Elementary events and probabilities in time-dependent quantum transport

Wolfgang Belzig[1] and Mihajlo Vanevic[2]

[1]*Department of Physics, University of Konstanz, D-78457 Konstanz*
e-mail address: Wolfgang.Belzig@uni-konstanz.de
[2]*Department of Physics, University of Belgrade, Studentski trg 12, 11158 Belgrade, Serbia*
e-mail address: mihajlo.vanevic@gmx.com

I. INTRODUCTION

A central problem of quantum transport is to find the statistics of electron transfer in a coherent quantum point contact (QPC) driven by an arbitrary time-dependent voltage. For the dc-case this question has been first addressed and theoretically answered by Levitov and Lesovik[1], who calculated the cumulant generating function (CGF) of probability distribution of the transferred charge $S_{t_0}(\chi) = \ln \sum_N P_{t_0}(N) e^{i\chi N}$. The astonishingly simple result in the zero-temperature limit was that the statistics is just binomial

$$S(\chi) = \frac{2eVt_0}{h} \sum_n \ln\left[1 + T_n\left(e^{i\chi} - 1\right)\right], \qquad (1)$$

where V is the applied bias voltage, t_0 is the observation time, and T_n are the transmission coefficients of the individual quantum channels. It should be emphasized that the result is nontrivial in that the Fermi sea of both contacts enters into the calculation.

The question of time-dependent driving has been addressed only recently. The CGF of a quantum contact can be expressed by the extended Keldysh action[2]

$$S(\chi) = \mathrm{Tr}\ln\left[1 + T_n\left(\left\{\check{G}_L(\chi), \check{G}_R\right\} - 2\right)/4\right], \qquad (2)$$

where now $\check{G}_{L/R}$ are the Keldysh Green's functions of the two leads. The counting field χ is introduced in Keldsyh space via $\check{G}(\chi) = \exp(-i\chi\check{\tau}_K/2)\check{G}\exp(i\chi\check{\tau}_K/2)$, where $\check{\tau}_K$ is the current operator matrix. This formulation is valid for a all possible quantum contacts comprising superconducting leads or time-dependent driving fields.

II. TIME-DEPENDENT VOLTAGE DRIVE

In time-dependent problems the Green's functions have to be considered as operators in a two-time (or two-energy) space, however the general formulation remains valid. Based on this observation it was shown that the electric current due to a voltage drive $V(t)$ with period T is composed of elementary events of two kinds[3]: unidirectional one-electron transfers determining the average current and bidirectional two-electron processes contributing to the noise only. These events are sketched in Fig. 1. As consequence the CGF takes the form

$$S(\chi) = S_{1p}(\chi) + S_{2p}(\chi). \qquad (3)$$

Here the first term describes so-called uni-directional one-particle processes, shown in Fig. 1(c) and is essentially

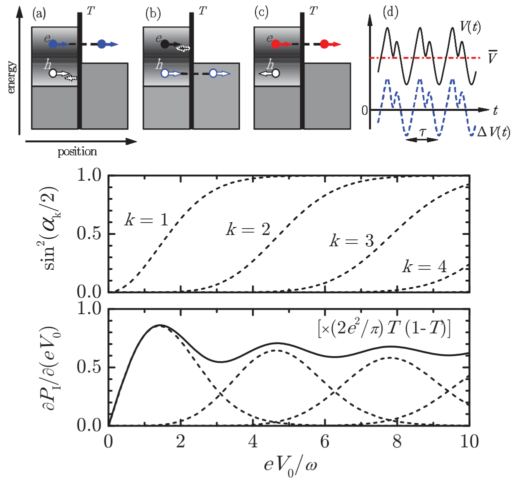

FIG. 1. Elementary events of a quantum point contact driven by a time-dependent voltage $V(t)$ with average $\bar{V}$ shown in (d). (a) and (b) depict the two-particle events in which an electron-hole pair is created and either the electron or the hole is transmitted (and the other is reflected). (c) are the unidirectional events in which simply one electron is transferred. The lower two plots show the elementary event analysis for a harmonic voltage drive (see text for an explanation).

given by Eq. (1), in which V is the average voltage $\bar{V}$. The two particle events contribute as

$$S_{2p}(\chi) = \omega t_0 \sum_k \ln\left[1 + 2p_k TR\left(\cos(\chi) - 1\right)\right]. \qquad (4)$$

Here $p_k = \sin^2(\alpha_k/2)$ is the probability of an elementary event. The above formulation allows to decompose the transport statistics of a voltage-driven QPC into its elementary constituents. An example is shown in Fig. 1 for a harmonic voltage drive $V_0\cos(\omega t)$. The dependence of the probabilities on the amplitude of the drive are shown in the upper plot and the corresponding shot noise $P = -\partial^2 S(\chi)/\partial\chi^2|_{\chi=0}/t_0$, which shows characteristic oscillations as function of the oscillation amplitude.

III. CONTROLLING ELEMENTARY EVENTS

One question emerging is how the elementary events can be controlled. A very interesting possibility was explored in[6], where the authors have applied a biharmonic time dependent drive at the contact. The experimental result was that by adding an in-phase second harmonic of

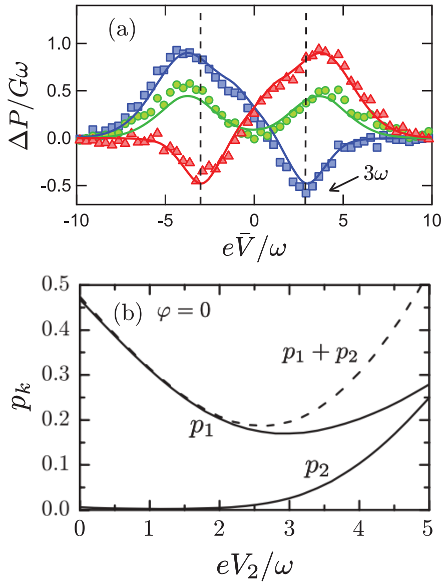

FIG. 2. (a) Difference ΔP between the noise generated by biharmonic drive $V(t) = \bar{V} + V_1 \cos(\omega t) + V_2 \cos(2\omega t + \varphi)$ and the noise generated by the first harmonic only. Parameters are $eV_1/\omega = 5.4$, $eV_2/\omega = 2.7$ and $\varphi = 0$ (blue), $\pi/2$ (green), and π (red curve). (b) Probabilities p_k of the electron-hole pair creations as a function of the amplitude V_2 for the biharmonic voltage drive with $e\bar{V}/\omega = 3$, $eV_1/\omega = 5.4$, and $\varphi = 0$.

a certain amplitude, it is possible to minimize the overall noise in the junction below the noise level produced by the first harmonic only, see Fig. 2 (a). This is because of a complex response of the Fermi sea upon time-dependent perturbation: the time-dependent drive mixes the electronic states of different energies which, in combination with the Pauli principle, gives a non-trivial charge transfer statistics. The explanation in terms of electron-hole pairs created in the system has been put forward in[5]. For a simple harmonic drive there is only one electron-hole pair generated per period with probability p_1. When the second harmonic is in phase with the first one, the probability p_1 of the pair generation decreases as the amplitude V_2 of the second harmonic is increased, see Fig. 2 (b). As V_2 is increased further, the second electron-hole pair is generated with an increasing probability p_2 per period. For the amplitude $eV_2/\omega \approx 2.6$, the total probability $p_1 + p_2$ of the electron-hole pair creation exhibits a minimum. This gives the minimal excess noise which has been observed in[6]. More fundamentally, it means the elementary excitations are controlled dynamically by shaping the time dependence of the driving voltage.

Another approach to control the excitations is to shape the voltage pulses as a Lorentzian, which should lead to special excitation states in which an exact integer number of charges is excited - so-called levitons[7]. As shown in Eq. 3, however this holds only for pulses of integer flux $\int dt V(t) = nh/e$ with the Planck quantum h, because otherwise additional electron-hole pairs are excited. Due to the finite temperature in the experiment[8], however, the minimal excitations are not found exactly for integer flux. A full analysis in terms of elementary events in this case is still missing.

IV. UNSOLVED PROBLEMS

So far the analysis of the current fluctuations is restricted to the zero-frequency current correlators. This quantity only contains information on the probabilities of the charge transfer, viz. contained in $\bar{V}$ and p_k. However the open problem is how to access the temporal shape of the excited wave function. On the theoretical side, the unsolved problem is how to approach these correlations in a quantum point contact between two many-body Fermi systems. The problem is complicated by the fact that a time-dependent voltage severely disturbs the Fermi see even at zero temperature. Hence, it is in general not possible to resort to a simple single-particle picture. One way to attack this problem theoretically is to make use of the extended Keldysh-Greens function formalism[2] summarized in Eq. (2) and reformulate the methods developed for the time-dependent voltage-driven full counting statistics[3] for time-resolved detection schemes. In this contribution we will formulate this unsolved problem of the theory of quantum transport and present first steps towards a solution.

ACKNOWLEDGMENTS

We thank J. Gabelli for useful discussion. WB acknowledges financial support by DFG through SFB 767 and the German Excellence Initiative through CAP. MV acknowledges MES project No. 171027.

[1] L. S. Levitov and G. B. Lesovik, Pis'ma Zh. Eksp. Teor. Fiz. **58**, 225 (1993) [JETP Lett. **58**, 230 (1993)].
[2] W. Belzig and Yu. V. Nazarov, Phys. Rev. Lett. **87**, 197006 (2001).
[3] M. Vanevic, Y. V. Nazarov, and W. Belzig, Phys. Rev. Lett. **99**, 076601 (2007).
[4] M. Vanevic, Y. V. Nazarov, and W. Belzig, Phys. Rev. B **78**, 245308 (2008).
[5] M. Vanevic and W. Belzig, Phys. Rev. B **86**, 241306(R) (2012).
[6] J. Gabelli and B. Reulet, Phys. Rev. B **87**, 075403 (2013)
[7] L. S. Levitov, H.-W. Lee, and G. B. Lesovik, J. Math. Phys. **37**, 4845 (1996).
[8] J. Dubois, T. Jullien, F. Portier, P. Roche, A. Cavanna, Y. Jin, W. Wegscheider, P. Roulleau, and D. C. Glattli, Nature **502**, 659 (2013).