Is the peculiar behavior of $1/f$ noise in graphene the result of the interplay between band-structure and inhomogeneities?

B. Pellegrini, P. Marconcini, M. Macucci, G. Fiori, G. Basso
Dipartimento di Ingegneria dell'Informazione,
via G. Caruso 16, 56122 Pisa, Italy
e-mail address: macucci@mercurio.iet.unipi.it

I. INTRODUCTION

Flicker noise in graphene based devices has attracted significant interest[1] because of the very peculiar features it exhibits, in comparison with what is observed in more traditional materials. In particular, the behavior of the noise power spectral density (PSD) as a function of carrier concentration has turned out to be rather puzzling, especially in bilayer graphene, and several authors[2–6] have made an effort to find an explanation for it. For example, it has been observed that the PSD of flicker noise in bilayer graphene, and sometimes also in monolayer graphene, has a minimum around the Dirac point, where charge concentration also reaches a minimum, while in conductors obeying Hooge's empirical formula[7] the opposite is expected. Attempts have been made to justify the particular dependence of the flicker PSD on carrier concentration on the basis of the known presence of electron and hole puddles in the graphene sheet[2], of a supposed variation[3] of the Hooge parameter with gate voltage, which would prevail on the effect of the carrier number decrease, of effects linked to mobility fluctuations[4], of a charge-noise model[5], or of the bandstructure of single layer and bilayer graphene[6]. Experiments have also shown[8] that the noise factor in monolayer graphene nanoribbons is independent of the resistance to length ratio, while a clear dependence on such a quantity is observed for bilayer graphene nanoribbons. However, a comprehensive model, capable of explaining all observed features, is still lacking. In the present contribution, we shall try to provide a framework within which a more general understanding of flicker noise in graphene sheets and nanoribbons can be derived.

II. CURRENT FLUCTUATIONS

Our aim is to find an expression for the PSD of flicker current noise in graphene-based devices. We assume that flicker noise is the result of charges moving into and out of traps that have an electrostatic coupling with the channel where the current flows. We also assume that such fluctuations occur on a time scale much longer than that of carrier scattering events (such as phonon scattering). Thus the contribution of each elementary area of the device is due to the fluctuation of the local value of the drift current; this can be related to the current at the terminals via the Ramo-Shockley-Pellegrini[9–11] theorem. In particular, if, for the sake of simplicity, we do not enter into the specific details of the device geometry and assume the electric field E to be somewhat constant across the device, we can write the current at the terminals, as long as we are interested just in the low-frequency fluctuations, in the form

$$I = \frac{1}{L}\int_A \mu n E dx dy\,, \tag{1}$$

where μ is the mobility, $A = WL$ the area of the device, L being its length and W its width. Let us now move on to the evaluation of the fluctuations of the current. We are interested only in the fluctuations due to charges moving into and out of traps, therefore we can consider just the action of such traps. In principle, trapping and detrapping events have an action not only on the number of carriers available for conduction, but also on the mobility, and on the local electric field. These two latter contributions are in general negligible with respect to the former[12], so that the relative fluctuation for each trap can be written

$$\frac{\Delta i}{I} = \frac{1}{A}\int_A \frac{\Delta n_c}{n_c} dx dy\,, \tag{2}$$

where $n_c = n_n + n_p$, i.e. the total concentration of carriers, while we also define $n = n_n - n_p$, which, multiplied by the electron charge, gives the total charge density.

Assuming for each trap a random telegraph signal χ for the occupancy (with a value of 1 when the trap is occupied and 0 when it is empty), the relative variation of the current due to a charge moving into the trap can be written[12]

$$\frac{\Delta i}{I} = -\frac{1}{A}\left(\frac{a_c}{a n_c}\right)\Delta\chi, \tag{3}$$

where $a_c = \partial n_c/\partial U$ (with U being the electrostatic potential energy), $a = \partial n/\partial U$. The superposition of the Lorentzian spectra associated with the traps leads to a PSD[13]

$$\frac{S}{I^2} = \frac{n_t B}{A}\left(\frac{a_c}{a n_c}\right)^2 \frac{1}{f^\gamma}\,, \tag{4}$$

where n_t is the trap density, B is a proper coefficient, and γ is usually 1 (flicker noise). We must consider that the potential is not at all uniform across a graphene sheet, but, rather, it fluctuates, due to the presence of impurities and of defects. Thus the PSD of Eq. (4) has to be weighed with the distribution function $P(U)$ of the potential energy:

$$\frac{\langle S\rangle}{I^2} = \frac{\langle n_t\rangle}{A f^\gamma}\int B\left(\frac{a_c}{a n_c}\right)^2 P(U) dU\,, \tag{5}$$

where we assume a Gaussian form for $P(U)$, with a standard deviation σ^*.

III. RESULTS

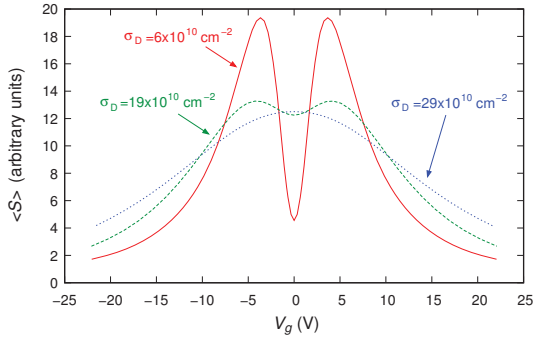

FIG. 1. Behavior of the PSD in monolayer graphene as a function of the applied gate voltage V_g, for three values of the standard deviation σ_D.

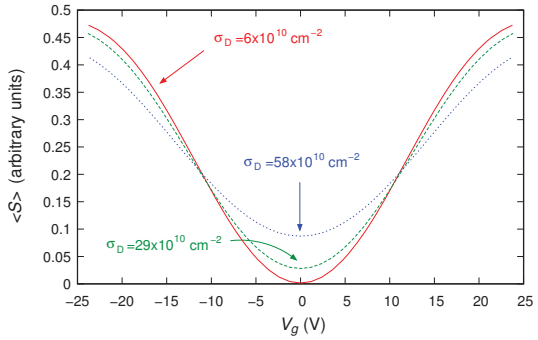

FIG. 2. Behavior of the PSD in bilayer graphene as a function of V_g, for different values of σ_D.

The result for single-layer graphene is shown in Fig. 1 for 3 values of σ_D, where $\sigma_D = \sigma^* \hat{a}$, with $\hat{a} \propto n_0$ the value of a for $U = 0$. The quantity n_0 is the reference density, given by $n_0 = (1/\pi)[k_B T/(\hbar v)]^2 \approx 7 \times 10^{10}$ cm^{-2}, with v being the in-plane velocity. For the lowest value of the standard deviation we notice a minimum of the noise around the Dirac point as a result of a suppression of current fluctuations resulting from the presence of an equal concentration of carriers with charges of opposite sign. We see that, for increasing σ_D, the behavior of the noise power spectral density as a function of the applied gate voltage moves from an M shape to a Λ shape, as a result of the increasing smoothing effect of $P(U)$. In Fig. 2 we report instead the behavior of the PSD as a function of the gate voltage for bilayer graphene, for a choice of 3 values of the standard deviation. We notice that now the shape is always of the V type, as a result of the much smoother variation of the energy dispersion relationship around the Dirac point with respect to what happens for

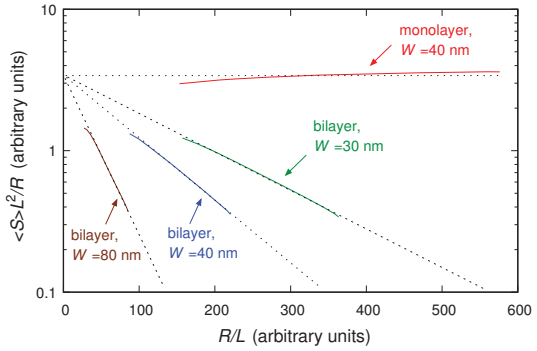

FIG. 3. Noise factor $\psi = \langle S \rangle L^2/R$ for monolayer (topmost curve) and for bilayer (lower curves) graphene nanoribbons, as a function of R/L.

monolayer graphene. The proposed approach can be applied also to nanoribbons, which are characterized by a peculiar bandstructure. If we consider armchair nanoribbons and compute the quantity $\psi = \langle S \rangle L^2/R$, where R is the nanoribbon resistance, we get the results reported in Fig. 3, as a function of R/L. We notice that for monolayer graphene nanoribbons there is no significant variation (at least in the considered range of resistances) of the noise factor, while for bilayer graphene, in a semilogarithmic scale, there is a clear linear dependence on the R/L ratio, with a different slope for different nanoribbon widths. This is in agreement with the experimental results by Lin and Avouris[8].

[1] A. A. Balandin, Nature Nanotechnology **8**, 549 (2013).
[2] G. Xu, C. M. Torres Jr., Y. Zhang, F. Liu, E. B. Song, M. Wang, Y. Zhou, C. Zeng, and K. L. Wang, Nano Letters **10**, 3312 (2010).
[3] Y. Zhang, E. E. Mendez, and X. Du, ACS Nano **5**, 8124 (2011).
[4] S. Rumyantsev, G. Liu, W. Stillman, M. Shur, and A. A. Balandin, J. Phys.: Condens. Matt. **22**, 395302 (2010).
[5] I. Heller, S. Chatoor, J. Maennik, M. A. G. Zevenbergen, J. B. Oostinga, A. F. Morpurgo, C. Dekker, and S. G. Lemay, Nano Letters **10**, 1563 (2010).
[6] A. N. Pal, S. Ghatk, V. Kochat, E. S. Sneha, A. Sampathkumar, A, Raghavan, and A. Ghosh, ACS Nano **5**, 2075 (2011).
[7] F. N. Hooge, Physics Letters A **29**, 139 (1969).
[8] Y. Lin and P. Avouris, Nano Letters **8**, 2119 (2008).
[9] S. Ramo, Proceedings of the IRE **27**, 584 (1939).
[10] W. Shockley, J. Appl. Phys. **9**, 635 (1938).
[11] B. Pellegrini, Phys. Rev. B **34**, 5921 (1986).
[12] B. Pellegrini, Eur. Phys. J. B **86**, 373 (2013).
[13] B. Pellegrini, Microel. Reliab. **40**, 1775 (2000).