# Heat and charge current fluctuations in a thermoelectric quantum dot


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## I. INTRODUCTION

The adage "*The noise is the signal*" [1] has been proved many times. In the domain of quantum electric transport for example, the zero-frequency noise gives the conductance in the linear regime via the fluctuation-dissipation theorem, or gives the charge in the weak transmission regime via the Schottky relation. One of the beautiful illustrations of this latter fact was the measurement of the fractional charge of a two dimensional electron gas in the fractional quantum Hall regime[2,3].

In the recent wave of quantum heat transport studies, some works are devoted to heat noise[4,5,6,7,8,9] but very few to the fluctuations between the charge and heat currents[10,11] that we call the *mixed noise*[12]. In addition to the characterization of such a "new" quantity, our objective was to find which kind of information can be extracted from it. For this, we have calculated the correlator between the charge current and the heat current for a two terminal quantum dot system using the scattering theory and we have studied its possible relation with quantities such as the thermoelectric efficiency and the figure of merit.

## II. SYSTEM AND DEFINITION

We considered a single level quantum dot connected to two reservoirs with distinct chemical potentials $\mu_{L,R}$ and temperatures $T_{L,R}$ as depicted on Fig. 1.

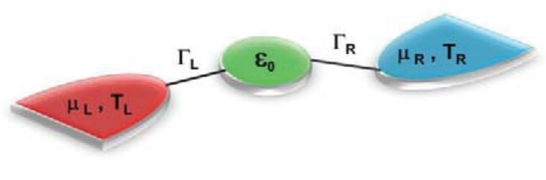


FIG. 1. Schematic picture of a single level quantum dot connected to two reservoirs with distinct chemical potentials and temperatures.

In a similar way to the standard definitions of the charge current noise, $\mathcal{S}_{pq}^{\mathrm{II}}$, and heat current noise, $\mathcal{S}_{pq}^{\mathrm{JJ}}$, we define the mixed noise as the zero-frequency Fourier transform of the correlator mixing the charge and the heat currents:

$$\mathcal{S}_{\mathrm{pq}}^{\mathrm{IJ}} = \int_{-\infty}^{\infty} \left\langle \delta \hat{I}_p(0) \delta \hat{J}_q(t) \right\rangle dt \tag{1}$$

where $\delta\hat{I}_p$ refers to the charge current and $\delta\hat{J}_p$ refers to the heat current. The indices $p$ and $q$ designate the left (L) or the right (R) reservoirs.

## III. MIXED NOISE

Using the Laudauer-Büttiker scattering theory, which applies in the absence of interactions, we obtain the expression of the mixed noise in terms of the transmission coefficient $\mathcal{T}$ and Fermi-Dirac distribution functions $f_p$:

$$\mathcal{S}_{\mathrm{pq}}^{\mathrm{IJ}} = \pm \frac{1}{h} \int_{-\infty}^{\infty} \left(\varepsilon - \mu_q\right) F(\varepsilon) d\varepsilon \tag{2}$$

with

$$\begin{aligned} F(\varepsilon) = {} & \mathcal{T}(\varepsilon) \left[ f_L(\varepsilon)\left(1 - f_L(\varepsilon)\right) + f_R(\varepsilon)\left(1 - f_R(\varepsilon)\right) \right] \\ & + \mathcal{T}(\varepsilon)\left[1 - \mathcal{T}(\varepsilon)\right] \left[ f_L(\varepsilon) - f_R(\varepsilon) \right]^2 \end{aligned} \tag{3}$$

In the linear response regime, the fluctuation-dissipation theorem holds and leads to the following relation between the noises and the electrical conductance $G$, the Seebeck coefficient $S$ and the thermal conductance $\kappa = \tilde{\kappa} - S^2 T_0 G$ :

$$\begin{aligned} \mathcal{S}^{\mathrm{II}} &= 2k_B T_0 G \\ \mathcal{S}^{\mathrm{JJ}} &= 2k_B T_0^2 \tilde{\kappa} \\ \mathcal{S}^{\mathrm{IJ}} &= \mathcal{S}^{\mathrm{JI}} = -2k_B T_0^2 S G \end{aligned} \tag{4}$$

where $T_0$ is the average temperature of the sample. The $p$ and $q$ indices have been removed in Eq. (4) since the noises are identical in amplitude in both reservoirs in the linear response regime. From these results, one shows that the thermoelectric figure of merit, defined as $ZT_0 = S^2 T_0 G/\kappa$, can be expressed fully in terms of noises such as:

$$ZT_0 = \frac{\left(\mathcal{S}^{\mathrm{IJ}}\right)^2}{\mathcal{S}^{\mathrm{II}}\mathcal{S}^{\mathrm{JJ}} - \left(\mathcal{S}^{\mathrm{IJ}}\right)^2} \tag{5}$$

When one leaves the linear response regime, the figure of merit is not more the relevant parameter to quantify thermoelectric conversion and one has rather to consider directly the efficiency which is defined as the ratio between the output and input powers. According to the thermoelectric engine that one wants to build, the powers are given either by the product between charge current and voltage or directly by the heat current. In general, it is not possible to connect these quantities to the noises. However, in the Schottky regime, i.e., in the weak transmission regime, with the help of the proportionality relations between the noises and the currents, we have shown that[12]:

$$\eta = \frac{\left(\mathcal{S}_{\mathrm{LR}}^{\mathrm{IJ}}\right)^2}{\left(\mathcal{S}_{\mathrm{LR}}^{\mathrm{IJ}}\right)^2 - \mathcal{S}_{\mathrm{LR}}^{\mathrm{II}}\mathcal{S}_{\mathrm{LR}}^{\mathrm{JJ}}} \tag{6}$$

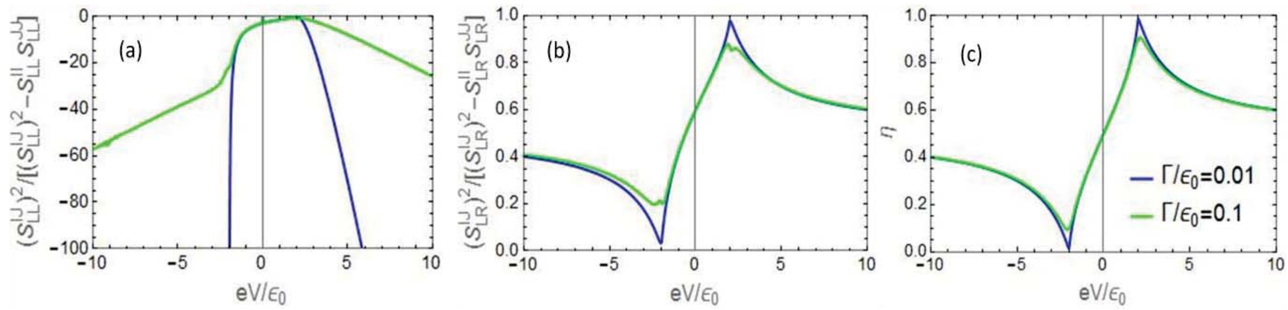


FIG. 2. (a) Auto-ratio of noises, (b) cross-ratio of noises, and (c) thermoelectric efficiency as a function of voltage.

A crucial point in Eq. (6) is the fact that it is needed to consider the ratio of cross-noises, i.e., the correlators between distinct reservoirs; otherwise the value that we get does not correspond to the efficiency. As a graphical proof, three quantities are shown on Fig. 2: the auto-ratio of noises, the cross-ratio of noises and the efficiency. As stated in Eq. (6), the cross-ratio of noises and the efficiency coincide at low transmission $\Gamma$, i.e., in the Schottky regime, whereas the profile of the auto-ratio of noises has nothing to do with these two first quantities. From this result, one can conclude that the mixed cross-noise is a measure of the efficiency. From Eq. (6) we also see that a vanishing mixed cross-noise cancels the thermoelectric efficiency.

Outside the linear response regime or the Schottky regime, i.e., in the intermediate regime, one needs to perform numerical calculations in order to characterize the behavior of the mixed noise. In Fig. 3 are shown the absolute values of the noises as a function of voltage and dot energy level at $\Gamma = T_{L,R}$, taken in the same reservoir ($p = q$) or in distinct reservoirs ($p \neq q$). Whereas the charge and heat noises present some symmetry, the mixed noise is fully asymmetrical. One knows that the asymmetry in noise gives information on the system, such as for example the asymmetry in the frequency noise spectrum which is directly related to the ac conductance through a generalized Kubo formula[13]. Thus, the identification of the information that may be contained in the asymmetry of the mixed noise and their possible relations to the thermoelectric conversion is a problem which remains to be addressed.

## IV. CONCLUSION

The mixed noise has fulfilled much of its promises since we showed that it is related to the thermoelectric figure of merit in the linear response regime or directly to the thermoelectric efficiency in the Schottky regime. We think that this quantity deserves to be studied on the same level than the charge and heat noises, both theoretically and experimentally. At the experimental side, the challenge is to find a way to measure such a quantity, whereas at the theoretical side, it is needed to calculate it using more realistic approaches that include, among others, interactions and many terminals.

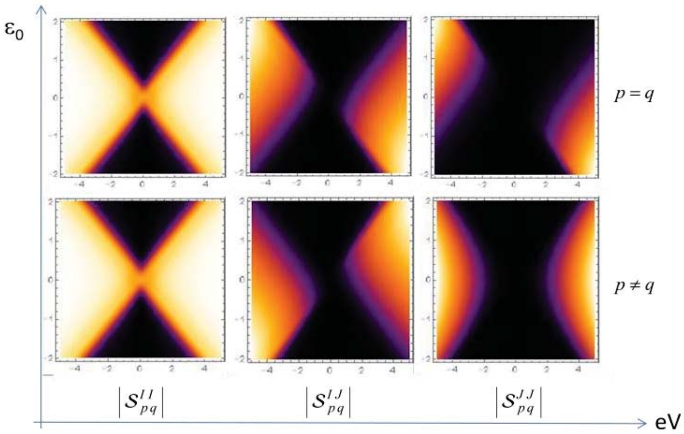


FIG. 3. Absolute value of the charge noise (left column), mixed noise (central column) and heat noise (right column) as a function of voltage and dot energy level. The black regions correspond to the lowest values (close to zero) and the bright regions to the highest values of the noises.

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