Is There an Optimal Search Strategy?

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I. INTRODUCTION

Bayesian analysis changes probability estimates as information is acquired. If you are searching for something and after some time you do not find it, that information can be used to estimate if the object of the search is present or not. Assume a searcher has finite resources and finite time to carry out a search and the searcher can search different areas and can use a variety of search methods. The unsolved question is when is it optimal to give up searching an area with a single method and switch methods or switch areas of search. Examples could be as simple as looking for a piece of paper between two cluttered desks, an animal foraging for food, or looking for a lost person, ship, or aircraft. Noise will cause fluctuations in the model parameters.

II. NET GAIN EQUATION

Let's start with the equation for searching two separate areas, 1 and 2, or the same area with two different methods also denoted by 1 and 2,

$$\begin{aligned} N(T,\tau) = {} & G_1 p_1 \Phi_1(T-\tau) + G_2 p_2 \Phi_2(\tau) \\ & - C_1(T-\tau) - C_2\tau \end{aligned} \tag{1}$$

where N is the net gain or reward, T is the allotted search time, τ is the time spend in region 2 or with search method 2, G is the gain, p is the probability that the target is present, $\Phi(t)$ is the probability, if the target is present, that it will be found by time t, and C is the cost of the search, per unit time.

III. GENERALIZATIONS: THIN OR FAT TAILS

Several possibilities can be explored, e.g. exponential or algebraic probability distributions,

$$\Phi(t) = 1 - \exp(-\lambda\, t)$$

or (2)

$$\Phi(t) = \frac{\tau_0}{\tau^2{}_0 + t^2/n}$$

where larger n corresponds to a fatter tail.

IV. BROWNIAN OR BALLISTIC

In the exponential case, λ can be studied for Brownian motion as

$$\lambda \propto \frac{D}{\langle d\rangle^2} \tag{3}$$

where D is the diffusion constant and < d > is the mean distance between targets.

For ballistic motion with velocity v, the success rate λ can be,

$$\lambda \propto \frac{\mathrm{v}R}{\langle d\rangle^2} \quad \text{(2D)}$$

$$\lambda \propto \frac{\mathrm{v}R^2}{\langle d\rangle^3} \quad \text{(3D)} \tag{4}$$

where R is the range at which a target can be sighted. If the targets are on a fractal set of dimension α, then d is replaced by $d^{1/\alpha}$. In the second case, larger n values mean fatter tails with successful search relegated probabilistically to longer times.

V. SPLITTING RESOURCES

With finite resources available, the success rate might need to be split between search of type 1 and type 2, i.e. there is a maximum Λ and it gets split between 1 and 2, $\Lambda = (\Lambda - \lambda) + \lambda$ where the term $\Lambda - \lambda$ is the success rate allotted to search of type 1 and λ is for the type 2 search.

VI. THE TARGET FLEES

Another generalization is that the target moves away to avoid detection, say in the manner,

$$p \Rightarrow p\exp(-\beta t) \qquad (5)$$

VII. CHASE OR AMBUSH

The cost,over a time t, of a constant velocity search might be related to kinetic energy expended so $C \propto m\mathrm{v}^2$ t, but then the success rate might increase proportional to v. One can contrast a predator practicing ambushing prey so the cost is C=0 while waiting, against chasing prey with a velocity v.

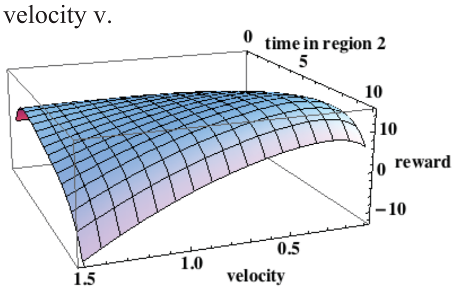

FIG. 1A representative example of maximizing the gain as a reward (measured in the z direction) as a function of time spent in region 2 (along the y axis) and velocity of the searcher in region 2 whose success rate λ is proportional to velocity, but the cost of searching is proportional to kinetic energy expended, i.e. v^2.

VIII. DO NOT PICK LOW HANGING FRUIT

Or perhaps the gain G = G(λ) is proportional to 1/λ, soeasier to find targets will typically have less value.

IX. CONCLUSIONS

The factors for a net gain can be generalized in several directions, and each type of search will have its own trade space of variables.

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