Non-zero probability of detecting identical electrons at the same position: How does it affect the Landauer-Büttiker noise expression at high temperatures?

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I. INTRODUCTION

Is it possible to detect two electrons at the same position with the same energy? Initially one would answer negatively due to the Pauli principle[1]. However, strictly speaking the Pauli principle, which is just a consequence of the exchange interaction (for the indistinguishability of quantum particles), only forbids common positions when electrons share exactly the same state. Therefore, if two particles are described by slightly different states, one cannot neglect the possibility of detecting both at the same location.

The modeling of quantum noise in mesoscopic systems is normally studied within the (energy) scattering (eigen)states[2], where only one state is available for each energy, precluding the detection of two particles at the same position. However, in this conference we show that quasi-particle wave packets do not preclude such possibility. The detection of two particles with identical energies at the same position are possible with such time-dependent states, because transmitted and reflected wave packets are not exactly identical. These new two-particle scattering probabilities leads to new terms in the usual Landauer-Büttiker[2] quantum noise expression.

II. TWO-PARTICLE SCATTERING

A new quantum noise formalism is developed for many-electron systems described by quasi-particle wave packets. For simplicity, the relevant effects are discussed in the two-particle scenario depicted in Fig. 1. The generalization to a realistic many-particle system will be mentioned in the conclusions. We analyze two identical wave packets that are located at each side of the barrier (at the same distance) and with opposite momentum (i.e. same central energy). During the interaction with the barrier, the initial wave packets split into a transmitted and a reflected part. At the final time, apart from the obvious probabilities of detecting a particle at each side of the barrier (see Fig. 1a and b), the time-dependent numerical solution constructed from quasi-particle wave packets shows that it is possible to find both electrons at the same place, i.e. both at the left side or at the left side (see Fig. 1c and d). The ultimate reason of these unexpected probabilities is the fact that the reflected and transmitted wave packets are not equal at the final time, even if they have identical energy at the initial time[3,4]. It is remarkable that our many-particle wave packet formalism provides simple physical explanations for some relevant and still unexplained experimental results[5,6].

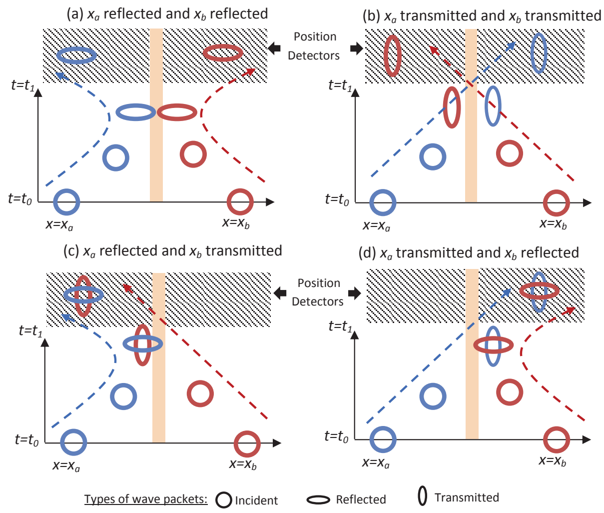

FIG. 1. Two identically injected wave packets from the left x_a and from the right x_b of a scattering barrier. Solid regions represent the barrier region and shaded regions represent the particle detectors. (a) and (b) each particle is detected on a different side of the barrier at final time t_1 when the interaction with the barrier has almost finished. (c) and (d) both particles are detected on the same side of the barrier.

For example, the possibility of finding both quasi-electrons at the left side ($\mathcal{P}_{\mathcal{L}\mathcal{L}}$) is

$$\mathcal{P}_{\mathcal{L}\mathcal{L}} = \int_{-\infty}^{0} dx_1 \int_{-\infty}^{0} dx_2 \ |\Phi|^2 = R_a T_b - |I_{a,b}^{r,t}|^2. \quad (1)$$

where Φ is the antisymmetric two-particle wave function. R_a and T_b are the reflection and transmission coefficients of the single wave packets. The term $|I_{a,b}^{r,t}|^2$ accounts for the overlapping among the reflected wave packet a and the transmitted wave packet b.

As mentioned above, the probabilities $\mathcal{P}_{\mathcal{L}\mathcal{L}}$ and $\mathcal{P}_{\mathcal{R}\mathcal{R}}$ are different from zero and their values fluctuate between $\mathcal{P}_{\mathcal{L}\mathcal{L}} = 0$ and $\mathcal{P}_{\mathcal{L}\mathcal{L}} = RT$. This is seen in Fig. 2, where the usual zero probability is recovered for large spatially extended wave packets (close to time-independent scattering sates) with energies far from the resonant energy. On the contrary, the maximum values of $\mathcal{P}_{\mathcal{L}\mathcal{L}}$ are achieved for not infinitely-extended wave packets with energies closer to the resonance.

Obviously, in scenarios with more scattering probabilities (the ones showed in Fig. 1c and d for both electrons at the same place), the quantum noise is enlarged. This

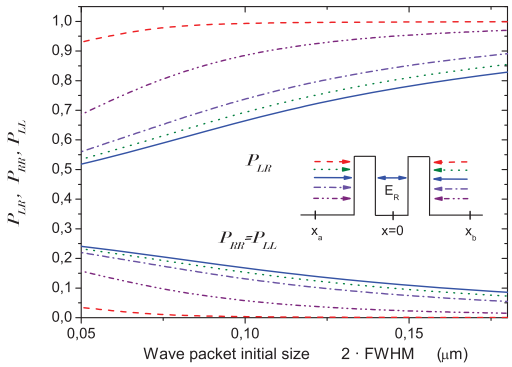

FIG. 2. Probability of detecting two electrons at the same side of the barrier ($\mathcal{P}_{\mathcal{LL}}$ and $\mathcal{P}_{\mathcal{RR}}$) and of detecting one electron at each side ($\mathcal{P}_{\mathcal{LR}}$) depending on the wave packet initial size for three different energies.

is reflected in the following expression for quantum noise which has a new term in our two-particle scenario (in a many-particle scenario more terms are added):

$$\langle S \rangle = \frac{4q^2}{h} \int_0^\infty dE \, \{T[f_a(1-f_a) + f_b(1-f_b)] + T(1-T)(f_a - f_b)^2 + 2\mathcal{P}_{\mathcal{LL}} f_a f_b\}. \quad (2)$$

The new term $2\mathcal{P}_{\mathcal{LL}} f_a f_b$ shows that the well-established Landauer-Büttiker[2] expression (obtained using scattering states) can be violated in same scenarios. On the contrary, when wave packets are close to scattering states, then $\mathcal{P}_{\mathcal{LL}} = 0$, and usual results are recovered.

III. CONCLUSIONS AND DISCUSSIONS

We generalize the Landauer-Büttiker noise expression by considering many-particle states constructed from an antisymmetric combination of quasi-particle wave packets. In the particular two-particle scenario, the results in equation (2) recover also the usual scattering states results when using infinitely-extended states.

A realistic scenario for quantum transport implies the consideration of a many-particle case. Then, new more terms appear in the quantum noise expression accounting for two-, three-, etc wave packets correlations. At low temperatures, when the phase-space is full, the mentioned new terms added in the quantum noise expression tends to zero (see Fig. 3) and the quantum noise is zero, satisfying the fluctuation-dissipation theorem[7,8]. Nevertheless, at high temperatures, these news terms can not be neglected and quantum noise is increased over what is usually predicted. We emphasize that the formalism presented in this conference provides a physical explanation for surprising experimental results[5,6], which are normally attributed to spurious effects. Finally, we remark that the increment of quantum noise discussed here is very robust and it is present (even magnified) when time-dependent potentials or (non-separable) Coulomb interactions are considered[3].

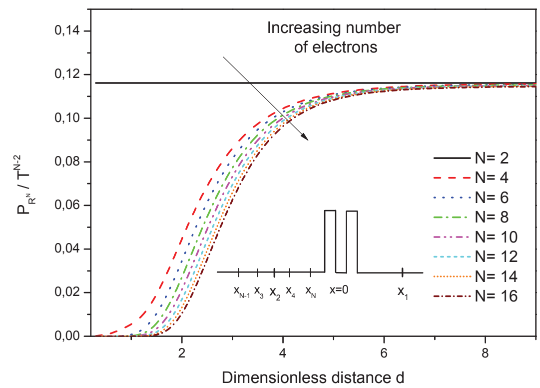

FIG. 3. Each curve reflects the new term added in the quantum noise expression for different number of involved electrons. The phase space gets filled as we increase the number of electrons and we decrease the dimensionless distance d. We see how probabilities decrease.

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