Dissipative dynamics of a quantum particle strongly interacting with a super-Ohmic heat bath

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I. INTRODUCTION

Realization of devices displaying quantum behavior is within the reach of present day experimental capabilities. Experimental and theoretical results on superconducting quantum devices has made them among the main candidates for the realization of quantum computing[1]. In these devices the magnetic flux quantum associated to the current of the superconducting circuit is usually subject to a bistable potential. Bistability is also present in systems such s single high-spin molecule magnets[2]. These molecules tunnel through the potential barrier of the effective bistable potential given by the interaction of the spin with the molecular field. Both single molecule magnets and superconducting devices are subject to environmental fluctuations. In quantum regime the dynamics of a particle interacting with the environment can be described by the celebrated Caldeira-Leggett model[3], which allows to analyze the dynamics of a particle coupled by a linear interaction to a reservoir of N independent quantum harmonic oscillators. The interaction with the bath can affect the system dynamics in a significant way since, even if the coupling with the individual oscillator is weak, the dissipation regime may be strong. In the thermodynamical limit $N \to \infty$ the reservoir is called a *heat bath* and its spectral density function $J(\omega)$, describing the frequency dependence of the coupling to the system, is taken to be of the form $J \propto \omega^s$, with a high-frequency cut-off. The special case $s = 1$ describes the so-called Ohmic dissipation. The quantum Langevin equation for the particle's coordinate in the Ohmic case is characterized by a memoryless damping kernel (frequency independent friction) and in the classical limit $\hbar \to 0$ corresponds to the case of white noise source.

Despite the circumstance that in most cases the Ohmic dissipation gives a good description of the effects exerted by the thermal bath, super-Ohmic environments ($s > 1$) are of interest on both the theoretical and the experimental point of view. Moreover, the system dynamics can be significantly affected by the value of the cut-off frequency present in $J \propto \omega^s$.

In this work we intend to answer two questions: i) how the dynamics of a M-level quantum particle changes when a heat bath with a super-Ohmic spectral density is present instead of an Ohmic reservoir; ii) how varying the cut-off frequency in the spectral density function affects the system dynamics. The study is carried out by using an integro-differential equation within the path integral formalism, following the approach used in Refs.[4,5].

II. THE MODEL

The model of dissipation used here is the Caldeira-Leggett model. It allows for a *microscopic* derivation of dissipation in the reduced dynamics. The system, a particle of mass M, coordinate $\hat{q}$, and momentum $\hat{p}$ subject to a potential V_0, is linearly coupled to the environment, a reservoir of N independent quantum harmonic oscillators of masses m_j, frequencies ω_j, coordinates $\hat{x}_j$, and momenta $\hat{p}_j$. The reservoir is also called, in the thermodynamical limit $N \to \infty$, bosonic *heat bath*, since its excitations obey the Bose-Einstein statistics. The full Hamiltonian is the sum of a free system term, a free reservoir term and a system-reservoir interaction term

$$\hat{H} = \frac{\hat{p}^2}{2M} + V_0(\hat{q}) + \sum_{j=1}^{N} \frac{1}{2} \left[\frac{\hat{p}_j^2}{m_j} + m_j \omega_j^2 \left(\hat{x}_j - \frac{c_j}{m_j \omega_j^2} \hat{q} \right)^2 \right]. \tag{1}$$

The bistable asymmetric potential V_0 used in this work is depicted in Fig. 1. In the general case of continuous bath the spectral density function is modeled as a power of ω, characterized by the exponent s, with an exponential cutoff at ω_c

$$J(\omega) = M\gamma\omega_{\mathrm{ph}}^{1-s}\omega^s e^{-\omega/\omega_c}. \tag{2}$$

The bath is said sub-Ohmic for $0 < s < 1$, Ohmic for $s = 1$ and super-Ohmic for $s > 1$. The so-called *damping constant* γ is a measure, in the continuous limit, of the system-bath coupling. The *phonon* frequency ω_{ph} is introduced in such a way that γ has the dimension of a frequency also in the non-Ohmic case ($s \neq 1$).

The dynamics of the reduced density matrix (RDM) $\rho_{qq'} = \langle q|\rho|q'\rangle$ is given by the exact formal expression

$$\rho_{qq'}(t) = \int dq_0 \int dq_0' G(q, q', t; q_0, q_0', t_0)\rho_{q_0 q_0'}(t_0), \tag{3}$$

where the propagator G is a double path integral in the left/right coordinate q/q'. The amplitudes in this sum-over-paths are weighted by the Feynman-Vernon influence functional $\mathcal{F}_{FV}$, which accounts for the effects of the environment.

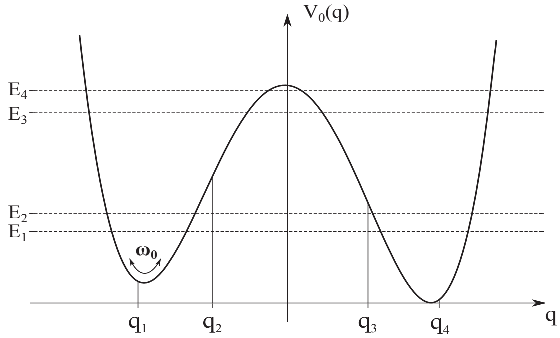

FIG. 1. Potential V_0, energy levels considered, and position eigenstates. The frequency ω_0 is the oscillation frequency around the minima and is of the order of the average inter-doublet spacing: $\hbar\omega_0 \sim (E_4 + E_3 - E_2 - E_1)/2$.

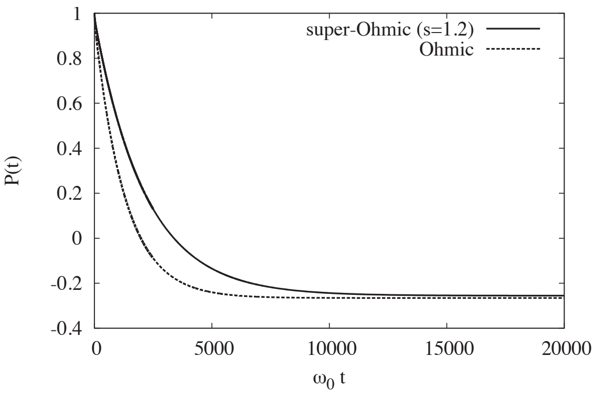

FIG. 2. Population difference $P(t) = P_R - P_L$, where $P_L = \rho_{11} + \rho_{22}$ and $P_R = \rho_{33} + \rho_{44}$ at damping strength $\gamma = 0.1\ \omega_0$ and temperature $T = 0.2\ \hbar\omega_0/k_B$. Comparison between the Ohmic and the super-Ohmic ($s = 1.2$) regime.

Here we consider the so-called *double-doublet* system, where only the first 4 levels of the potential V_0 are considered. The continuum of position states turns into a discrete set of states localized around a grid of 4 position eigenvalues $q_1, \ldots, q_4$, where $\hat{q}|q_j\rangle = q_j|q_j\rangle$. The set $\{q_i, |q_i\rangle\}$ constitutes the discrete variable representation (DVR). The system dynamics is studied through the time evolution of the populations $\rho_{ii} = \langle q_i|\rho|q_i\rangle$. Finally, within a NIBA-like approximation scheme the generalized master equation (GME) for the populations in the DVR reads[4]

$$\dot{\rho}_{ii}(t) = \sum_{j=1}^{4} \int_{t_0}^{t} dt' K_{ij}(t-t')\rho_{jj}(t'). \tag{4}$$

Solving Eq. (4) in the intermediate temperature/damping regime, with the initial condition $\rho_0 = |q_1\rangle\langle q_1|$, we obtain the results shown in Fig 2 (Ohmic and sub-Ohmic case with a high frequency cutoff at $\omega_c = 50\omega_0$) and Fig. 3 (Ohmic regime for different cutoff frequencies). We notice that the equilibrium configuration in the super-Ohmic case is reached later with respect to the Ohmic case, even if the time evolution of the individual populations (not shown) displays similar features (transient intra-well oscillations and incoherent tunneling). Changing the cutoff frequency in the Ohmic regime has an influence both on the relaxation dynamics and on the stationary configuration.

A major unsolved problem in the context of the influence of quantum noise on multi-state systems is the description of the decoherence in this intermediate dissipation regime, using a fully non-Markovian approximation scheme.

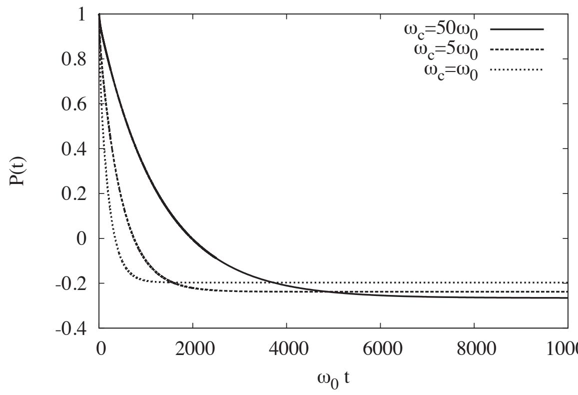

FIG. 3. Population difference $P(t) = P_R - P_L$, where $P_L = \rho_{11} + \rho_{22}$ and $P_R = \rho_{33} + \rho_{44}$ at damping strength $\gamma = 0.1\ \omega_0$ and temperature $T = 0.2\ \hbar\omega_0/k_B$. Ohmic regime with different cutoff frequencies.

ACKNOWLEDGMENTS

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