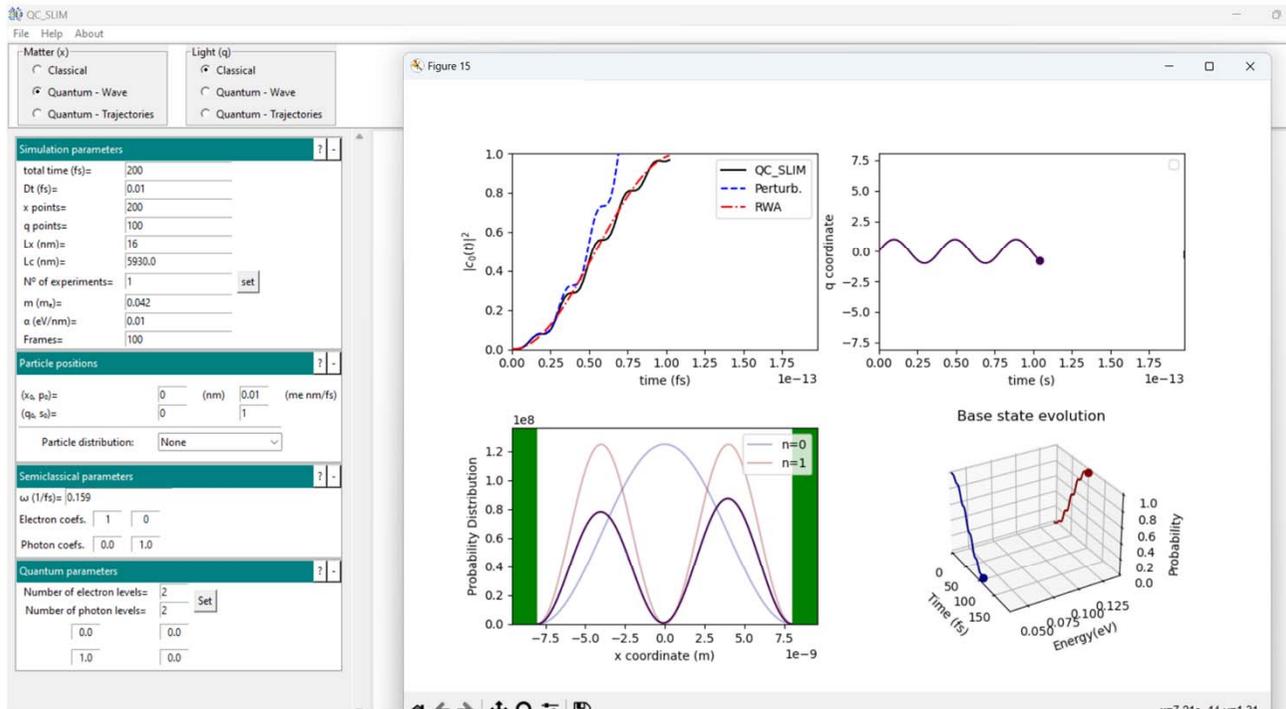
$$H = -\frac{p^2}{2m_e} + V(x) + \frac{\hbar\omega}{2}(q^2 + s^2) + \alpha \cdot x \cdot q$$

Electron in the well + light in a cavity + **interaction**

<i>electron</i>		<i>Light</i>		
$p$	$x$	$q$	$s$	
$p \rightarrow -i\hbar \frac{\partial}{\partial x}$	$x \rightarrow x$	$q$	$s$	<b>Classical</b>
$p \rightarrow -i\hbar \frac{\partial}{\partial x}$	$x \rightarrow x$	$q \rightarrow q$	$s \rightarrow -i \frac{\partial}{\partial q}$	<b>Semi-classical</b>
$p \rightarrow -i\hbar \frac{\partial}{\partial x}$	$x \rightarrow x$	$q \rightarrow q$	$s \rightarrow -i \frac{\partial}{\partial q}$	<b>Quantum</b>

# 6.1.- The QC slim software

User-friendly interface:



User's arbitrary selection of parameters

Graphical visualization of the results

Classical, semiclassical, quantum simulations

## 6.1.- The QC slim software

### Concrete simulated examples

*Rabi oscillations*

*Graphical visualization of the interchange of probabilities between the different components of the electrons and photons states, satisfying conservation rules.*

*Spontaneous emission  
(vacuum fluctuations)*

*No spontaneous emission for (zero) classical light*

*Stimulated emission and absorption*

*Creation and annihilation of photons as a transition between energy eigenstates*

*Test accuracy of perturbative and rotating wave approximations*

*Compare analytical approximate results with exact simulated results.*

*Weak values of electrons/light*

*The quantum trajectories of electrons/light corresponds to the empirical accessible weak values of the momentum*

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## 7.- Conclusions

Although quantum and classical mechanics differ, the differences are not as radical as they may first appear. Emphasizing their similarities can help one understand quantum electrodynamics as a natural extension of the classical description.

In ordinary quantum mechanics of matter, valuable insight is obtained by visualizing dynamical behavior through wave functions (or quantum trajectories). A similar strategy can enhance our understanding of quantum electrodynamics.

Being familiar with multiple formulations and understanding how they relate to each other is especially beneficial.

For a comfortable understanding of quantum electrodynamics (and any physical theory), it is useful to identify the (ontologically) real elements in the theory—in our case, the nature of electrons, electromagnetic fields, and photons.

*The QCslim software has been designed according to this pedagogical approach.*