D'yakonov-Perel' spin decay in the weak scattering regime and the case of graphene

Xavier Cartoixà[1]

[1] *Departament d'Enginyeria Electrònica, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain*
e-mail address: Xavier.Cartoixa@uab.cat

I. INTRODUCTION

In the past two decades, there has been a strong interest in spin effects in semiconductors for applications in spintronics[1,2]. The use of electron spin for information processing, storage and transmission naturally demands a detailed knowledge of the spin dynamics within the host material.

When a spin imbalance is created in a semiconductor, typically there are three mechanisms that will try to restore equilibrium, namely (a) the Elliot-Yafet (EY) mechanism[3,4], related to the fact that states with a general $\mathbf{k}$ are not strict spin eigenstates, and thus there is a finite probability for a spin flip at an otherwise spin-independent scattering event; (b) the Bir-Aronov-Pikus mechanism[5], related to a hyperfine-like interaction between electrons in the conduction band and holes in the valence band; and (c) the D'yakonov-Perel'[6] (DP) mechanism, related to spin precession about a random magnetic effective field caused by the spin-orbit interaction in a non-centrosymmetric system.

As of lately, the interest in the dynamics of spins in 2DEGs has revived due to the special characteristics of graphene[7]. In the traditional analysis of the DP mechanism[6,8], expressions are obtained for the spin lifetime tensor in the strong scattering regime, meaning that $\omega_L\tau_p \ll 1$, where ω_L is a characteristic Larmor frequency for spin precession and τ_p is the momentum relaxation time. Under these conditions, it can be shown[6,8–11] that the spin lifetime τ_s is proportional to τ_p^{-1}, a feature that is extensively used to discriminate the DP mechanism from other forms of spin decay. Experimental observations in graphene have found that at low temperatures $\tau_s \propto \tau_p$, as characteristic for the EY mechanism[12]; however, results at low carrier densities suggest that a combination of both mechanisms is at play[13].

We will present in this conference theoretical evidence that the results in graphene can be understood from the DP mechanism operating in the *weak* scattering regime, e.g. $\omega_L\tau_p \gg 1$. There have been some passing comments[14,15] that $\tau_s \propto \tau^p$ when $\omega_L\tau_p \gg 1$, and here we will present a series expression for the spin lifetime valid in the general $\omega_L\tau_p$ case confirming the weak scattering behavior $\omega_L\tau_p \gg 1$. Obtaining a closed (or few-term) form for the general case remains an open question.

II. RESULTS

The Hamiltonian describing the spin-orbit interaction for electrons in the conduction band of an n-dimensional semiconductor can usually be written as $H(\mathbf{k}) = \hbar/2\ \mathbf{\Omega}(\mathbf{k})\cdot\boldsymbol{\sigma}$, where $\mathbf{\Omega}$ is a precession vector, which depends on the electron momentum $\mathbf{k}$, and $\boldsymbol{\sigma}$ is a vector composed of the three Pauli matrices. This Hamiltonian indicates that spins evolve deterministically until the electron scatters, when a new $\mathbf{k}$ is obtained probabilistically. Thus, the overall evolution of a single spin is a stochastic problem.

The time evolution of a single spin can be cast quantically as

$$i\hbar\frac{\partial|\psi\rangle}{\partial t} = H[\mathbf{k}(t)]|\psi\rangle, \tag{1}$$

where $|\psi\rangle$ is the spinor describing the state of an individual state. The spinor univocally determines an expectation value; and the converse is also true except for an overall phase of the spinor. If we define $\mathbf{s} \equiv \langle\psi|\boldsymbol{\sigma}|\psi\rangle$, a standard derivation from Eq. (1) readily yields the classical expression

$$\dot{\mathbf{s}} = \mathbf{\Omega}[\mathbf{k}(t)] \times \mathbf{s} = \overline{\overline{\Omega}}[\mathbf{k}(t)]\,\mathbf{s}, \tag{2}$$

where $\overline{\overline{\Omega}}$ is an antisymmetric matrix providing the vector product of $\mathbf{\Omega}$ with whatever follows.

Since, in general, Hamiltonians at different times do not commute, the time evolution operator cannot be written as $U(t,t_0) = \exp\left\{-i/\hbar\int_{t_0}^{t} H(t')\,dt'\right\}$. Most commonly, the solution to Eq. (1) is written as a Dyson series[16], though lately the Magnus expansion[17] applied to Eq. (2) has gained favor[18] because it preserves the unitarity of the evolution to all orders in the series.

The techniques from the preceding paragraph and others[8,11] are well suited for the study of spin dynamics in the strong scattering regime. In the conference, we will present a highly intuitive approach naturally suited to the study of the weak scattering regime. Let us study the evolution of a *single* spin, initially at $\mathbf{s}_i$ from an initial time t_0 to a final time t. Class n processes will be those where the electron momentum has scattered n times during the interval $(t-t_0)$, and the spin has evolved into a value $\mathbf{s}_n(t)$ given by

$$\mathbf{s}_n(t) = \prod_{j=0}^{n} \frac{1}{t_{j+2}-t_0}\int_{t_0}^{t_{j+2}} dt_{j+1}\, R\left[(t_{j+1}-t_j)\mathbf{\Omega}(\mathbf{k}_j)\right]\mathbf{s}_i, \tag{3}$$

where t_j is the time at which an individual scattering event takes place, $t_{j\geq n} = t$ and proper averaging of the times of the scattering events has been made[19]. The probability that evolution from t_0 to t has actually been of class n will be given by a Poisson distribution with mean τ_p, the momentum scattering time. From

Eq. (3) the following recurrence relationship immediately follows:

$$\mathbf{s}_n(t) = R\left[(t-t_n)\mathbf{\Omega}(\mathbf{k}_n)\right]\mathbf{s}_{n-1}(t_n)\ ,\ p_n = \frac{(t/\tau_p)^n e^{-t/\tau_p}}{n!}, \tag{4}$$

where $R[t\mathbf{\Omega}]$ is a rotation of angle $t|\mathbf{\Omega}|$ about the vector $\mathbf{\Omega}$, and with the Poisson distribution explicitly stated. The time evolution of an ensemble will necessarily involve additional integrations over the distribution of momenta of the particles.

The results above are valid for any values of $\omega_L\tau_p$, but it is hard to extract any information valid for all regimes. However, under weak scattering conditions only the first terms in the series will need to be taken into account. In the case of a 2D material with spherical bands and isotropic elastic scattering, for a given energy one readily has

$$\begin{aligned}\langle \mathbf{s}(t)\rangle \simeq\ & e^{-t/\tau_p}\left\{\frac{1}{2\pi}\int_0^{2\pi} d\theta_0 R\left[(t-t_0)\mathbf{\Omega}(k,\theta_0)\right] + \right.\\ & \frac{1}{(2\pi)^2\tau_p}\int_0^{2\pi} d\theta_0 \int_0^{2\pi} d\theta_1 \int_0^t dt_1 R\left[(t-t_1)\mathbf{\Omega}(k,\theta_1)\right]\times \\ & \left. R\left[(t_1-t_0)\mathbf{\Omega}(k,\theta_0)\right]\right\}\mathbf{s}_i,\end{aligned} \tag{5}$$

where $\langle\rangle$ denotes an ensemble average.

Equation (5) clearly shows that in the weak scattering regime the spin lifetime will be exactly the momentum relaxation time, ie. $\tau_s = \tau_p$, even though we are in a situation where only the DP spin relaxation mechanism is allowed. Such conditions are expected to take place in high quality graphene, with mean free paths of the order of 1 μm at room temperature and 100's of μm's at low T[20], which translate into momentum relaxation times of the order of 1-100 ps, for a precession frequency of $\sim$120 THz for graphene on Au(111)[21].

Discussion – Among the many surprising properties of graphene, one of them might be that it is the first material where the DP spin relaxation mechanism manifests itself in the weak scattering regime, with the spin lifetime having the same temperature dependence as the momentum relaxation time. The treatment that will be presented at the conference allows obtaining results under a few specific conditions, but a more general formalism is still missing. The study of the information that can be provided by the characterization of the fluctuations in the spin signal is in its infancy[22,23]. Finally, it should always be kept in mind that a problem very similar to DP spin relaxation is studied in Nuclear Magnetic Resonance, and any insight from one field would have an immediate impact on the other.

ACKNOWLEDGMENTS

The author acknowledges financial support by the Spanish Ministerio de Economa y Competitividad under Project No. TEC2012-31330. Also, the research leading to these results has received funding from the European Union Seventh Framework Programme under grant agreement No. 604391 Graphene Flagship.

1 G. A. Prinz, Phys. Today **48**, 58 (1995).
2 S. A. Wolf, D. D. Awschalom, R. A. Buhrman, J. M. Daughton, S. von Molnár, M. L. Roukes, A. Y. Chtchelkanova, and D. M. Treger, Science **294**, 1488 (2001).
3 R. J. Elliott, Phys. Rev. **96**, 266 (1954).
4 Y. Yafet, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, New York, 1963), vol. 14, pp. 1–98.
5 G. L. Bir, A. G. Aronov, and G. E. Pikus, Zh. Eksp. Teor. Fiz. **69**, 1382 (1975), [Sov. Phys. JETP **42**, 705 (1976)].
6 M. I. D'yakonov and V. I. Perel', Fiz. Tverd. Tel. **13**, 3581 (1971), [Sov. Phys. Solid State **13**, 3023 (1972)].
7 D. V. Tuan, F. Ortmann, D. Soriano, S. O. Valenzuela, and S. Roche, Nature Physics **10**, 857 (2014).
8 G. E. Pikus and A. N. Titkov, in *Optical Orientation*, edited by F. Meier and B. P. Zakharchenya (North Holland, Amsterdam, Netherlands, 1984), vol. 8, pp. 73–131.
9 M. I. D'yakonov and V. Y. Kachorovskiĭ, Fiz. Tekh. Poluprovodn. **20**, 178 (1986), [Sov. Phys. Semicond. **20**, 110 (1986)].
10 W. Knap, C. Skierbiszewski, A. Zduniak, E. Litwin-Staszewska, D. Bertho, F. Kobbi, J. L. Robert, G. E. Pikus, F. G. Pikus, S. V. Iordanskii, et al., Phys. Rev. B **53**, 3912 (1996).
11 N. S. Averkiev and L. E. Golub, Phys. Rev. B **60**, 15582 (1999).
12 W. Han and R. K. Kawakami, Phys. Rev. Lett. **107**, 047207 (2011).
13 P. J. Zomer, M. H. D. Guimarães, N. Tombros, and B. J. van Wees, Phys. Rev. B **86**, 161416 (2012).
14 D. Culcer and R. Winkler, Phys. Rev. B **76**, 195204 (2007).
15 M. Dyakonov, in *Spin Physics in Semiconductors*, edited by M. I. Dyakonov (Springer Berlin Heidelberg, 2008), vol. 157 of *Springer Series in Solid-State Sciences*, pp. 1–28, ISBN 978-3-540-78819-5.
16 F. J. Dyson, Phys. Rev. **75**, 1736 (1949).
17 W. Magnus, Comm. Pure and Appl. Math. **7**, 649 (1954), ISSN 1097-0312.
18 S. Blanes, F. Casas, J. Oteo, and J. Ros, Phys. Rep. **470**, 151 (2009), ISSN 0370-1573.
19 Of course, there is no integral over the final time t.
20 L. Wang, I. Meric, P. Y. Huang, Q. Gao, Y. Gao, H. Tran, T. Taniguchi, K. Watanabe, L. M. Campos, D. A. Muller, et al., Science **342**, 614 (2013).
21 D. Marchenko, A. Varykhalov, M. Scholz, G. Bihlmayer, E. Rashba, A. Rybkin, A. Shikin, and O. Rader, Nature Comm. **3**, 1232 (2012).
22 G. M. Müller, M. Oestreich, M. Römer, and J. Hübner, Phys. E **43**, 569 (2010), ISSN 1386-9477.
23 J. Hübner, F. Berski, R. Dahbashi, and M. Oestreich, phys. stat. sol. (b) **251**, 1824 (2014), ISSN 1521-3951.