Non-Gaussian Stochastic Diffusion: Accounting Fourth Cumulant

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RESUME

In this report I discuss the non-Gaussian stochastic diffusion in electrochemical circuit of alternating current. I found the Fokker-Planck equation with the spatial derivative of second order that takes into account the excess function of the Poissonian electrochemical noise. The open question is formulated.

EQUATIONS

Let us consider the electrochemical circuit of alternating current showed on Fig. (1).

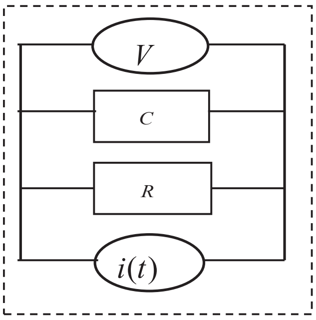

FIG. 1. Non-Gaussian Markov's electrochemical noise circuit

The circuit on Fig. (1) contains the double layer capacity C, the resistance of electrochemical discharge R, and the non-Gaussian current noise source $i(t)$ (t is time). The noise $i(t)$ describes the random character of electrochemical discharge. The voltmeter V measures the random voltage $e(t)$ on capacity C.

Let us introduce the random quantity ψ as the electrochemical analog of the random displacement of free Brownian particle:

$$\psi = \int_{-\theta}^{0} dt \varepsilon(t) \tag{1}$$

In Eq. (1) the symbol θ stands for observation time. At large observation time ($\theta >> RC$) we have for the symmetric (Poissonian) electrochemical discharge [1]:

$$\psi^{(2)} = 2D\theta \tag{2}$$

$$\psi^{(3)} = 0 \tag{3}$$

$$\psi^{(4)} = 4! D_4 \theta \tag{4}$$

Left-hand side of Eqs. (2), (3), and (4) equals the second, third and fourth cumulant of quantity ψ correspondingly. The symbols D and D_4 stand for the coefficients.

Eqs. (2) - (3) are the corollaries of the Einstein stochastic diffusion equation (5) for the probability density function $W(\psi, \theta)$:

$$[\frac{\partial}{\partial \theta} - D \frac{\partial^2}{\partial \psi^2}] W(\psi, \theta) = 0 \tag{5}$$

But Eq. (4) is not a corollary of Eq. (5).

Our main result is the Fokker—Planck equation (6):

$$[\frac{\partial}{\partial \theta} - (D + \frac{D_4}{D} \frac{\partial}{\partial \theta}) \frac{\partial^2}{\partial \psi^2}] W(\psi, \theta) = 0 \tag{6}$$

Eqs. (2), (3), and (4) are the corollaries of Eq. (6).

OPEN QUESTION

In general case of the any duration of the observation time, the Einstein stochastic diffusion equation (5) must be replaced by the Cattaneo equation (7) (telegrapher's equation type) [2, 3]:

$$[(1 + RC \frac{\partial}{\partial \theta}) \frac{\partial}{\partial \theta} - D \frac{\partial^2}{\partial \psi^2}] W(\psi, \theta) = 0 \tag{7}$$

My open question is:

How I can find the analog of Cattaneo equation (7) for Eq. (6)?

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