Finite-frequency noise in a non-interacting quantum dot

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I. INTRODUCTION

In this work we study a one channel quantum dot connected to two reservoirs. We calculate the non-symmetrized finite frequency noise in the framework of the Keldysh Green's function formalism. The transmission processes are introduced using a transmission amplitude defined with the help of the hoping parameter and the Green's functions of the dot. The expression of the non-symmetrized finite frequency noise is obtained. When we symmetrize our result, it coincides with the expression of the Büttiker formula of the finite frequency noise.

II. MODEL

We consider a one channel quantum dot as depicted in Fig. (1). The Hamiltonian reads as $H = H_L + H_R + H_T + H_{cen}$, where:

$$H_{\alpha=L,R} = \sum_{k\in\alpha} \varepsilon_k c_k^\dagger c_k \tag{1}$$

$$H_T = \sum_{\alpha=L,R}\sum_{k\in\alpha} V_k c_k^\dagger d + h.c. \tag{2}$$

$$H_{dot} = \varepsilon_0 d^\dagger d \tag{3}$$

The $c_{k,p,\sigma}^\dagger$ and $c_{k,p,\sigma}$ are respectively the creation and annihilation operators in the reservoirs. The $d_\sigma^\dagger$ and d_σ are respectively the creation and annihilation operators in the dot. V_k is the hopping parameter. In our work, we focus on the spinless case.

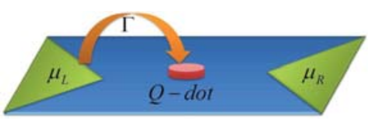

FIG. 1. Schematic representation of the quantum dot and the leads. The tunneling process occurs with a strength Γ. $\mu_{L,R}$ are the chemical potential of the left and right reservoirs. In the following, we take $\mu_{L,R} = \pm eV/2$.

Next, we define the non-symmetrized finite frequency noise in the left reservoir:

$$S(\omega) = \int_{-\infty}^{\infty} \langle \delta\hat{I}_L(0)\delta\hat{I}_L(t)\rangle e^{i\omega t} dt \tag{4}$$

where $\delta\hat{I}_L(t) = \hat{I}_\alpha(t) - \langle I_L\rangle$, where $\langle I_L\rangle$ is the average left current and $\hat{I}_L$ the current operator in the left reservoir, which is given by[1]:

$$I_L(t) = \frac{ei}{\hbar}\sum_k \left(V_k c_{k,L}^\dagger d - V_k^* d^\dagger c_{k,L}\right) \tag{5}$$

The next step is the evaluation of the current-current correlator $\langle\delta\hat{I}_L(0)\delta\hat{I}_L(t)\rangle$. For this we need first to rewrite the Hamiltonian in the interaction representation. Then, using an $\mathcal{S}$-matrix expansion one can rewrite the current-current correlator in the interaction representation. The resulting expression is a function of four-points Green's functions of the dot $G_1^{dd}(\tau,\tau',\tau_1,\tau_2) = i^2\langle T_C d(\tau)d(\tau')d^\dagger(\tau_1)d^\dagger(\tau_2)\rangle$, where τ is time variable in this representation.

Now we use a Wick theorem in order to factorize the four-points Green's functions in a product of two points Green's functions[2]:

$$G_1^{dd}(\tau,\tau',\tau_1,\tau_2) = G(\tau,\tau_2)G(\tau',\tau_1) - G(\tau,\tau_1)G(\tau',\tau_2) \tag{6}$$

The results contains two parts, a disconnected part which is equal to the square of the average current, and a connected part which contains fifteen contributions. To rewrite the correlator as a function of the time variable t, we use the Keldysh formalism[3]. Applying a Fourier transform to the result and after some algebras, one finds the non-symmetrized finite frequency noise expression.

III. RESULTS

In the case of symmetric barriers, the non-symmetrized finite frequency noise reads as:

$$\begin{aligned}S(\omega) = \frac{e^2}{h}\int d\varepsilon\Bigg[&\Big[\mathsf{T}(\varepsilon-\omega)\mathsf{T}(\varepsilon) + |t(\varepsilon) - t(\varepsilon-\omega)|^2\Big] f_{LL} \\ &+\mathsf{T}(\varepsilon-\omega)\mathsf{T}(\varepsilon) f_{RR} + \mathsf{T}(\varepsilon)\Big[1-\mathsf{T}(\varepsilon-\omega)\Big] f_{RL} \\ &+\mathsf{T}(\varepsilon-\omega)\Big[1-\mathsf{T}(\varepsilon)\Big] f_{LR}\Bigg]\end{aligned} \tag{7}$$

where $f_{\alpha\beta} = n_\alpha(\varepsilon)[1 - n_\beta(\varepsilon-\omega)]$ with n the Fermi-Dirac distribution function and $\alpha,\beta = L,R$. The transmission amplitude and the transmission coefficient are respectively given by:

$$\mathsf{t}(\varepsilon) = \frac{i\Gamma}{\varepsilon - \varepsilon_0 + i\Gamma} \tag{8}$$

$$\mathsf{T}(\varepsilon) = \frac{\Gamma^2}{(\varepsilon - \varepsilon_0)^2 + \Gamma^2} \tag{9}$$

where ε_0 is the dot energy level, $\Gamma = (2\pi)^{-1}\rho(\varepsilon)|V_k|^2$ is the barriers strength and ρ the density of states of the reservoirs which are considered as identical. The symmetrized noise is obtained from the expression $S_{sym}(\omega) = [S(\omega) + S(-\omega)]/2$. Doing this, we get the Büttiker formula of the symmetrized finite frequency noise[4,5]:

$$\begin{aligned} S_{sym}(\omega) = \frac{e^2}{h}\int d\varepsilon \Bigg[& \Big[\mathsf{T}(\varepsilon-\omega)\mathsf{T}(\varepsilon) + |t(\varepsilon) - t(\varepsilon-\omega)|^2\Big] F_{LL} \\ & + \mathsf{T}(\varepsilon-\omega)\mathsf{T}(\varepsilon) F_{RR} + \mathsf{T}(\varepsilon-\omega)\Big[1-\mathsf{T}(\varepsilon)\Big] F_{LR} \\ & + \mathsf{T}(\varepsilon)\Big[1-\mathsf{T}(\varepsilon-\omega)\Big] F_{RL} \Bigg] \end{aligned} \tag{10}$$

To see the evolution of the non-symmetrized finite frequency noise, we plot the non-symmetrized excess noise $\Delta S(\omega, V) = S(\omega, V) - S(\omega, 0)$ as a function of frequency for different values of the temperature and for different impurity strengths. In Fig. (2), we plot the non-symmetrized excess noise as a function of frequency for fixed impurity strength at different values of temperature. What we see first is the fact that the spectrum is symmetric in frequency whatever the temperature is, then the intensity of the excess noise decreases with the temperature. At low temperature, the noise presents a singularity in the vicinity of $\pm eV/\hbar$. Then the cancellation occurs beyond $\pm eV/\hbar$ because of the cancellation of the thermal noise contribution. In Fig. (3), we plot the non-symmetrized excess noise as a function of frequency for fixed temperature and different values of the impurity strength in the weak impurity regime. The noise here becomes anti-symmetric with a singularity in the vicinity of $\pm eV/2\hbar$. In Fig. (4) we plot the non-symmetrized excess noise as function of the frequency for fixed temperature in an intermediate impurity regime. What we see here is that the noise becomes asymmetric.

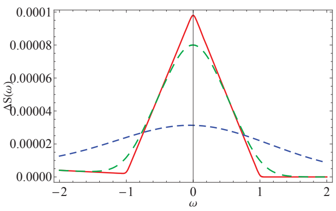

FIG. 2. Non-symmetrized excess noise in units of $e^3V/\hbar$ as a function of frequency in units of $eV/\hbar$ for $\varepsilon_0/eV = 100$, and for fixed $\Gamma/eV = 1$. Solid red line corresponds to $k_BT/eV = 0.01$, dashed green line to $k_BT/eV = 0.1$ and dotted blue line to $k_BT/eV = 0.5$.

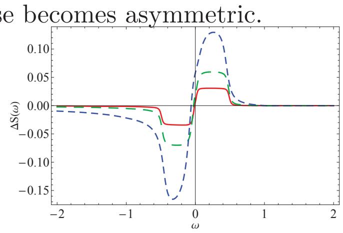

FIG. 3. Non-symmetrized excess noise in units of $e^3V/\hbar$ as a function of frequency in units of $eV/\hbar$ for $\varepsilon_0/eV = 0.01$, and for fixed $k_BT/eV = 0.01$. Solid red line corresponds to $\Gamma/eV = 0.01$, dashed green line to $\Gamma/eV = 0.02$ and dotted blue line to $\Gamma/eV = 0.05$.

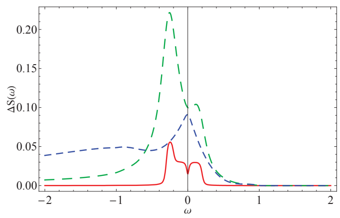

FIG. 4. Non-symmetrized excess noise in units of $e^3V/\hbar$ as a function of frequency in units of $eV/\hbar$ for $\varepsilon_0/eV = 0.3$, and for fixed $k_BT/eV = 0.01$. Solid red line corresponds to $\Gamma/eV = 0.01$, dashed green line to $\Gamma/eV = 0.1$ and dotted blue line to $\Gamma/eV = 1$.

IV. CONCLUSION

In this work we calculated the non-symmetrized finite frequency noise for a single level quantum dot. We used the Keldysh formalism to evaluate the current-current correlator and then we performed a Fourier transform to get the expression of the finite frequency noise. Our result is consistent with the Büttiker formula of the symmetrized finite frequency noise obtained using the scattering theory since the symmetrization of our expression give the formula obtained by Büttiker. Varying the temperature, the dot energy and the barrier strength, the profile of the noise spectrum changes from symmetric behavior, to asymmetric or to anti-symmetric behaviors.

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