Stochastic enhancement of absolute negative mobility

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I. INTRODUCTION

Classical, dynamical systems have been analysed for ages. And they still are intensively studied in all contexts of nowadays science. They are the core part of applied maths, biology, chemistry, financial markets, physics (still!) and all the possible mixtures like econophysics, biophysics, biochemistry to name but a few. In general, one can write down short equation that creates endless possibilities for defining and later investigation of system in question

$$\ddot{x}(t) = F(x, \dot{x}, t). \tag{1}$$

Here x and t is the position and time respectively. Dot means the differentiation with respect to time. The F on the r.h.s. stands for any force that can act in the system like classical, non-classical, constant, time-dependent, deterministic or stochastic.

One of the global understanding of the Newton's Second Law of Motion is that the particle described by the eq. (1) follows the force it feels. If we push the coffee-cup lying on the table to the left it will move to the left. We cannot imagine other picture than that. It all happens because the forces are linear and we all drink coffee in the macro–world. In the micro–world, however, the linearities happen rarely. For finite temperatures noise effects appear, not just contributing to the loss of the system's dynamical abilities, but many times enhancing it[1]. On the other hand, for very low temperatures chaos seems to be inevitable. This all ingredients makes the equation (1) alive and well, still attracting strong attention.

So, what is typical? If the linear systems are in minority, noise doesn't destroy the dynamics, chaos seems to be everywhere, and even positive mobility is not so distinctly common[2].

The absolute mobility can be defined as the ratio of the velocity of the particle to the force $\mu = \dot{x}/F$. This ratio is commonly positive. There are situations, however, where it can become negative and the system shows counterintuitive dynamics. This phenomenon is called the absolute negative mobility. It has been reported for the classical, non-linear system with periodic potential driven by the periodic time-dependent force, constant force f in the presence of the equilibrium thermal noise, typically modelled by the δ-correlated Gaussian white noise ξ of zero mean[3]

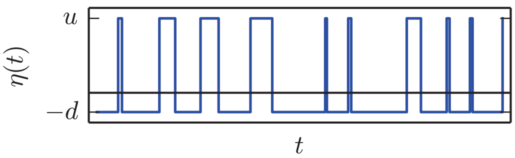
$$\ddot{x} = \sin(x) + a\cos(\omega t) + f + \xi(t). \tag{2}$$

For a certain conditions the absolute mobility can be negatively valued. The original numerical findings has been later confirmed experimentally[4].

What if one would test the system not against constant bias force f, but instead apply stochastic force, with the same *average* value as the constant force above? The dynamics would change drastically, but surprisingly the effect stays if one would consider biased Poissonian white shot noise[5]. The question if the effect would be similar for other type of non-equilibrium random forces appears naturally. Maybe even the *enhancement* of the ANM effect can be possible for random noise? One of the natural candidates to test the idea seem to be the two–state noise also known as a random telegraph noise

$$\eta(t) = \{-d, u\}, \quad d, u > 0, \tag{3}$$

The probabilities of transition per unit time from state $-d$ to u and back is given by μ_d and μ_u respectively.



Over the talk we'll try to address those questions, formulate the problem and discuss the dynamics of the classical system (2) where the constant bias will be replaced by the mean value of the dichotomous noise

$$f = \langle \eta(t) \rangle = \frac{u\mu_d - d\mu_u}{\mu_d + \mu_u}. \tag{4}$$

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