Noise in graphene and carbon nanotube devices

G. Fève, J-M. Berroir, T. Kontos, C. Voisin, B. Plaçais

Laboratoire Pierre Aigrain – Ecole Normale Supérieure
24 rue Lhomond, 75231 Paris Cedex 05 France
www.lpa.ens.fr
Noise in genuine low-dimensional systems

From 2D graphene to 1D carbon nanotubes

From massless chiral Dirac Fermions to single mode electronic “fibers”

- Large Fermi velocity $v_F = 10^6 \text{ m/s}$
- Long and tunable wave length
- Low density of states

Castro-Neto et al. RMP-2009
Charlier et al., RMP 2007
Graphene as tunable 2D semi-metal

a) Quantum shot noise in graphene (a brief review)
b) Noise thermometry of hot electrons: electron-phonon in 2D
c) Applications: HEBs, LNAs, Photo-detectors,

Carbon Nanotubes as single mode nano-conductors (a review)

a) Quantum shot noise in carbon nanotube devices
b) Thermal noise in CNT wires and CNT-FETs: the noise conductance
Graphene as a tunable high-mobility metal

Landauer-Büttiker

\[ G_L = \frac{4e^2}{h} \times \sum_1^N T_n = \frac{1}{6450 \, \Omega} \times \sum_1^N T_n \]

\[ \frac{N}{W_{\mu m}} = \frac{k_F}{\pi} = 56 \sqrt{n/10^{12} \, \text{cm}^{-2}} = 5 - 500 \]

h-BN encapsulated graphene

L. Wang et al., Science 342, 614 (2013)
Quantum Shot Noise in graphene

**Conductance is transmission**

\[ G = \frac{e^2}{h} \sum_{n=1}^{N} T_n \]

**Quantum Shot Noise (QSN)**

\[ S_I = 2eI \frac{\sum T_n (1 - T_n)}{\sum T_n} = 2eI \times "Fano" \]

Ya.M. Blanter, M. Büttiker / Physics Reports 336 (2000) 1-166

**Dirac point : electronic transmission**

\[ T_{n,\text{Dirac}} = \cosh^{-2}\left[ \pi (n + \alpha) \frac{L}{W} \right] \]

**Noise**

\[ Fano_{\text{Dirac}} = \frac{1}{3} \]

« Conductivity »

\[ \sigma_{\text{Dirac}} \frac{L}{W} = \frac{4e^2}{h} \frac{L}{W} \int_{0}^{\infty} \frac{dk_y}{\cosh^2[k_yL]} = \frac{4e^2}{\pi h} \]

QSN in graphene junctions (Aalto exp.)

Ballistic graphene junctions (W=5L)  
F=1/3  at Dirac Point

QSN in graphene ribbons (Aalto exp.)

**Theory**

- **Graph 1:**
  - $\sigma [4 e^2/\pi \hbar]$ vs $W/L$
  - $\mu = 0$

- **Graph 2:**
  - $F$ vs $W/L$
  - $\mu = 0$

**Experiment**

- **Graph 3:**
  - $F$ vs $V_{bias}$ and $V_{gale}$

UPoN-2015, Barcelona, 15/7/2015
from quantum to classical

... on increasing sample length

from quantum to thermal

... on increasing bias voltage

\[ eV_{e-e} \sim \frac{E_F}{\sqrt{k_F L}} \]
\[ V_{e-ph} \sim \frac{L_0}{\sqrt{4L^2 \rho \Sigma_{ph}}} \]

\[ \sqrt{3}/4 \]

1/3

Hot electrons

Phonon cooling

\[ \frac{S_r}{2eI} \]

Semi-ballistic

Diffusive

\[ \leq \text{RF noise thermometry} \Rightarrow \]
RF current-noise measurement

current noise spectrum at high-bias

typical noise spectra in a 1kΩ resistor

- 1/f noise
- Shot (or Thermal) noise
- eV/h cutoff

GHz-setup (LPA)
Typical Fano-factor dependence

$$S_i = 4k_B \langle T_e \rangle / R$$

$$P = V^2 / R$$

Sample #1

Sample #2
Electron-Phonon interaction in graphene

- **OP-phonons** irrelevant
- **large AC-phonons** velocity 
  \( s = 2 \times 10^4 \text{ m/s} \)


---

*UPoN-2015, Barcelona, 15/7/2015*
Electron-Phonon in graphene

OP-phonons irrelevant

Very weak Phonon resistivity $\rho \approx 0.1 \times T$

Bloch-Grüneisen Temp. $\sim 50K$

large AC-phonons velocity $(s = 2 \times 10^4 \ m/s)$

AC-phonon resistivity

\[ T < T_{BG} \text{ (cold)} \]

Fermi surface

\[ \alpha_T \]

\[ 2k_F > q_{\text{max}} \]

\[ \sim 40 \, K \]

\[ \sim 1000 \, K \]

\[ T = T_{BG} = (2s/v_F)T_F \]

Available phonon space

\[ q = kT \]

\[ \Delta \rho(T_{ph} \ll \theta_{BG}) = \frac{8D^2k_F}{\rho_m e^2Sv_F^2} \times f\left(\frac{T_{BG}}{T_{ph}}\right) \sim T^4 \]

\[ \Delta \rho(T_{ph} \ll \theta_{BG}) \sim \text{const} \times T \] ; const. \approx 0.1\Omega/K

\[ \mu_{ph}(300K) = 1/ne\Delta\rho \approx 2 \times 10^5/n_{12} \]

\[ l_{ph}(300K) = \mu E_F/e v_F \approx 7 \mu m / \sqrt{n_{12}} \]

AC-phonon relaxation: \( T \leq T_{BG} \)

**Joule heating + phonon cooling**

Graphene:
\[ V_F = 10^6 \, \text{m/s} \]
\[ s/V_F = 0.02 \]

Electric field + scattering
acoustic-phonons only

\[ \Sigma_{e-ph} = \frac{\pi^2 D^2 k_B^4 \mu_F}{15 \rho_m \hbar^5 s^2 v_F^5} \leq 10 \text{mW/m}^2 K^4 \ll P_{Kapitza} \approx 10 \text{W/m}^2 K^4 \]

**Very weak AC-phonon coupling \( \Rightarrow \) very hot electrons**

\[ P = \sum_{e-ph} T^4 \]

\[ P = \sum_K T^4 \]

\[ P_{WF} \propto L_0 T^2 \]

\[ L_0 \approx 10 \, \text{W/cm}^2 K^4 \]
RF-noise thermometry

Thermal + 1/f noise

diffusive G/hBN sample

linear I-V’s (diffusive)

noise: from linear to sublinear


UPoN-2015, Barcelona, 15/7/2015
\[ \langle T_e \rangle \equiv R S_I / 4k_B \quad \text{with} \quad P = V \times I \]
RF noise thermometry

\[ \langle T_e \rangle \equiv R S_I / 4k_B \text{ with } P = V \times I \]

\[ T_e \equiv R \frac{S_I}{4k_B} \frac{1}{w_{\text{ch}}} P = V \times I \]

A. Betz et al. / Nat. Phys. 9 (2012) 109

\[ 2 k_F > q_{\text{max}} \propto T \]
Heat equation (steady state)

\[
\frac{V^2}{R} = LW \sum_{e-ph} T_e^4 - \frac{L_o}{2R} L^2 \frac{\partial^2 T_e}{\partial x^2}
\]

\[L_o = \frac{\pi^2 k_B^2}{3e^2} \text{ (Lorenz number)}\]
Heat equation (steady state)

\[
\frac{V^2}{R} = LW \sum_{e-ph} T_e^4 - \frac{L_o}{2R} \frac{L^2 \partial^2 T_e}{\partial x^2}
\]

L_o = \pi^2 k_B^2 / 3 e^2 (Lorenz number)


T^4-BG - hot electrons

T^2-WF - hot electrons
Hot-phonon regime (low doping)

![Graph showing temperature vs. power with various gate voltages](image)

- $V_g = +12 \text{ V (CNP)}$
- $0 \text{ V}$
- $-10 \text{ V}$
- $-20 \text{ V}$
- $-32 \text{ V}$
- $-43 \text{ V}$
- $-55 \text{ V}$

$T_e (\text{K})$ vs. $P (\text{mW} \left[ \mu \text{m} \right]^{-2})$

$T > T_{BG(\text{hot})}$

$2k_F < q_{max}$
The « supercollision » regime

Ordinary electron-phonon collision

![Graph showing ordinary electron-phonon collision and 3-body electron-phonon impurity](image)

3-body electron-phonon impurity

Exp. : Betz et al. / Nat. Phys. 9 (2012)
Th. : Song-Levitov / PRL (2013)

T3 A. Betz et al., Nat. Phys. 9 (2012) 109
T3 …/…
Hot electron Bolometers for single phonon detection:
tiny electronic heat capacity + weak electron-phonon relaxation

\[ \varepsilon \delta(t) = LW \sum T_e^4 - \frac{L_o L^2}{2R} \frac{\partial^2 T_e^2}{\partial x^2} + \gamma LW \frac{\partial T_e^2}{\partial t} \]

\[ P = \sum_{\text{e-ph}} T_e^4 \]
\[ = 100 \frac{aW}{\mu m^2} (T < 1K) \]
\[ = 1 \text{photon/100pS} \]

Electronics: Hot-electrons limit the resolution of RF charge detectors

*LPA graphene group:*

E. Pallecchi et al., JPAP 47, 094004 (2014)
Hot electrons reveal optical-phonons

OP-phonon energy $\approx 2000$ K

Use suspended BLG:
$\Rightarrow$ rid of substrate phonons
$\Rightarrow$ AC-phonon is suppressed
$\Rightarrow$ But a large WF++

Hot electrons and substrate-phonons

UPoN1: Benchmarking optical and Joule heating

UPoN2: Investigate interactions with substrate polar phonons (SPPs)

Low-frequency $1/f$ noise in graphene devices

Alexander A. Balandin

Low-frequency noise with a spectral density that depends inversely on frequency has been observed in a wide variety of systems including current fluctuations in resistors, intensity fluctuations in music and signals in human cognition. In electronics, the phenomenon, which is known as $1/f$ noise, flicker noise or excess noise, hampers the operation of numerous devices and circuits, and can be a significant impediment to the development of practical applications from new materials. Graphene offers unique opportunities for studying $1/f$ noise because of its two-dimensional structure and widely tunable two-dimensional carrier concentration. The creation of practical graphene-based devices will also depend on our ability to understand and control the low-frequency noise in this material system. Here, the characteristic features of $1/f$ noise in graphene and few-layer graphene are reviewed, and the implications of such noise for the development of graphene-based electronics including high-frequency devices and sensors are examined.

UPoN3: new clues on $1/f$ noise and Hooge’s law using tunable graphene?
See next talk by M. Macucci
UPoN4: investigate interplay between electrons and plasmons in 2D
See pm-talk by L. Varani
Graphene as tunable 2D semi-metal

- Quantum shot noise in graphene (a brief review)
- Noise thermometry of hot electrons: electron-phonon in 2D
- Applications: HEBs, LNAs, Photo-detectors

Carbon Nanotubes as single mode nano-conductors (a review)

- Quantum shot noise in carbon nanotube devices
- Thermal noise in CNT wires and CNT-FETs: the noise conductance
good contacts + ballistic carbon nanotube \implies \text{Fabry-Pérot electronic cavity}

\begin{align*}
\text{Checkerboard conductance pattern} &\implies \text{QSN suppression:} \quad S_I = 2eI(1 - T)
\end{align*}

Kondo CNT-devices

Medium contacts + interactions + odd e-number \implies Kondo effect

Kondo ridge \implies QSN suppression

Coulomb-blockade devices

Poor contacts + interactions = quantum dot => Coulomb blockade

Coulomb Blockade + inelastic cotunneling => superpoissonian noise

**Hot electrons in 1D carbon nanotubes**

\[ P_{ph} = \sum (T_e^{d+2} - T_{ph}^{d+2}) \]

Graphene: \[ P_{ph} = \sum (T_e^4 - T_{ph}^4) \]

Carbon nanotube: \[ P_{ph} = \sum (T_e^3 - T_{ph}^3) \]

*F. Wu et al., Appl. Phys. Lett. 97, 262115 (2010)*
The thermal noise conductance in FETs


Two-terminal conductors

\[ S_I(\omega) = 4k_B T_e \times G_{\text{diff}}(\omega) \]

Three-terminal conductors

\[ S_I(\omega) = 4k_B T_e \times G_{\text{noise}} \]

See also: talk on noise temperature fluctuations and the Noise Thermal Impedance by E. Pinsolle and B. Reulet
The noise conductance of nano-FETs

\[ G_{\text{noise}} = G_{ds} + G_m \frac{C_Q}{2C_{gs}} \]

\[ G_{ds} = \frac{4e^2}{h} \times f_d(\Delta) \]

\[ G_m = \frac{4e^2}{h} \times [f_s(\Delta) - f_d(\Delta)] \times \frac{C_{gs}}{C_Q} \]

\[ G_{\text{noise}} = \frac{4e^2}{h} \times [f_s(\Delta) + f_d(\Delta)] \]

Thank you for your attention