Randomness and Earth's climate variability

Levinshtein M.E,[1] Dergachev V. A[2], Dmitriev A. P[3]., and Smakov P. M[4]
[1]*Ioffe Institute, St. Petersburg, Russia*
e-mail address: melev@nimis.ioffe.ru
[2]*Ioffe Institute, St. Petersburg, Russia*
e-mail address: v.dergachev@mail.ioffe.ru
[3]*Ioffe Institute, St. Petersburg, Russia*
e-mail address: apd1812@gmail.com
[4]*Ioffe Institute, St. Petersburg, Russia*
e-mail address: pshmakov@yandex.ru

I. INTRODUCTION

The problem of studying and predicting the natural phenomena evolution of our planet is vital for all of us and for the Humanity as a whole. At present, there are steadily increasing changes of the natural conditions in various geospheres on the Earth's surface. It is widely believed that the rise in the average temperature on the Earth's surface in the past few decades is caused by the increasing human activity. Based on this assumption, trillions of dollars are spent to fight the global warming.

At the same time, the opponents of this concept argue that the periods of relative cooling and warming repeatedly alternated in the Earth's history without any human intervention. Suffice it to mention that only 20 000 years ago the whole territory of North Europe was occupied by a glacier with a thickness reaching 2.5 - 3 km. It is known for sure that approximately 9 – 10 thousand years ago this monstrous ice armour melted without any impact of human activity.

The only scientific way to try to predict the future trends of these changes is by making a correct analysis of the climatic changes in the past. The Paleo-Sciences have accumulated over the years of studies tens of thousands of very different records related to climatic changes. Such important parameters as solar insolation, temperature, content of the oxygen isotope ($\delta^{18}O$), atmospheric radiocarbon, deuterium, etc., have been recorded and analyzed. The researchers have tried to identify periodic and quasi-periodic processes in these paleoenvironmental records. In this paper, we show that this analysis is incomplete, and that *random processes*, namely *single-time-constant random processes* (noise with a Lorentzian noise spectrum) play a very important and, perhaps, a decisive role in numerous natural phenomena.

II. METHOD OF CALCULATIONS

We consider a paleoenvironmental record of some quantity φ as a random function ("noise") $\varphi(t)$. The spectral density of fluctuations $S^{\varphi}(f)$ is calculated as follows. Another random function $\varphi'(t)$ is introduced defined as $\varphi'(t) = \varphi(t) - \overline{\varphi}$, where $\overline{\varphi}$ is the average value of $\varphi(t)$ over a long time interval T:

$$\overline{\varphi} = \frac{1}{T}\int_{-T/2}^{T/2} \varphi(t)dt \qquad (1)$$

The average value of $\varphi'(t)$ is zero, so it describes the deviation of $\varphi(t)$ from the mean. The spectral noise density of fluctuations $S^{\varphi}(f)$ can be expressed through the Fourier transform $\tilde{\varphi}(f)$ of the $\varphi'(t)$ function:

$$S^{\varphi}(f) = \frac{1}{T}\left|\tilde{\varphi}(f)\right|^2, \qquad (2)$$

$$\text{where } \tilde{\varphi}(f) = \int_{-T/2}^{T/2} \varphi'(t)\exp(i2\pi ft)dt\,,\ T \to \infty \qquad (3)$$

The function $S^{\varphi}(f)$ was calculated numerically.

III. RESULTS AND DISCUSSION

Paleoenvironmental records have, at first sight, the form of noise (so called "grass"). As an example, Fig 1 shows the time dependence of the virtual axial dipole moment (VADM) for last 800000 years (800 Ka)[1]**.**

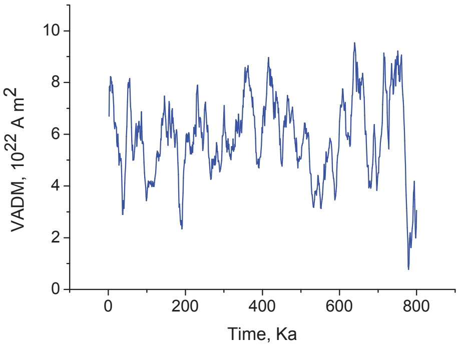

Fig. 1 The time dependence of the virtual axial dipole moment (VADM) for 800 Ka[1].

Of course, periodic and quasi-periodic processes can be (and should be) identified in the dependence shown in Fig. 1. At the same time, this dependence can be regarded as "noise". In this case, the frequency dependence of the spectral density *S*(*f*) of this "noise" should be calculated.

Fig. 2 presents the frequency dependence of the spectral density fluctuations $S^{DM}(f)$ for the time dependence of virtual axial dipole moment (VADM) presented in Fig. 1.

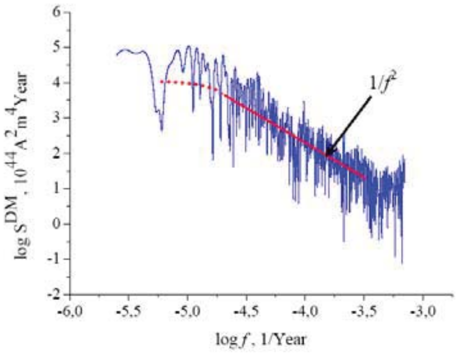

Fig. 2. The frequency dependence of spectral density fluctuations for the time dependence of VADM presented in Fig. 1. The lines are guidelines to the eye.

It can be seen in Fig. 2 that the overall run of S^{DM} curve in the frequency range $10^{-5} \leq f \leq 10^{-3.5}$ 1/year is quite well described by a single Lorentzian with a characteristic time constant $\tau_0 \sim 10^{4.7}/2\pi \approx$ 8000 years. This means that, along with the possible periodic and quasi-periodic processes in the time dependence of VADM, an important (and possible dominant) influence is exerted *by a random process* with a characteristic time constant $\tau_0 \sim 8000$ years.

We calculated spectral densities of fluctuations for several very important paleoenvironmental records.

Fig. 3 presents the frequency dependence of spectral density fluctuations, S^{2H} for the deuterium content (Vostok Ice Core Deuterium Data for 420,000 years[2]).

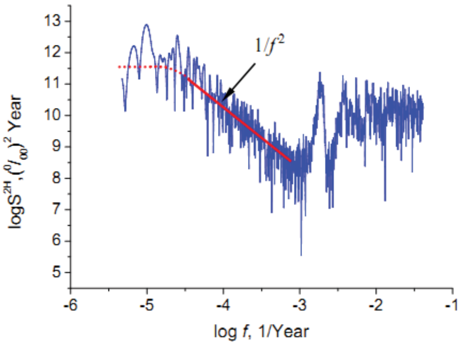

Fig. 3. The frequency dependence of spectral density fluctuations for the deuterium content[2].

It is seen from Figure 3 that the overall behavior of S^{2H} dependence in the frequency range $f \geq 10^{-5}$ 1/years is well described by a single Lorentzian with an characteristic time constant $\tau_0 \sim (10^{4.5}-10^{4.7})/2\pi \approx (5\div8)\times10^3$ years.

Very similar dependence was observed for the deuterium data for 740,000 years from EPICA Dome C Ice Cores[3].

It is interesting to compare the data presented in Figs. 2 and 3 with $S^{\delta 18O}$ dependence for the content of oxygen ^{18}O isotope[4] (Fig.4).

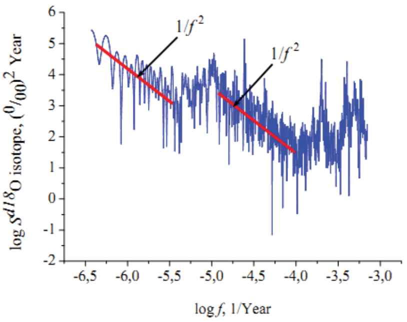

Fig. 4. The frequency dependence of spectral density fluctuations for globally distributed benthic δ^{18}O records[4].

The data presented in [4] span 5.3 Myr and are an average of 57 globally distributed benthic δ^{18}O records, aligned by an automated graphic correlation algorithm. It can be seen from Fig. 4 that there are two characteristic parts in the frequency dependence of spectral density, in which the dependence follows the law $S \sim 1/f^2$. For the part in the frequency range $10^{-4.7} \leq f \leq 10^{-4.0}$ 1/year, the value of τ_0 is close to the values of τ_0, found for the plots shown in Figs. 2 and 3. For the "low-frequency" part $10^{-6.3} \leq f \leq 10^{-5.3}$ 1/year, the value of τ_0 cannot be found from the data presented. It is obvious, however, that τ_0 for this random process, exceeds 10^5 years.

The above-mentioned random process with the same time constant $\tau_0 \sim (10^{4.5} - 10^{4.7})/2\pi$ years for several records is certainly not universal.

For example, the frequency dependence of the spectral density of the atmospheric radiocarbon content fluctuations[5] is quite well described by a Lorentzian with a characteristic time constant $\tau_0 \sim$ 300 years .

Obviously, the most intriguing problem is the physical interpretation of the data, i.e., the identification of the random processes with a single time constant which are responsible for the appearance of single Lorentzians in the *S*(*f*) dependences analyzed in this paper.

IV. CONCLUSION

The time dependences of several important paleoclimatic parameters are considered for the first time as random processes ("noise"). It is shown that *single-time-constant random processes* (noise with a Lorentzian noise spectrum) play a very important and, perhaps, a decisive role in some such important dependences.

To the best of our knowledge, this is the first indication of the role played by random processes in climate variations. Identification of the random processes responsible for the appearance of single Lorentzians in the *S*(*f*) dependences that were revealed in this work will allow a better understanding of the nature of climatic changes

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