The quest for the missing noise in a micro-mechanical system out of equilibrium

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I. INTRODUCTION

Equipartition principle plays a central role in the understanding of the physics of systems in equilibrium: the mean potential and kinetic energy of each degree of freedom equilibrates to $k_B T/2$, with k_B the Boltzmann constant and T the temperature. This equality is linked to the fluctuation-dissipation theorem: fluctuations of one observable are proportional to the temperature and dissipation in the response function associated to that observable. In non equilibrium situations however, such relations between fluctuations and response are not granted, and *excess* noise is usually expected to be observed with respect to an equilibrium state[3].

In this presentation, we show that the opposite phenomenon can also be experimentally observed: a system that fluctuates *less* than what would be expected from equilibrium ! Indeed, when we measure the thermal noise of the deflexion of a micro-cantilever subject to a strong stationary temperature gradient (and thus heat flow), fluctuations are much smaller that those expected from the system mean temperature.

We first present the experimental system, an atomic force microscope (AFM) micro-cantilever in vacuum heated at its free extremity with a laser. We show that this system is small enough to have discrete degrees of freedom but large enough to be in a non-equilibrium steady state (NESS). We then estimate its temperature profile with the mechanical response of the system, and observe that equipartition theorem can not be applied for this NESS: the thermal noise of the system is roughly unchanged while its temperature rises by several hundred degrees ! We conclude with a widely open question of the origin of this missing noise.

II. EXPERIMENT

Using a differential interferometer[2], we measure the thermal noise induced flexural deflexion of an AFM cantilever, a micro-mechanical beam clamped at one extremity and free at the other – see sketch in the inset of Fig. (1). No external forces are applied to the 500 μm long cantilever. As illustrated in Fig. (1), the power spectrum density (PSD) of the deflexion in vacuum is characteristic of a collection of independent quasi-harmonic oscillators corresponding to the normal modes of the cantilever. The resonance frequencies and spacial mode shapes are well described by an Euler-Bernoulli model for an elastic beam[1]. In equilibrium, the amplitude of the thermal noise can be used to deduce the stiffness k_n of each mode n using equipartition:

$$\frac{1}{2} k_n \langle d_n^2 \rangle = \frac{1}{2} k_B T \tag{1}$$

with $\langle d_n^2 \rangle$ the mean square deflexion measured for that mode (integral of the PSD around the resonance).

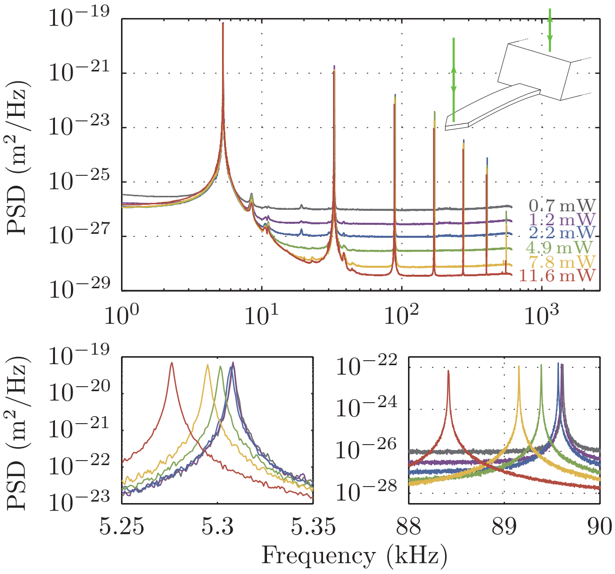

FIG. 1. Power spectrum density (PSD) of thermal noise induced deflection of a silicon micro-cantilever as a function of frequency, for various incident light power I on the lever in vacuum. The flat base line of the PSD is due to the shot noise on the photodetectors, it decreases for larger I. All the resonant frequencies of the cantilever decrease when I grows, as illustrated in the 2 bottoms figures for modes 1 and 3. The inset of the top figure illustrate the principle of the measurement: the fluctuations of deflection are recorded through the interference of two laser beams, one reflected on the cantilever free end, the other on the chip holding the cantilever[2].

To lower the contribution of the shot noise of the photodetectors to the measured spectrum, we increase the laser intensity. The sought noise reduction is obtained as illustrated in Fig. (1), but we also observe a lowering of the resonant frequencies of the normal modes. A simple argument can be used to understand this red shift: each mode can be pictured as an harmonic oscillator of mass m, whose stiffness k_n is proportional to the Young's modulus E of silicon (the cantilever material). The increase of temperature, due to light absorption, induces a softening of the cantilever: the temperature coefficient of E is negative

$$\alpha_E = \frac{1}{E}\frac{dE}{dT} \approx -64 \times 10^{-6}\,\mathrm{K}^{-1} \tag{2}$$

The resonant frequencies f_n of the normal modes should therefore decrease as light intensity I increases:

$$2\pi f_n = \sqrt{\frac{k_n}{m}} \propto E^{1/2} \tag{3}$$

$$\frac{\Delta f_n}{f_n} \approx \frac{1}{2}\frac{\Delta E}{E} \approx \frac{1}{2}\alpha_E \Delta T \propto I \tag{4}$$

In the presentation, a careful treatment of the interplay between the temperature profile and the spatial shape of the modes will show how we deduce from the frequency shifts the temperature gradient in the cantilever.

The deduced temperature at the free end of the cantilever is plotted in Fig. (2): we reach huge temperature gradients for a few tens of mW of the measuring laser. We could even melt silicon cantilevers in vacuum for $I = 20\,\mathrm{mW}$, when the melting point of silicon is 1410 °C !

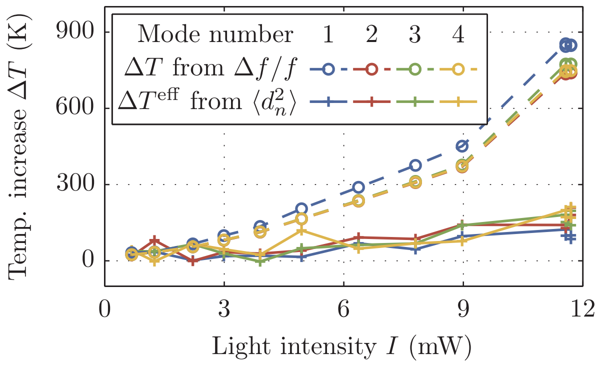

FIG. 2. Temperature increase at the free end of the cantilever deduced by two methods: upper curves (dashed) from the frequency shift of the resonances (Eq. 4), lower curves (plain) from an extension of the equipartition theorem to this NESS (Eq. 5). The latter is clearly under evaluated.

III. EFFECTIVE TEMPERATURE

Though the cantilever is in a NESS due to the heat flowing along its length, it is tempting to extend equipartition as in Eq. (1) to define an effective temperature of each mode. Indeed, the mean square deflection can be measured from the PSD, and the stiffness of the modes is known from a calibration in equilibrium and the frequency shift already discussed. We may thus extend Eq. (1) to define an affective temperature T_n^{eff} of each mode:

$$T_n^{\mathrm{eff}} = \frac{k_n}{k_B}\langle d_n^2 \rangle \tag{5}$$

Again, in the presentation, a careful treatment of the interplay between the temperature profile and the spatial shape of the modes will show how to define the effective temperature gradient in the cantilever from the measured thermal noise.

The deduced effective temperature at the free end of the cantilever is plotted in Fig. (2): though T^{eff} rises with I, its value is far smaller than that inferred from the response of the cantilever (frequency shift). The equipartition clearly fails for this NESS, but in a completely unexpected way: the level of noise is below what one would expect. And indeed, the area under the resonance peaks in the thermal noise spectra in Fig. (1) looks constant while the maximum temperature in the lever increases from 300 K to 1100 K. On the contrary, one would expect the fluctuations to rise even more due to the non equilibrium situation.

IV. OPEN PROBLEMS

Our system offers a nice and carefully controlled out of equilibrium system, where fluctuation and response can be measured with a high accuracy for several independent degrees of freedom. We demonstrate that in such a NESS, the thermal noise of all modes is close to the one expected for the lower temperature of the system, when one would expect it to be higher (of at least equal) to that of its mean temperature. This result is in strong contradiction with other experiments of mechanical systems subject to a stationary heat flow, where excess noise is observed[3].

In our quest for this missing noise, we have identified a few leads that will be discussed during the presentation, for instance:
> effect of a spatially non-uniform damping mechanism ?
> absence of coupling between longitudinal heat transport and flexural deflexion ?
Other suggestions will be welcomed during the discussion !

ACKNOWLEDGMENTS

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