Noise on resistive switching: a Fokker-Planck approach

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Stotland and Di Ventra[1] were the first to present an analysis of the influence of noise on memristors. The interplay between noise and resistive switching has been further characterized, both experimentally and through simulations, in several papers[2–4].

In this work we go back to the their seminal paper, where they analyzed the influence of additive white Gaussian noise on a simple model of a memristor put forth by Strukov *et al.*[5]. By means of numerical simulations, they showed that the contrast between low- and high-resistive states is enhanced by the addition of internal noise in the presence of a weak harmonic driving signal, and provided an explanation in terms of a stochastic resonance phenomenon.

In this contribution, we aim at extending the work of Stotland and Di Ventra. Motivated by the potential application of resistive switching in the area of non-volatile data storage, we consider the case of non-harmonic driving signals. Noise is also shown to enhance the contrast between resistive states, and we provide an alternative explanation of the observed behavior in terms of the associated Fokker-Planck equation.

According to the model by Strukov *et al.*[5], resistance in a memristor can be written as

$$R(x) = \alpha(1 - \delta R\, x), \tag{1}$$

where $\alpha, \delta R \in \mathbb{R}^+$ are adequate constants and $x \in [0, 1]$ is a state variable governed by the equation

$$\frac{dx}{d\tau} = \frac{4x(1-x)}{1 - \delta R\, x} v(\tau), \tag{2}$$

where τ is a suitably normalized time variable and $v(\tau)$ is the (normalized) external voltage drive. It is easy to verify that $x(\tau)$ can be found as a solution to the equation

$$x^{\beta}(\tau) + g^{\beta}(\tau)x(\tau) - g^{\beta}(\tau) = 0, \tag{3}$$

where $\beta = (1 - \delta R)^{-1}$ and

$$g(\tau) = \frac{x(0)}{(1 - x(0))^{\frac{1}{\beta}}} \exp\left\{4\int_0^{\tau} v(t)dt\right\}. \tag{4}$$

In this section, we consider the case in which equation (2) is modified by additive white Gaussian noise $\eta(\tau)$ such that $\langle\eta(\tau)\rangle = 0$ and $\langle\eta(\tau)\eta(\tau')\rangle = \Gamma\delta(\tau - \tau')$. We emulate an alternating 0-1 writing-pattern of a memory device by considering a non-harmonic drive $v(\tau)$ which consists of a sequence $+1 \to -1 \to +1 \to \cdots$ of pulses of width τ_b.

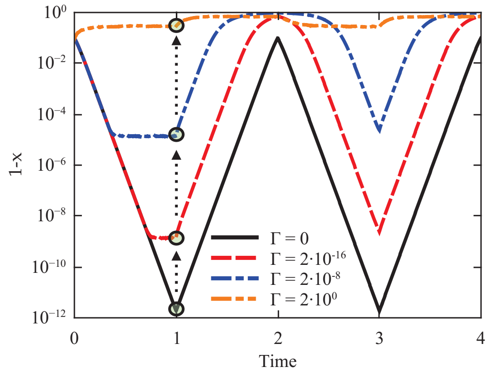

FIG. 1. Temporal evolution of the state variable x for several noise intensities. Results are the average of 1000 realizations.

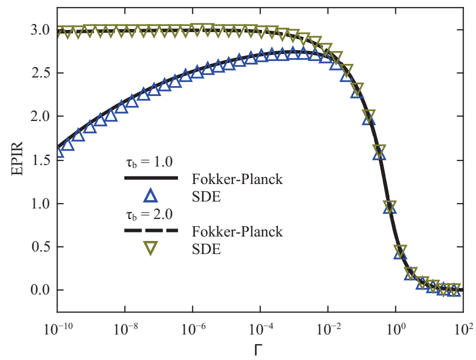

FIG. 2. EPIR ratio as a function of internal noise intensity. Solid and dashed lines correspond to quasi-analytic predictions developed in the text. Results corresponding to the average of 1000 realizations of the stochastic differential equation are represented by triangles.

Fig. 1 shows the temporal evolution of x for several noise intensities and $\tau_b = 1$. Observe that the maximum value that the state variable of x reaches after a $+1$ pulse is applied decreases as the noise intensity increases, as noted by circles and arrows. A usual way of quantifying the contrast between low (R_l) and high (R_h) resistance states is through the Electric Pulse Induced Resistance (EPIR) ratio given by $\frac{R_h - R_l}{R_l}$. As it can be observed in Fig. 2, the EPIR ratio is maximized for a certain optimal noise intensity and pulsewidth.

Results in Fig. 1 can be understood by resorting to the associated Fokker-Planck equation. Assuming that τ_b is

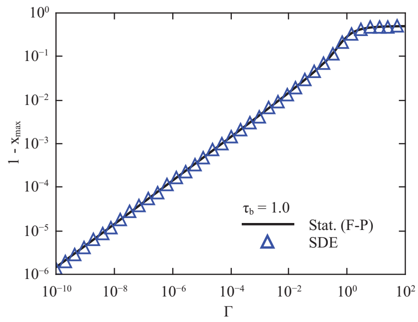

FIG. 3. Maximum value of x vs. noise intensity: approximation given by the stationary distribution (solid line) and by integration of the SDE (blue triangles).

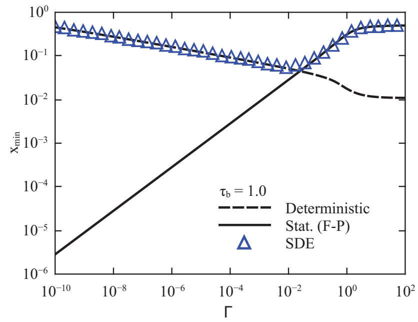

FIG. 4. Minimum value of x vs. noise intensity: approximation given by the stationary distribution (solid line), approximation given by the deterministic solution (dashed), and result of integrating the SDE (blue triangles).

large enough, we can work with its stationary solution

$$P(x, \tau_b) \approx P_s(x) \propto \exp\left\{\frac{2}{\Gamma}\int_0^x v(\tau_b)\frac{4y(1-y)}{1-\delta R y}dy\right\}, \quad (5)$$

and $\langle x(\tau_b)\rangle$ can be computed by numerical integration. Fig. 3 shows a good agreement between simulations of the stochastic differential equation (SDE) and results obtained through the stationarity hypothesis. As it is readily seen from equations (3)-(4), the deterministic evolution of $x(\tau)$ is highly dependent on the initial condition. One of the effects of noise is to erase the memory of the initial condition. Indeed, as expected, the stationary probability distribution in equation (5) does not depend on the initial condition. However, the time of convergence to stationarity does depend on the initial condition. In general, the convergence time decreases as the noise intensity increases, *i.e.*, a higher noise intensity erases the memory of the initial condition faster.

We can try to use the stationarity hypothesis to compute the minimum value attained by x after a -1 pulse is applied, *i.e.*, $x(2\tau_b)$. Fig. 4 shows $\langle x(2\tau_b)\rangle$ as a function of the noise intensity (the stationary probability is similar to that in equation (5)). The behavior for low noise intensities deviates from that predicted by the stationary distribution. Indeed, for the given initial condition ($\langle x(\tau_b)\rangle$ in Fig. 3, the x value at the end of the previous $+1$ pulse), the pulsewidth τ_b is not large enough to allow for the convergence to stationarity and higher noise intensities are needed to erase the memory of the initial condition. Moreover, when the noise intensity is low, the value of $x(2\tau_b)$ can be approximated by the deterministic solution in equations (3)-(4).

Using the predictions based on the stationary probability distribution and the deterministic solution (for low noise intensities) in Figs. 3-4, we can estimate the EPIR ratio. The result is shown in Fig. 2 and agrees very well with simulations. Intuitively, the main effect of the added noise is to lower the value of x at the end of the first $+1$ pulse in such a way that $\langle x(\tau_b)\rangle$ is smaller than expected from the deterministic solution. For low noise intensities, this 'new' initial condition for the differential equation for $\tau > \tau_b$ results in a mean value of $x(2\tau_b)$ smaller than that in the noiseless case and, thus, leads to an enhanced EPIR ratio. For high noise intensities, values of the state variable x attained at the end of each pulse are independent of the initial conditions and determined by the stationary solution of the corresponding Fokker-Planck equation. Furthermore, as Γ increases, the distribution in equation (5) broadens, $\langle x_s\rangle$ tends to 1/2 and the EPIR ratio goes to zero.

In summary, we introduced a Fokker-Planck approach to tackle the effect of internal noise on resistive switching, and used it to explain the resistive-contrast enhancement found in numerical simulations. It remains an open question whether such an approach can be applied to account for the beneficial role of external noise in resistive switching as it was reported in a previous work[2].

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