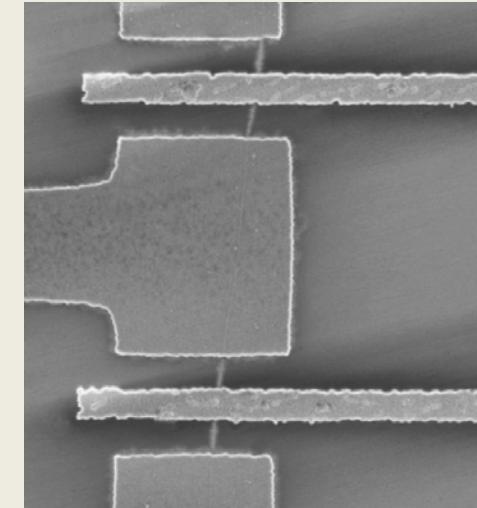
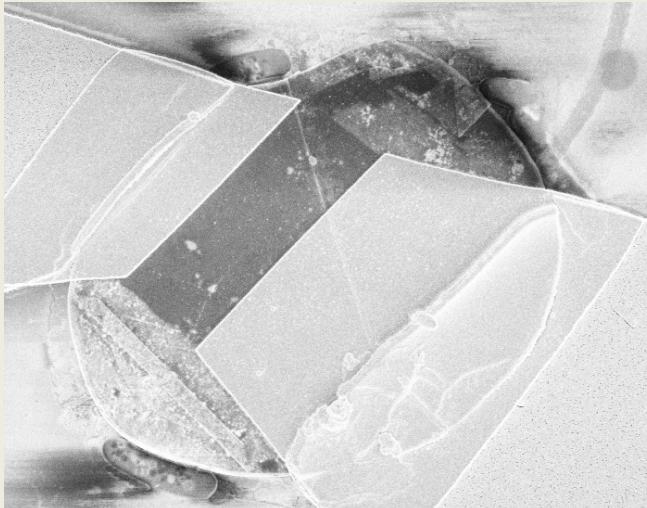


## Noise in graphene and carbon nanotube devices



G. Fève, J-M. Berroir, T. Kontos, C. Voisin, B. Plaçais

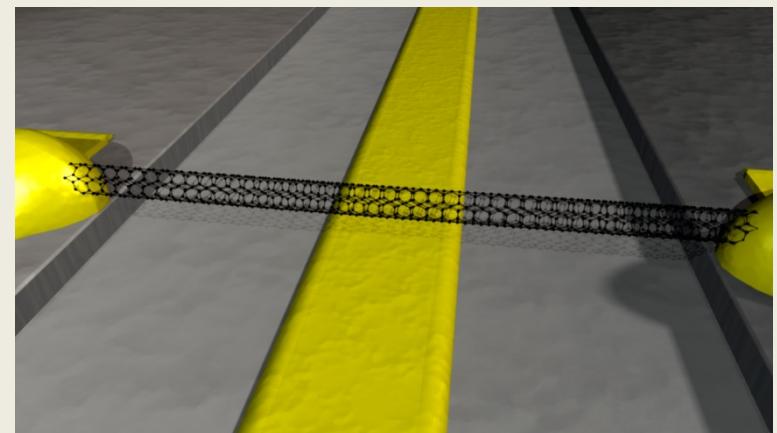
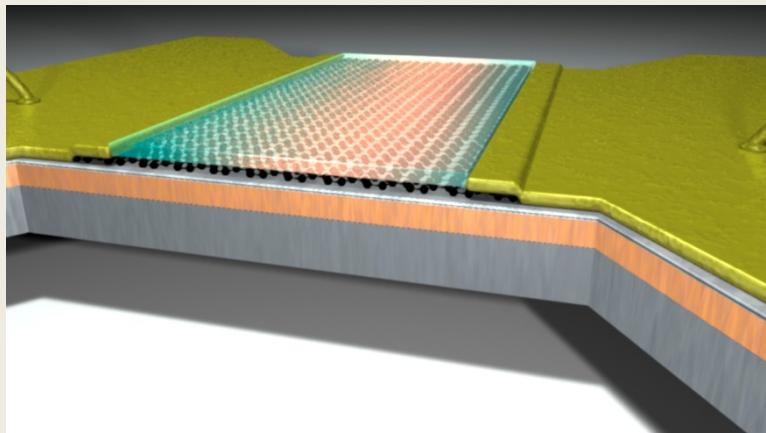
Laboratoire Pierre Aigrain – Ecole Normale Supérieure  
24 rue Lhomond, 75231 Paris Cedex 05 France  
[www.lpa.ens.fr](http://www.lpa.ens.fr)

From

2D graphene

to

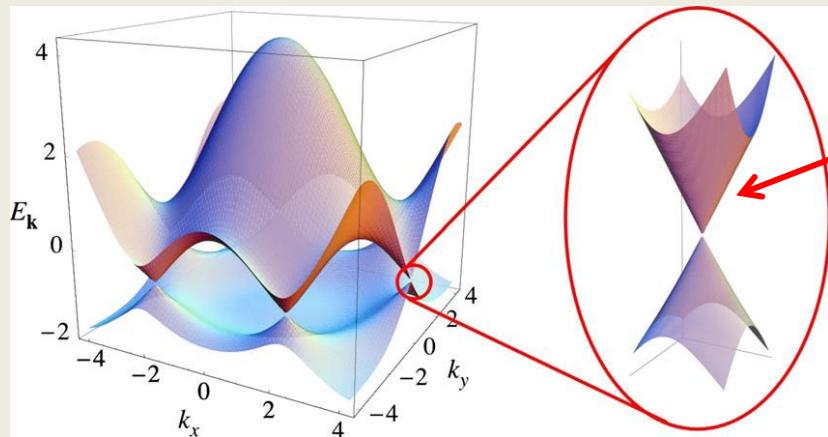
1D carbon nanotubes



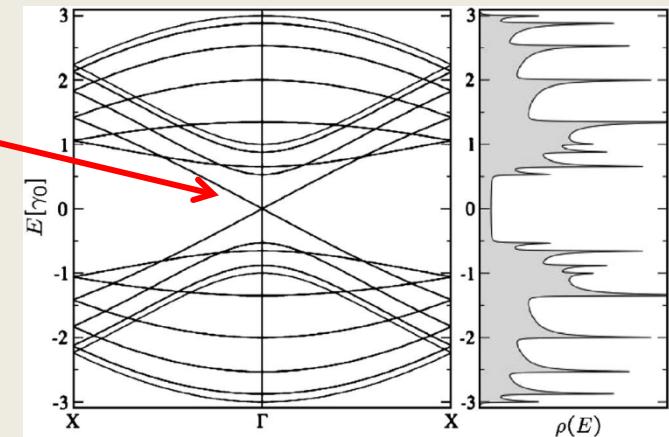
From massless chiral Dirac Fermions

to

single mode electronic “fibers”

**large Fermi velocity**

$$v_F = 10^6 \text{ m/s}$$

**Long and tunable  
wave length****Low density of  
states**

Castro-Neto et al. RMP-2009

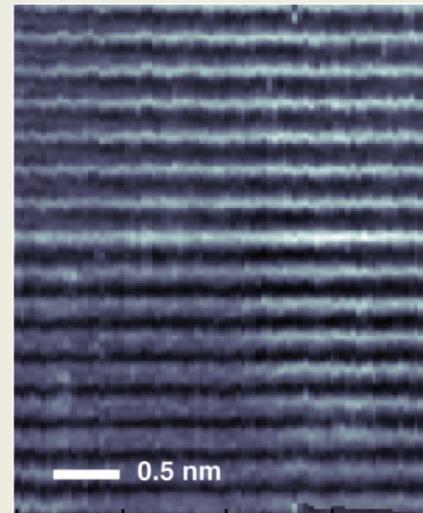
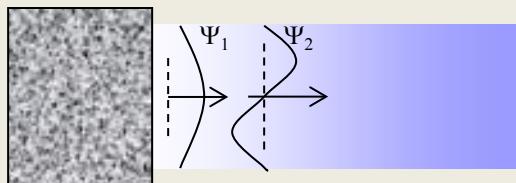
UPoN-2015, Barcelona, 15/7/2015

Charlier et al., RMP 2007

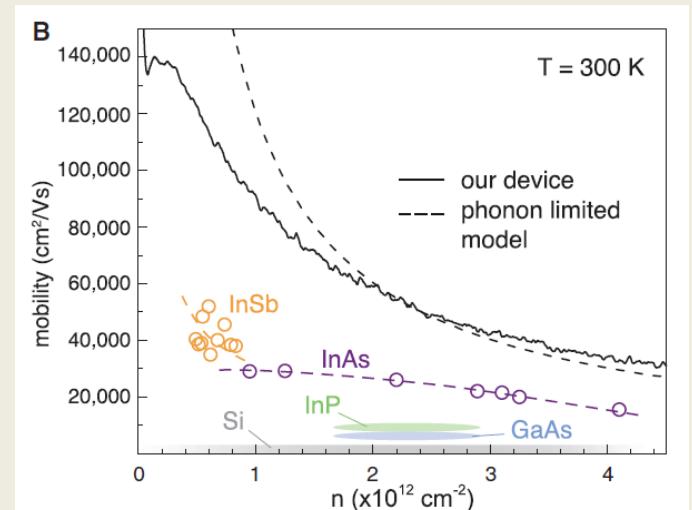
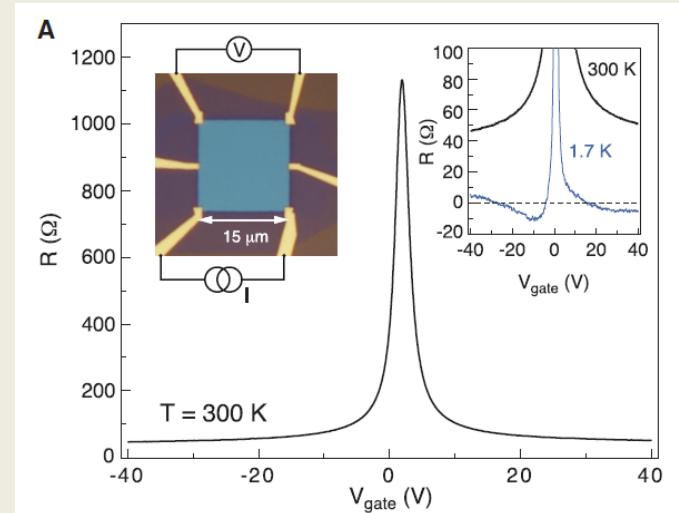
# Outline

- Graphene as tunable 2D semi-metal
  - a) Quantum shot noise in graphene (a brief review)
  - b) Noise thermometry of hot electrons : electron-phonon in 2D
  - c) Applications : HEBs, LNAs, Photo-detectors,
  
- Carbon Nanotubes as single mode nano-conductors (a review)
  - a) Quantum shot noise in carbon nanotube devices
  - b) Thermal noise in CNT wires and CNT-FETs: the noise conductance

## Landauer-Büttiker



## h-BN encapsulated graphene



$$G_L = \frac{4e^2}{h} \times \sum_1^N T_n = \frac{1}{6450 \Omega} \times \sum_1^N T_n$$

$$\frac{N}{W_{\mu m}} = \frac{k_F}{\pi} = 56 \sqrt{n / 10^{12} \text{ cm}^{-2}} = 5 - 500$$

# Quantum Shot Noise in graphene

Conductance is transmission

$$G = \frac{e^2}{h} \sum_1^N T_n$$

Quantum Shot Noise (QSN)

$$S_I = 2eI \frac{\sum T_n (1 - T_n)}{\sum T_n} = 2eI \times "Fano"$$

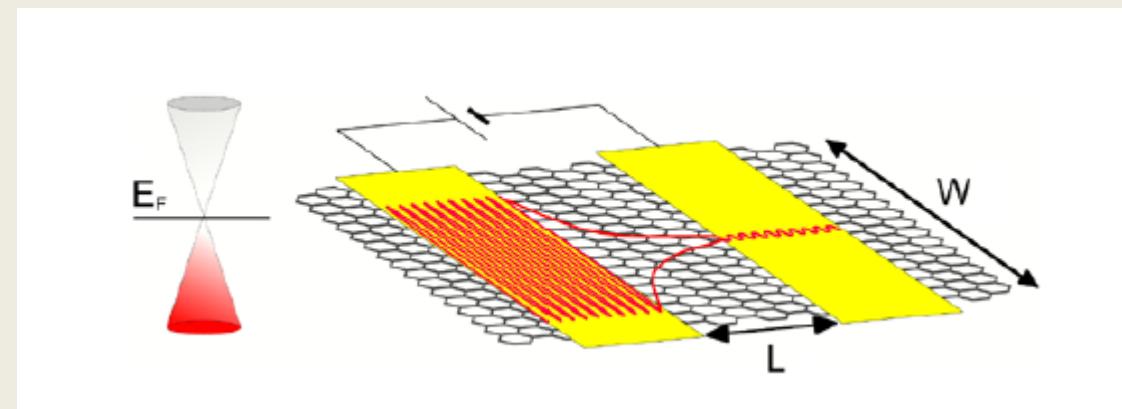
Ya.M. Blanter, M. Büttiker / Physics Reports 336 (2000) 1-166

Dirac point : electronic transmission

$$T_n^{Dirac} = \cosh^{-2} \left[ \pi(n + \alpha) \frac{L}{W} \right]$$

Noise

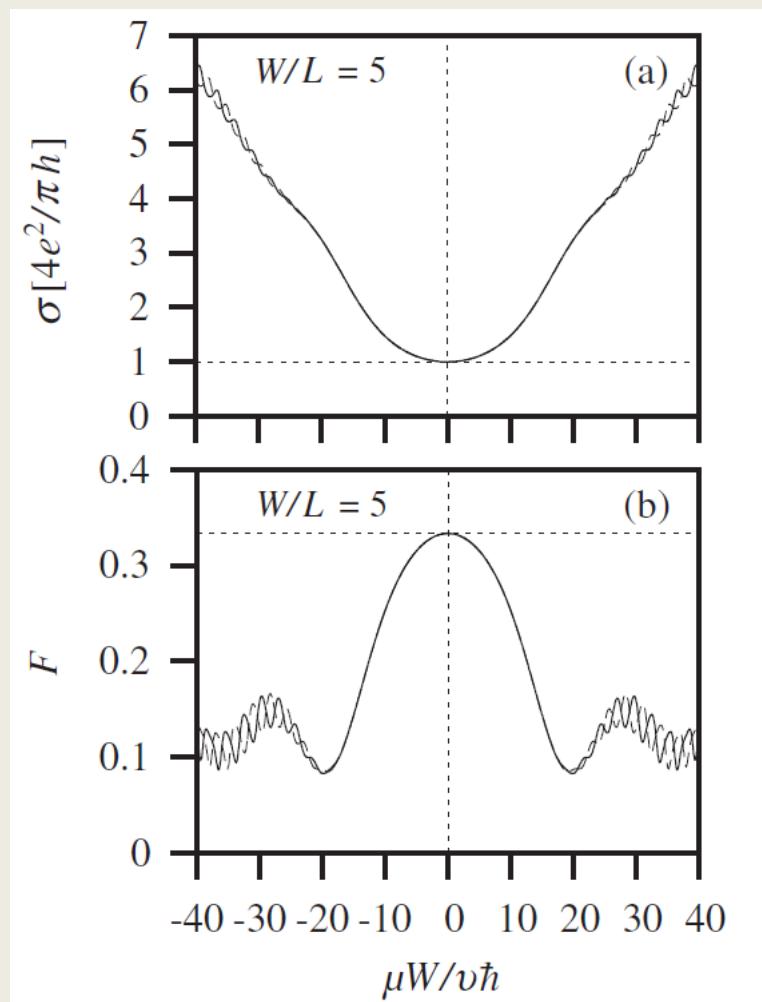
$$Fano^{Dirac} = \frac{1}{3}$$



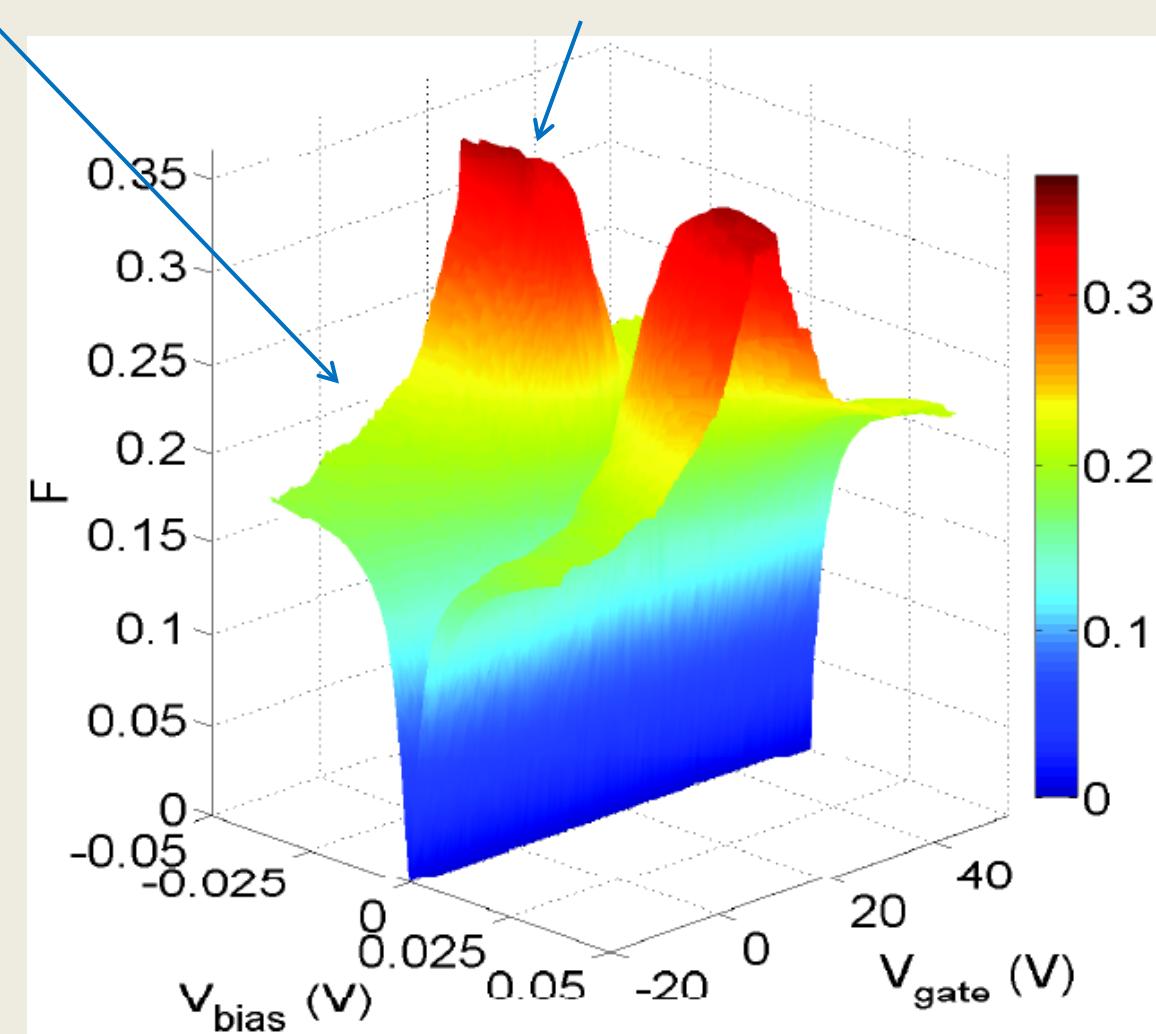
« Conductivity »

J. Tworzydlo et al. / Phys. Rev. Lett. 96 (2006) 246802

$$\sigma^{Dirac} = \sigma^{Dirac} \frac{L}{W} = \frac{4e^2}{h} \frac{L}{W} \int_0^\infty \frac{dk_y}{\cosh^2[k_y L]} = \frac{4e^2}{\pi h}$$

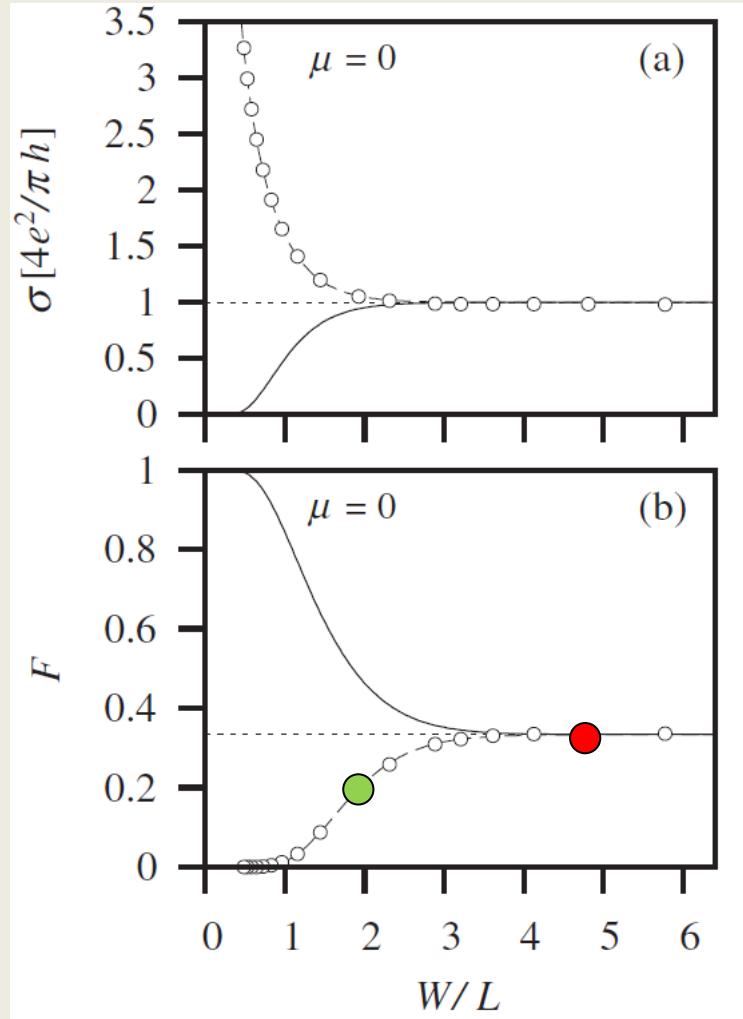
Ballistic graphene junctions ( $W=5L$ )

J. Tworzydlo et al. / Phys. Rev. Lett. 96 (2006)

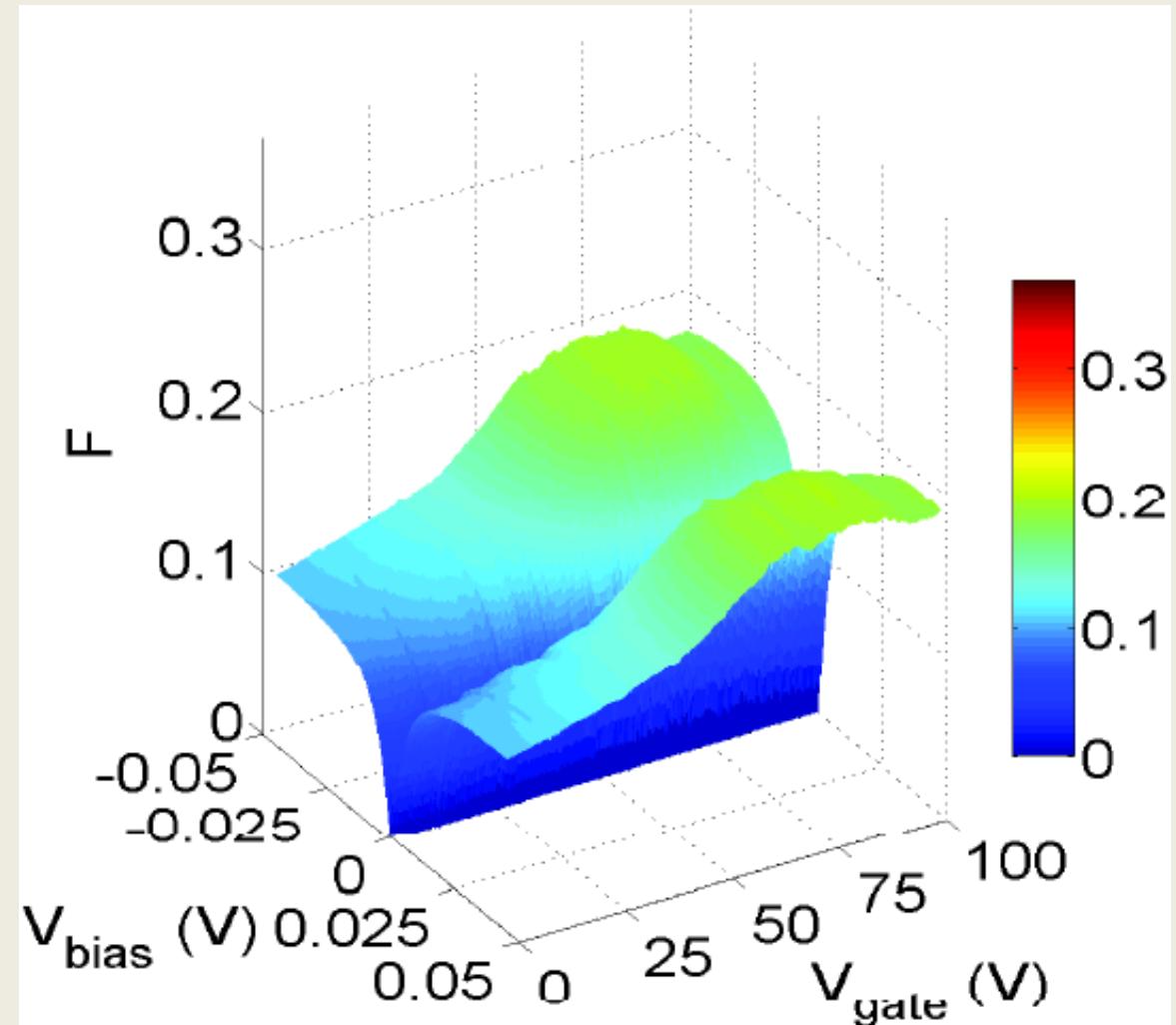
 $F=1/3$  at Dirac Point

Exp.: R. Danneau et al./ Phys. Rev. Lett. 100 (2008)

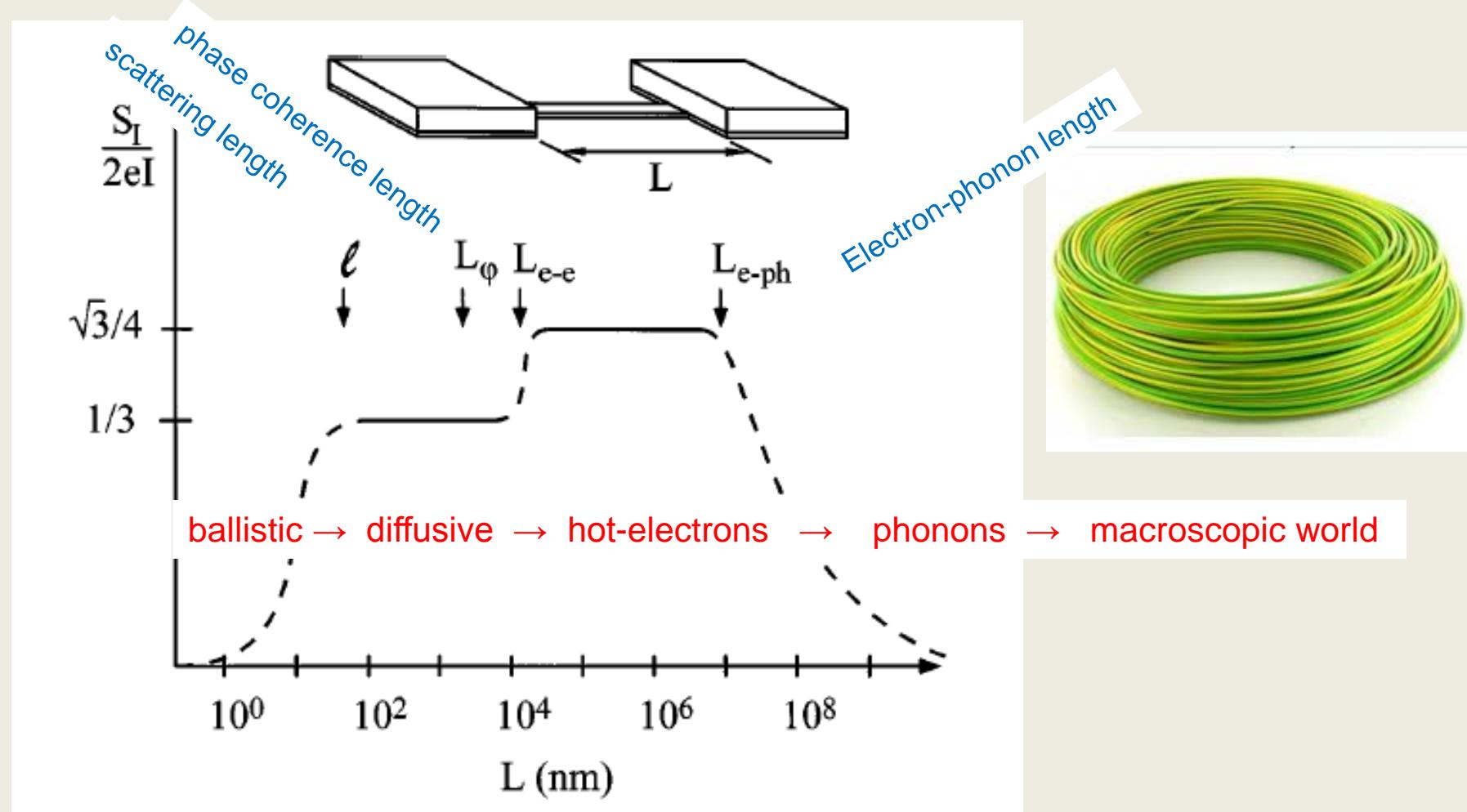
theory



experiment

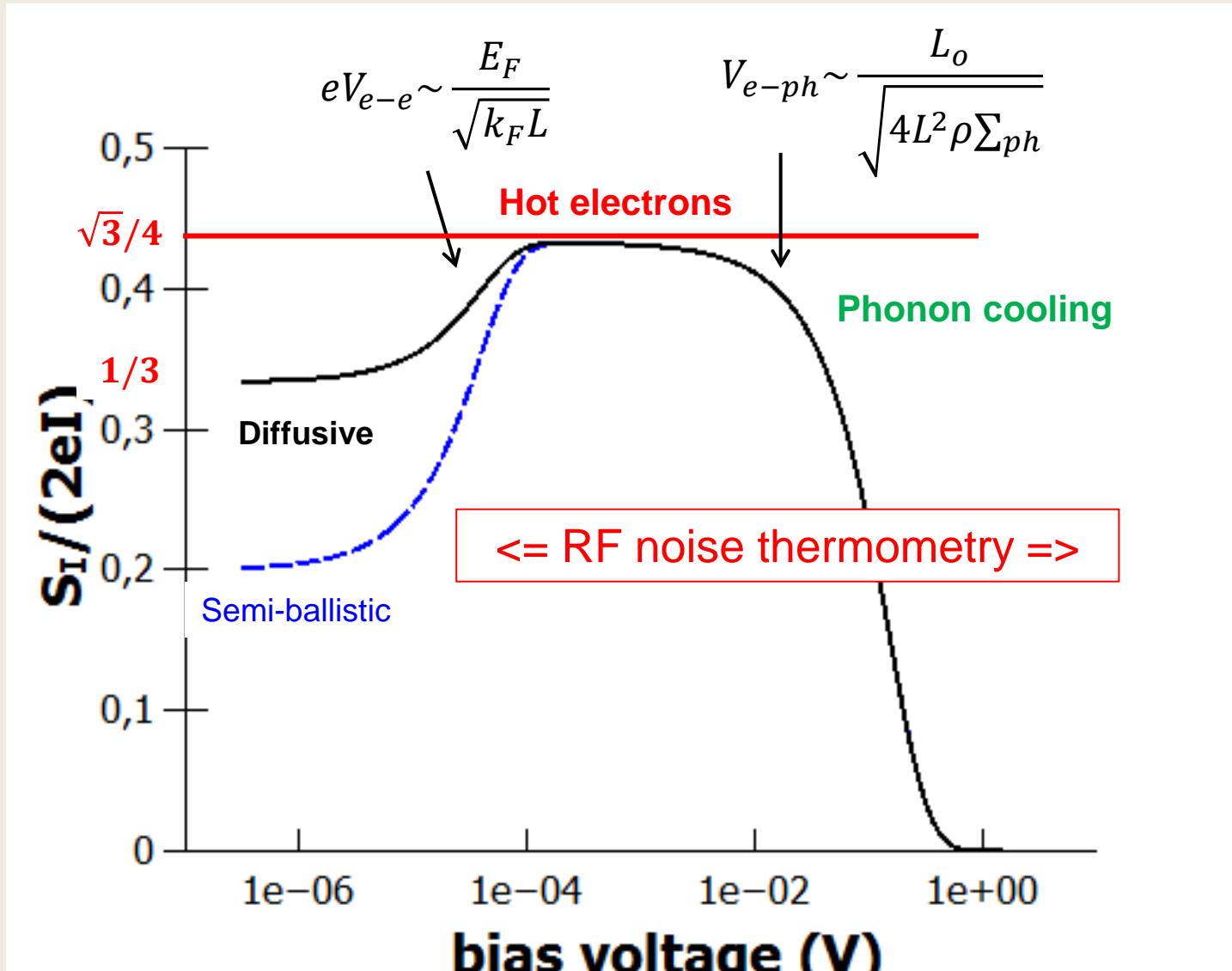


... on increasing sample length



A.H. Steinbach et al. / Phys. Rev. Lett. 76(1996) 3806

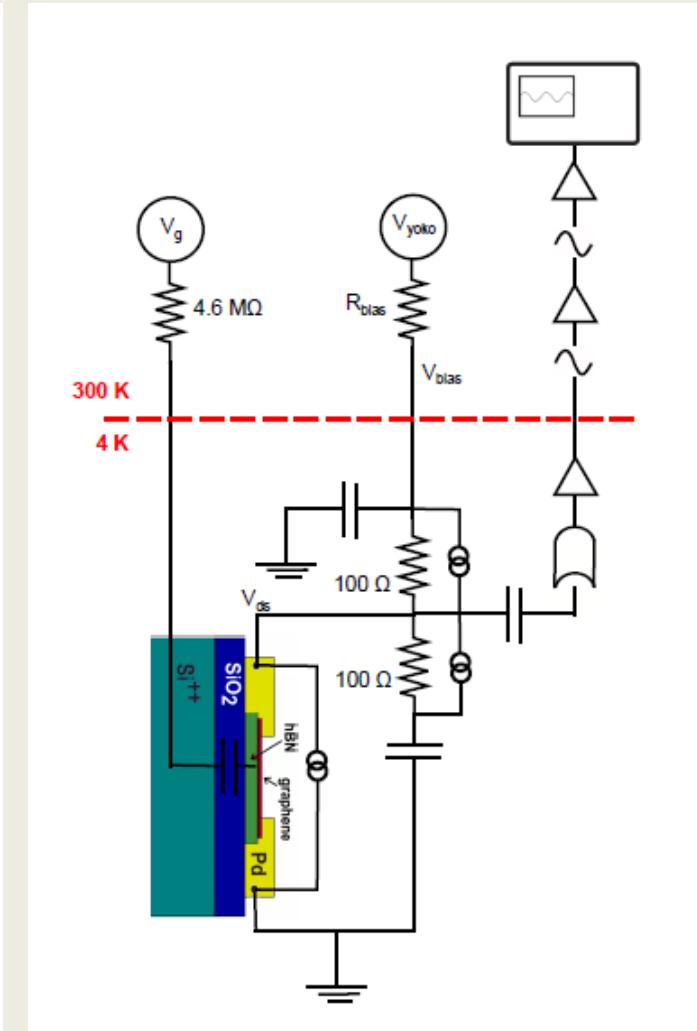
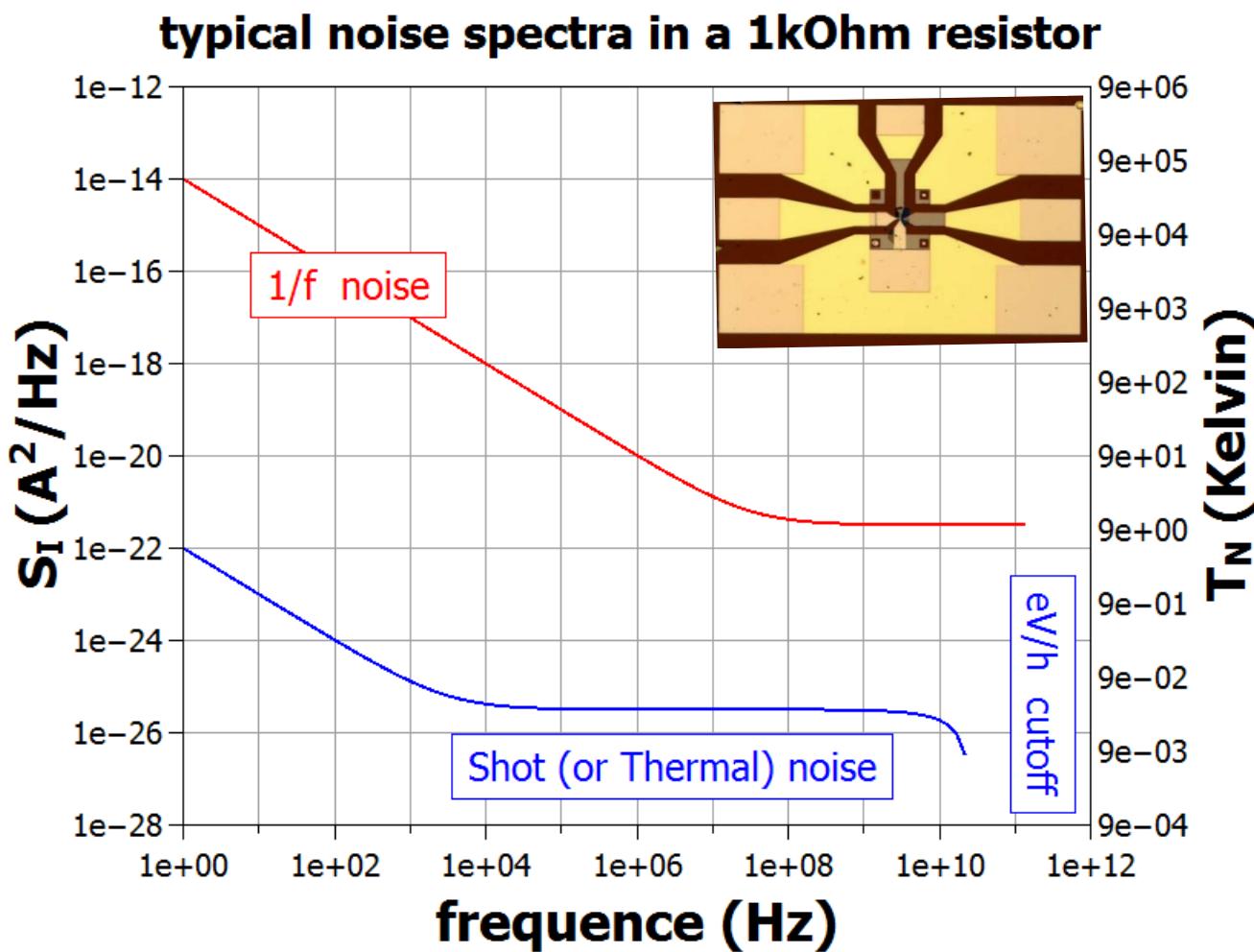
... on increasing bias voltage



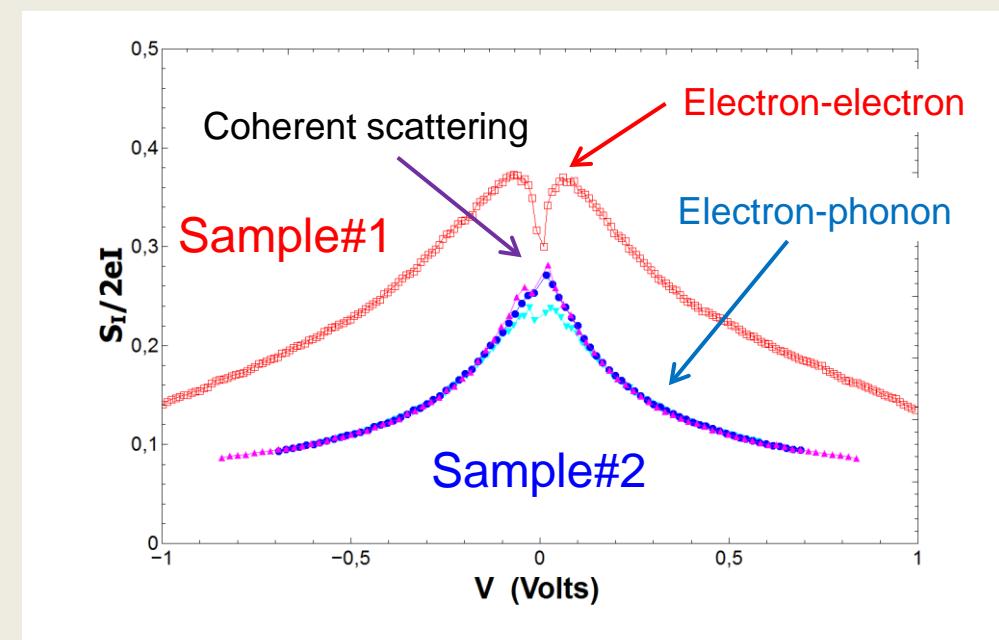
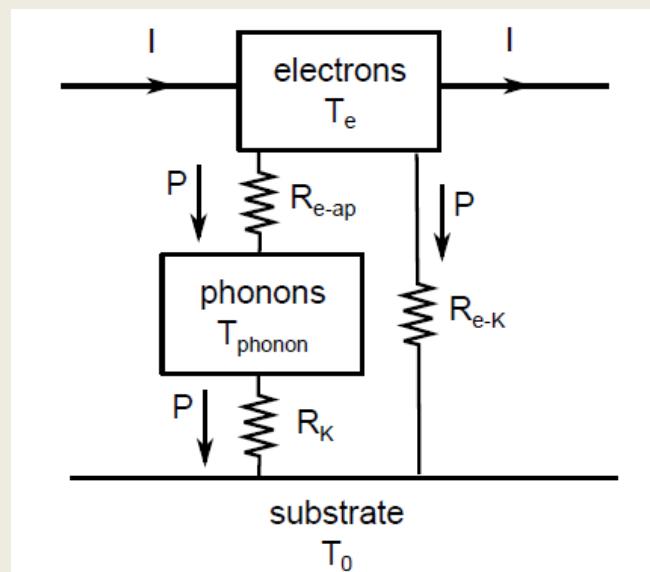
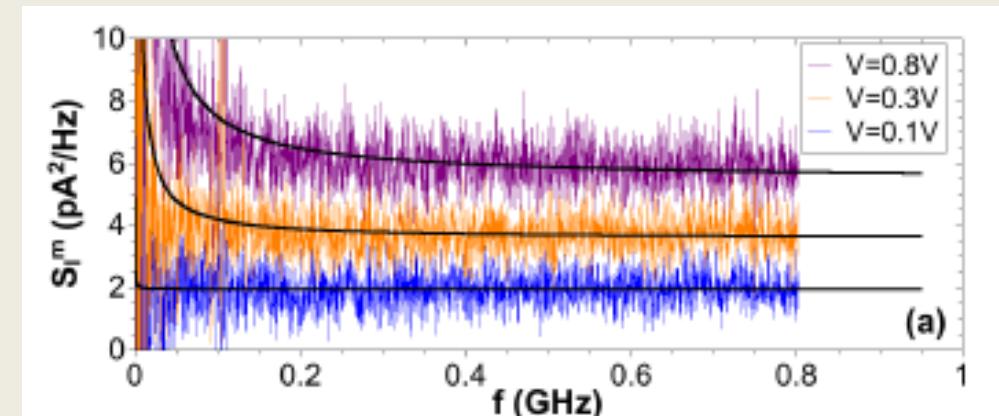
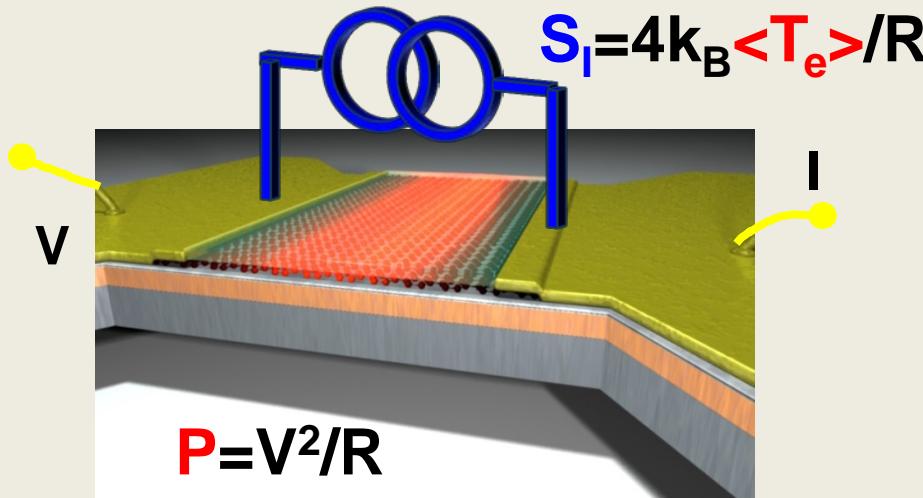
# RF current-noise measurement

current noise spectrum at high-bias

GHz-setup (LPA)

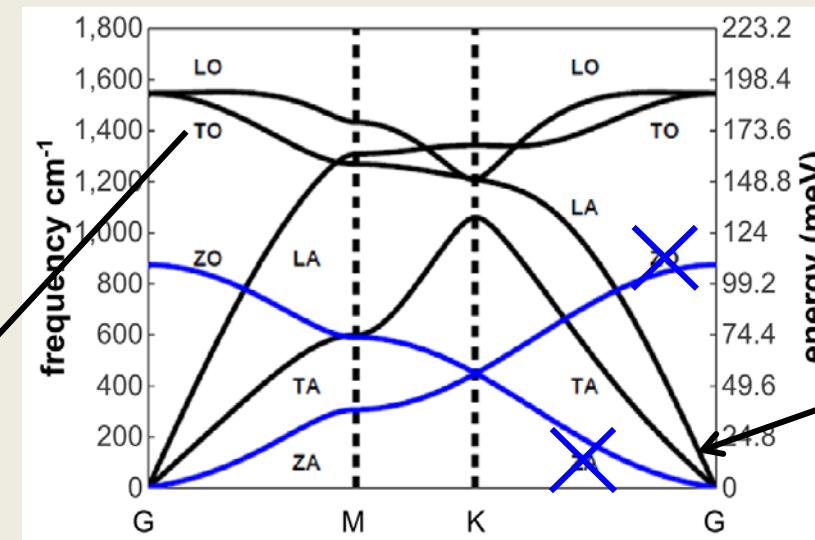


# Typical Fano-factor dependence

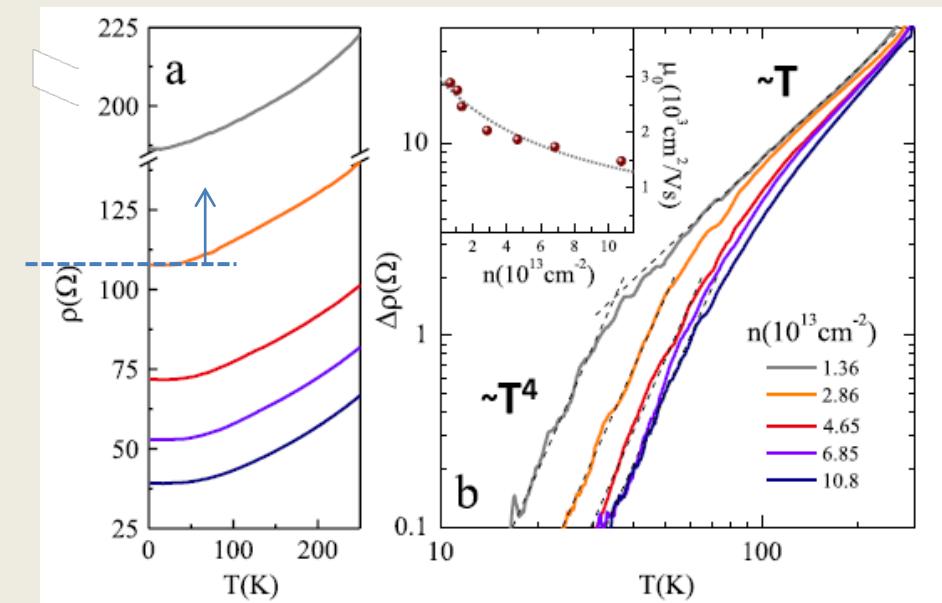
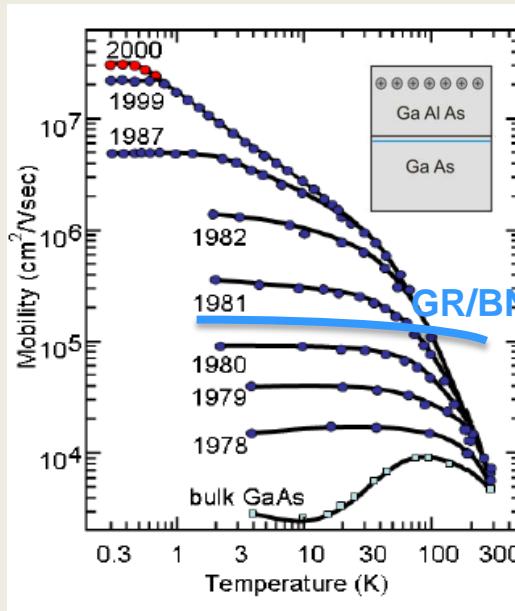


# Electron-Phonon in graphene

OP-phonons  
irrelevant



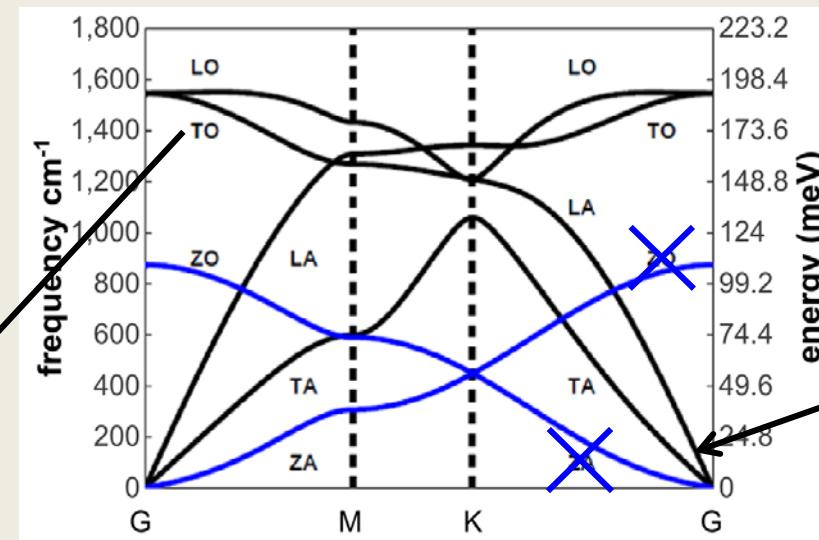
large AC-phonons  
velocity  
( $s = 2 \times 10^4 \text{ m/s}$ )



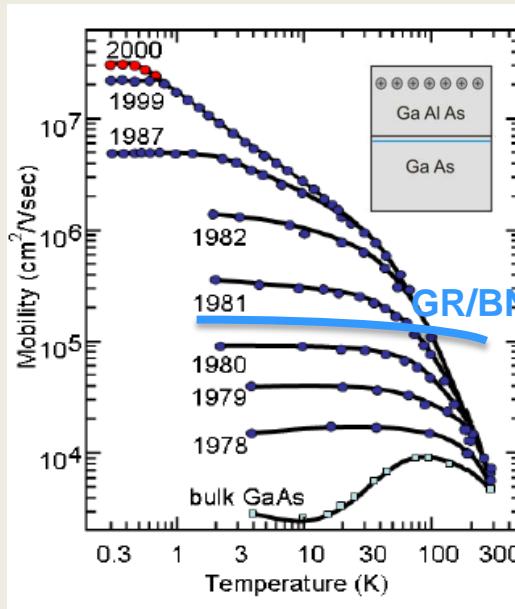
Chen-Fuhrer / Nat. Nano (2008) ; Efetov-Kim / Phys. Rev. Lett. (2010)

# Electron-Phonon in graphene

OP-phonons  
irrelevant

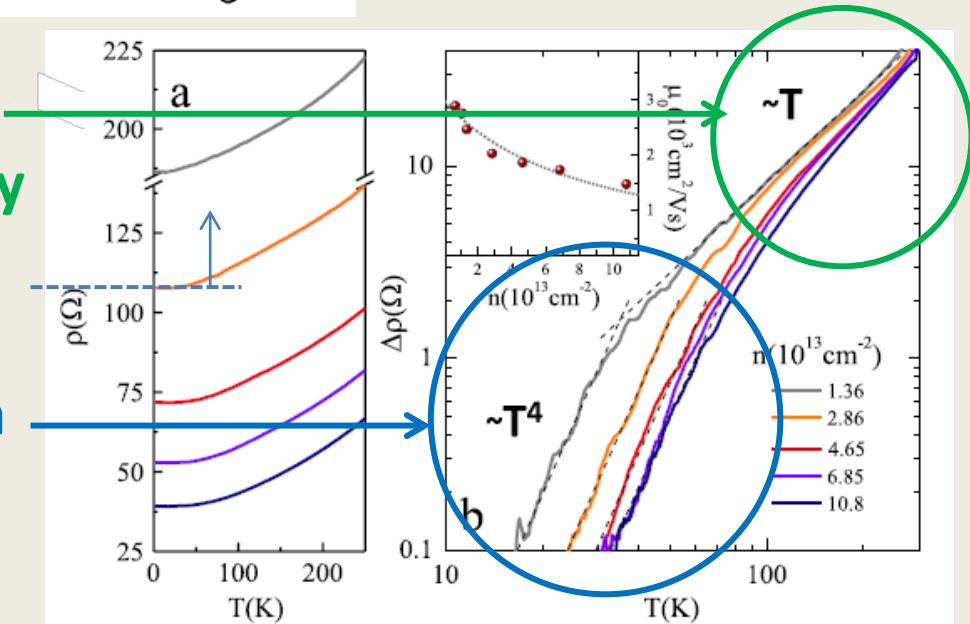


large AC-phonons  
velocity  
( $s = 2 \times 10^4 \text{ m/s}$ )



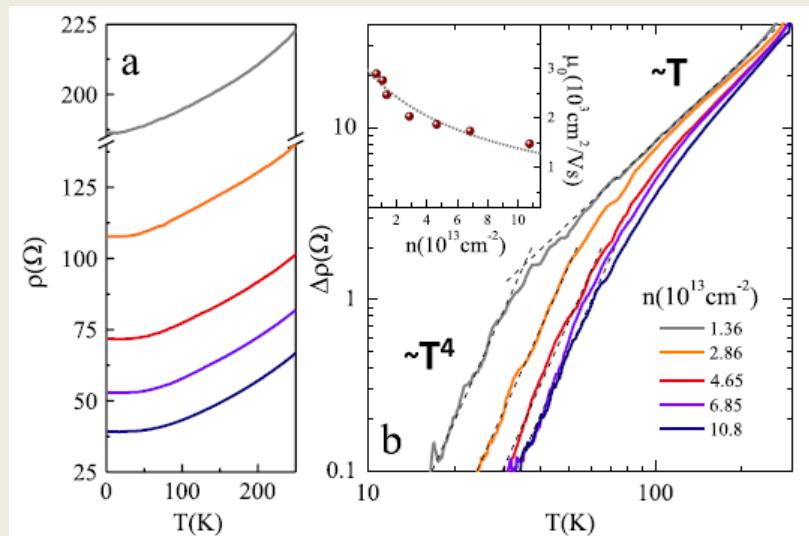
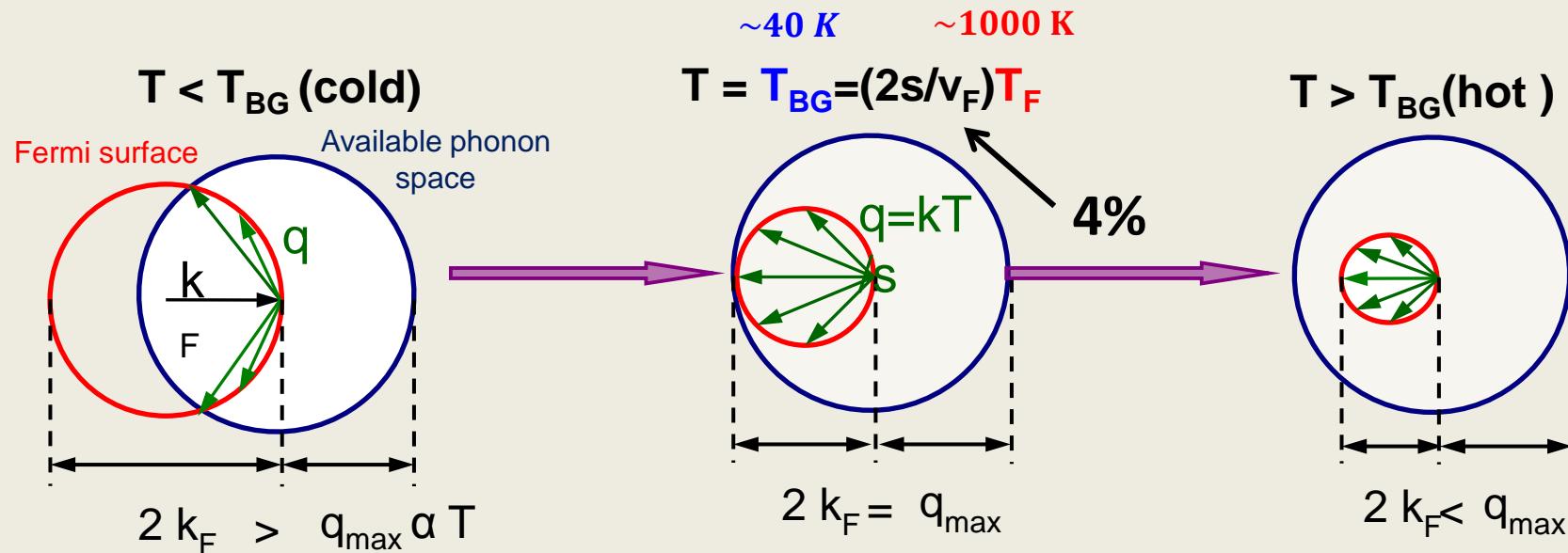
**Very weak  
Phonon resistivity**  
 $\rho \approx 0.1 \times T$

**Bloch-Grüneisen  
Temp.  $\sim 50\text{K}$**



Chen-Fuhrer / Nat. Nano (2008) ; Efetov-Kim / Phys. Rev. Lett. (2010)

# AC-phonon resistivity



$$\Delta\rho(T_{ph} \ll \theta_{BG}) = \frac{8D^2k_F}{\rho_m e^2 s v_F^2} \times f\left(\frac{T_{BG}}{T_{ph}}\right) \sim T^4$$

$\Delta\rho(T_{ph} \ll \theta_{BG}) \sim \text{const.} \times T !!! ; \text{ const.} \approx 0.1\Omega/K !!!$

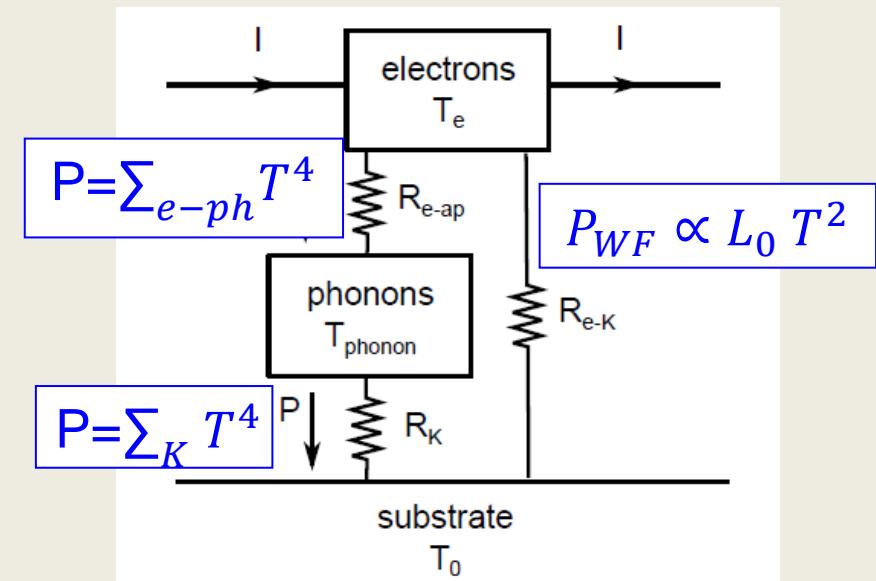
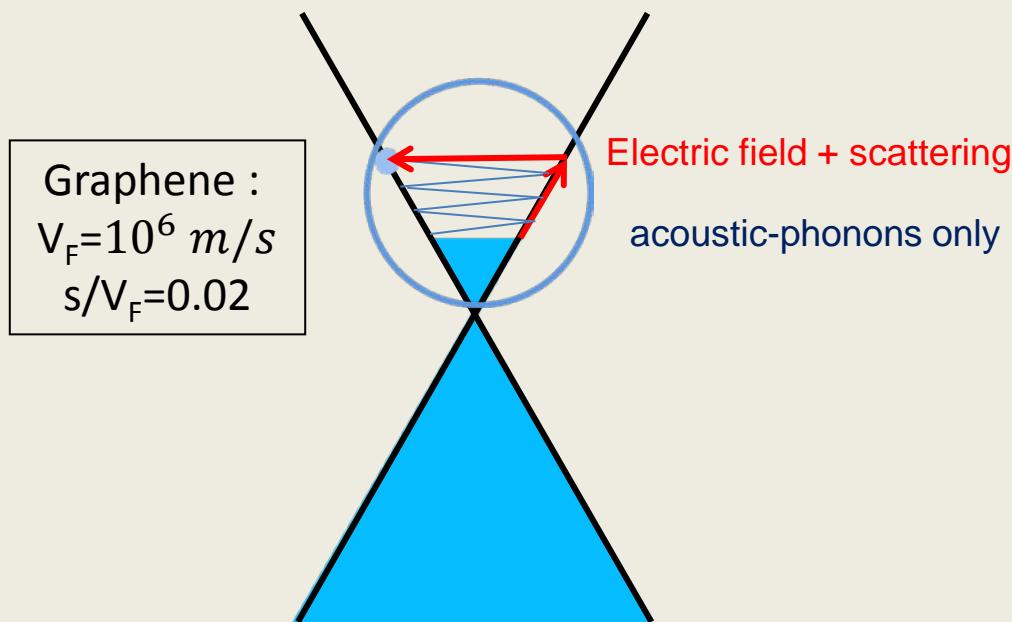
$$\mu_{ph}(300K) = 1/ne\Delta\rho \approx 2 \times 10^5/n_{12}$$

$$l_{ph}(300K) = \mu E_F/e v_F \approx 7 \mu m / \sqrt{n_{12}}$$

Chen-Fuhrer / Nat. Nano (2008) ; Efetov-Kim / Phys. Rev. Lett. (2010)

# AC-phonon relaxation : $T \leq T_{BG}$

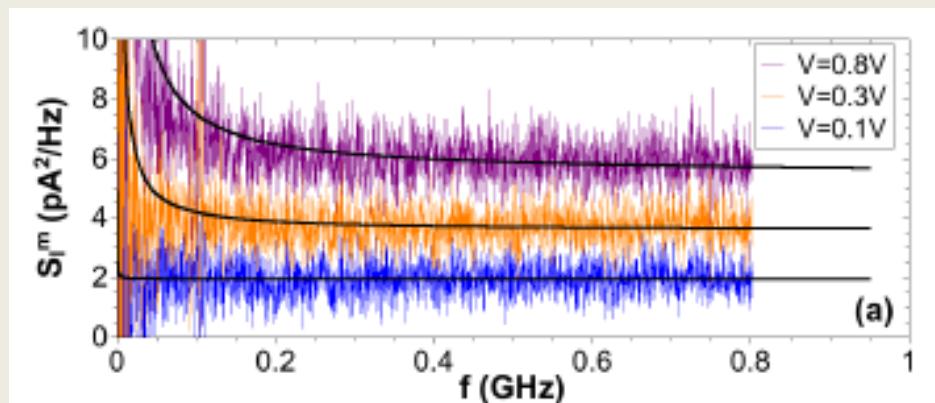
## Joule heating + phonon cooling



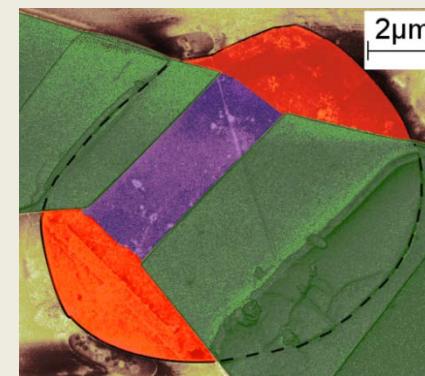
$$\sum_{e-ph} = \frac{\pi^2 D^2 k_B^4 \mu_F}{15 \rho_m \hbar^5 s^2 v_F^5} \leq 10 \text{ mW/m}^2 \text{ K}^4 \ll P_{Kapitza} \approx 10 \text{ W/m}^2 \text{ K}^4$$

**Very weak AC-phonon coupling => very hot electrons**

Thermal + 1/f noise

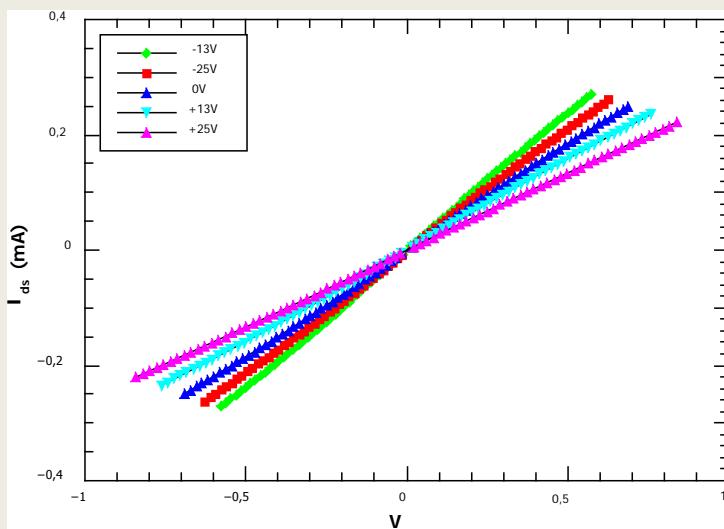


diffusive G/hBN sample

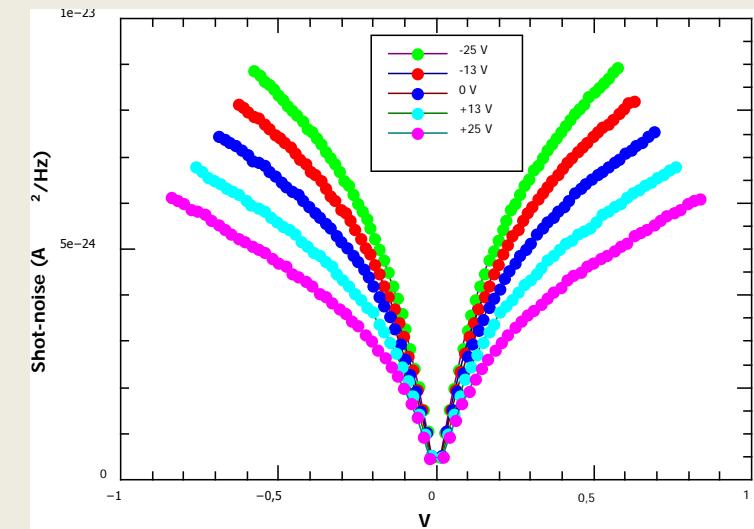


very-BN™  
hBN powder  
by St Gobain

linear I-V's (diffusive)



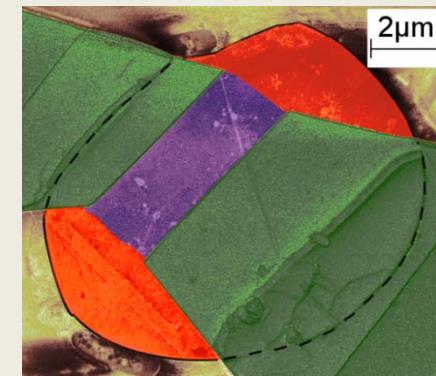
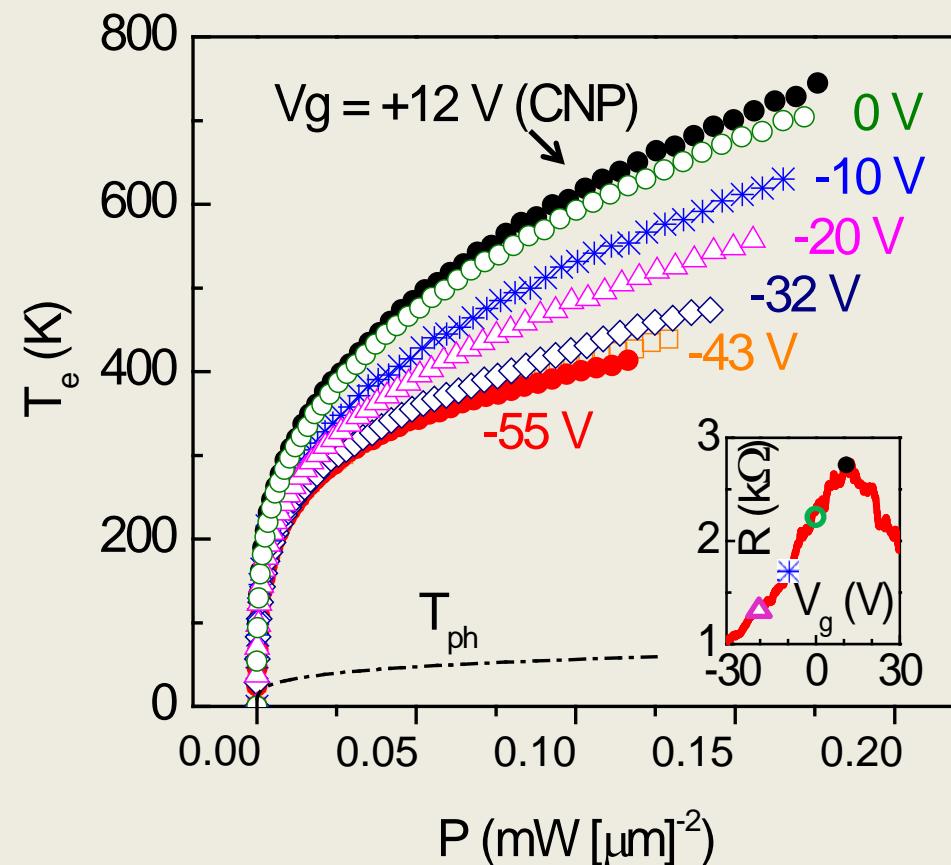
noise: from linear to sublinear



A. Betz et al. / Phys. Rev. Lett. 109 (2012) 056805

UPoN-2015, Barcelona, 15/7/2015

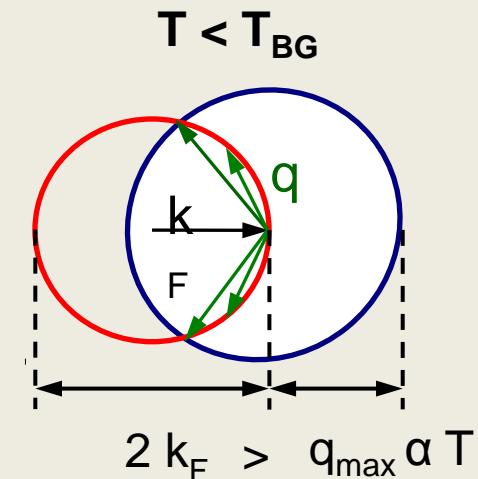
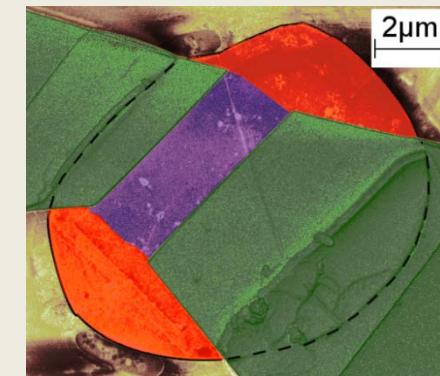
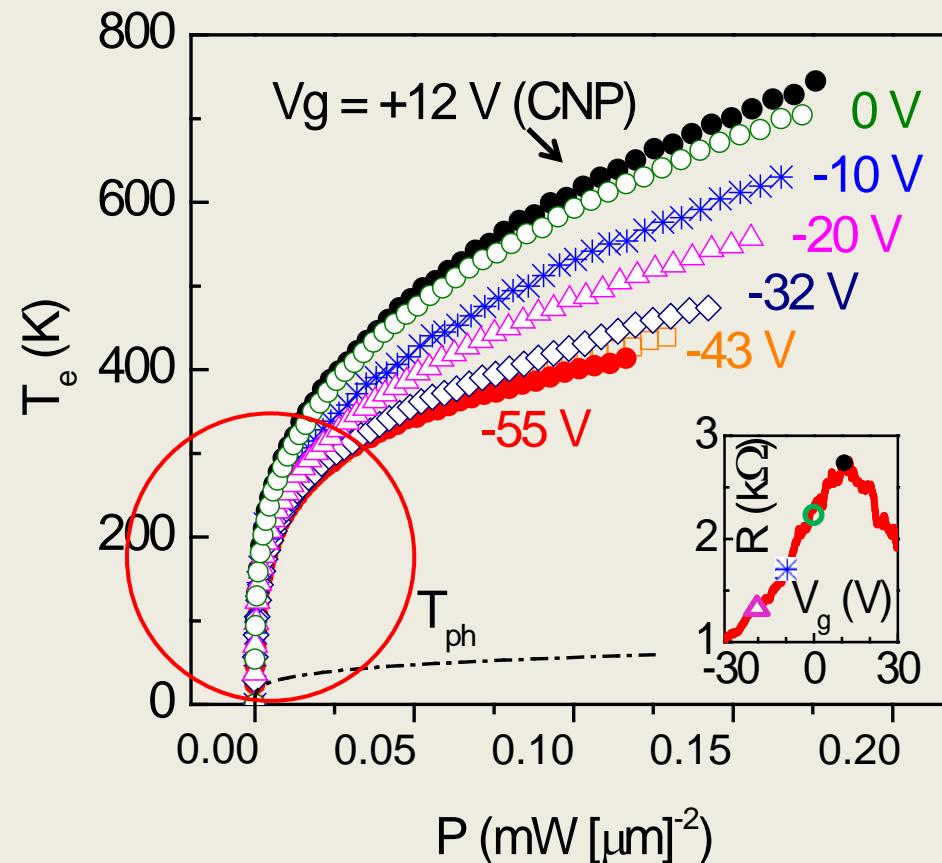
$$\langle T_e \rangle \equiv R S_I / 4k_B \quad \text{with} \quad P = V \times I$$



A. Betz et al. / Phys. Rev. Lett. 109 (2012) 056805

A. Betz et al. / Nat. Phys. 9 (2012) 109

$$\langle T_e \rangle \equiv R S_I / 4k_B \quad \text{with} \quad P = V \times I$$



A. Betz et al. / Phys. Rev. Lett. 109 (2012) 056805

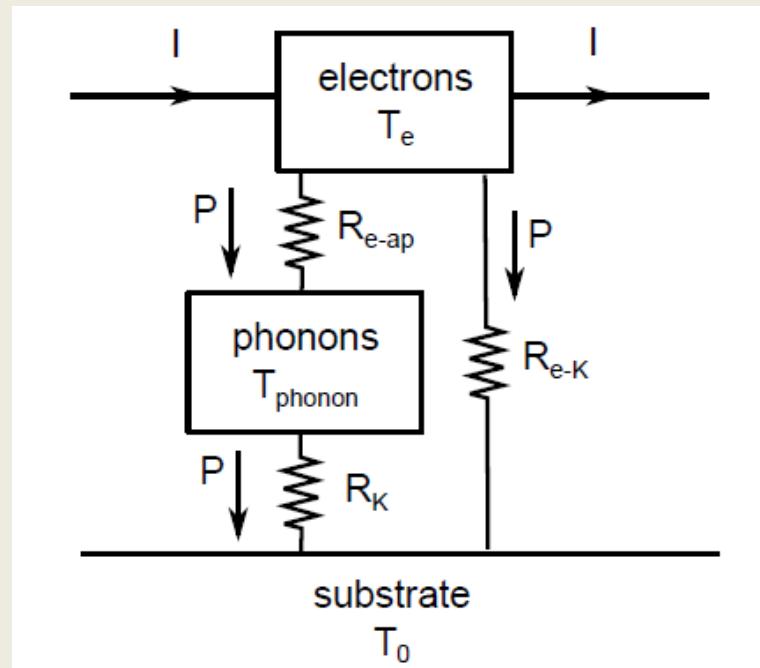
A. Betz et al. / Nat. Phys. 9 (2012) 109

# Bloch-Gruneisen regime (large doping)

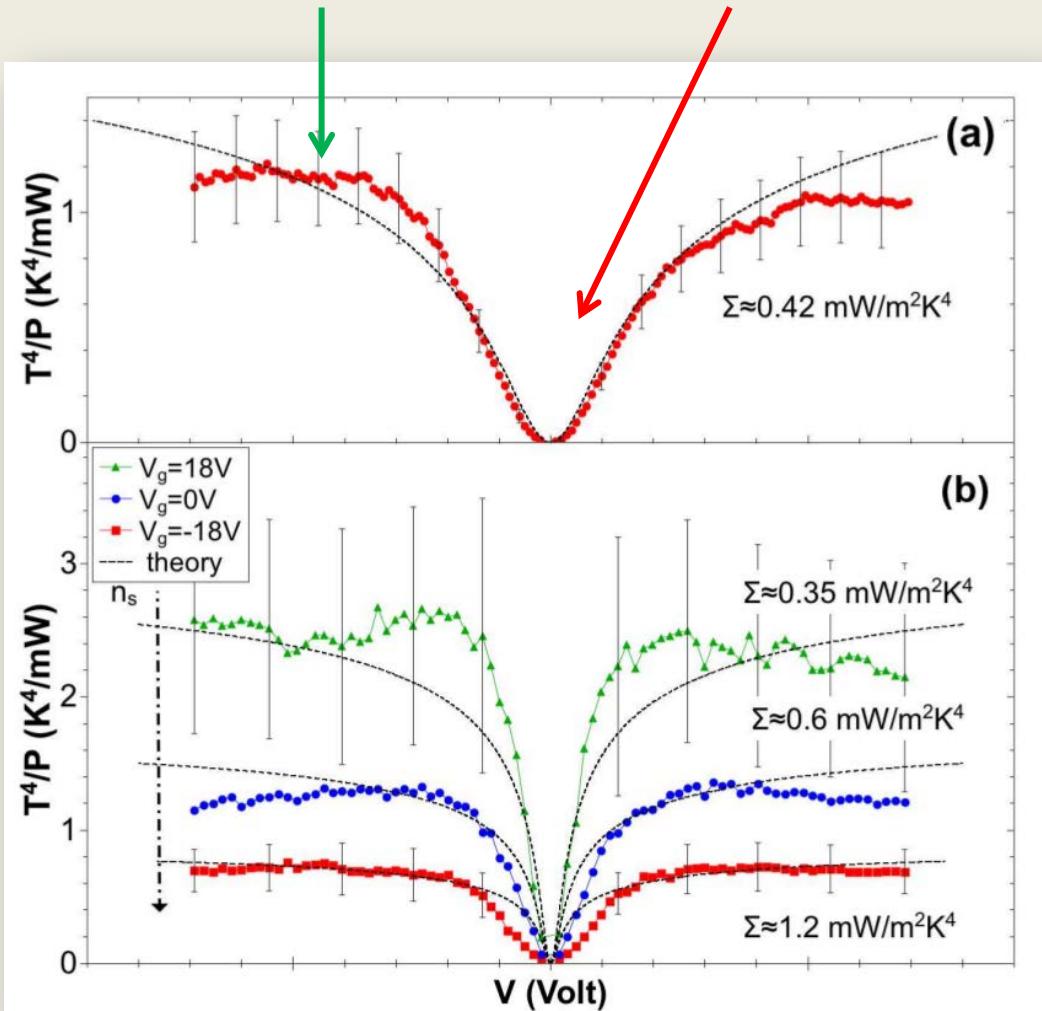
Heat equation (steady state)

$$\frac{V^2}{R} = LW \sum_{e-ph} T_e^4 - \frac{L_o}{2R} \frac{L^2 \partial^2 T_e^2}{\partial x^2}$$

$$L_o = \pi^2 k_B^2 / 3e^2 \text{ (Lorenz number)}$$



**T<sup>4</sup>-BG - hot electrons**



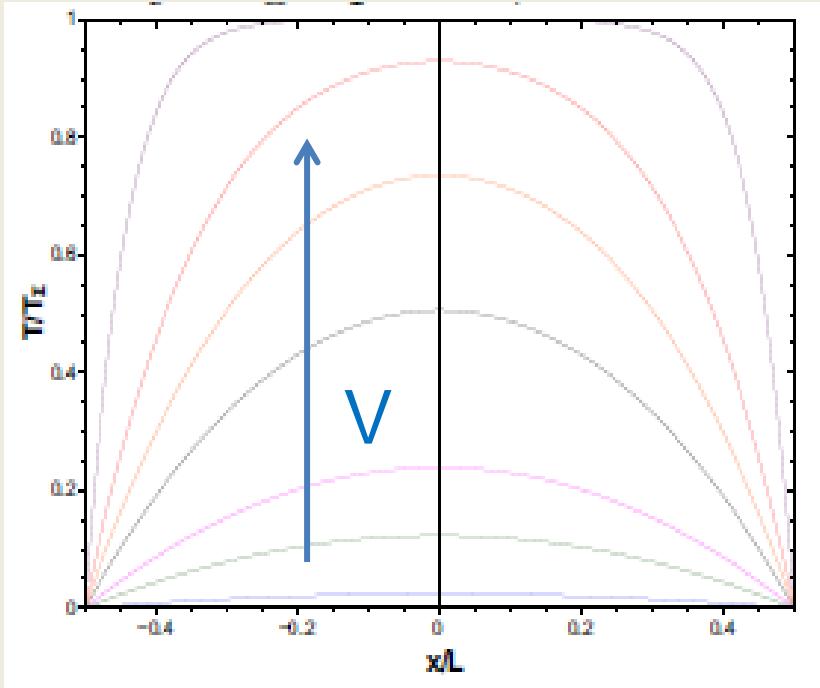
A. Betz et al. / Phys. Rev. Lett. 109 (2012) 056805

# Bloch-Gruneisen regime (large doping)

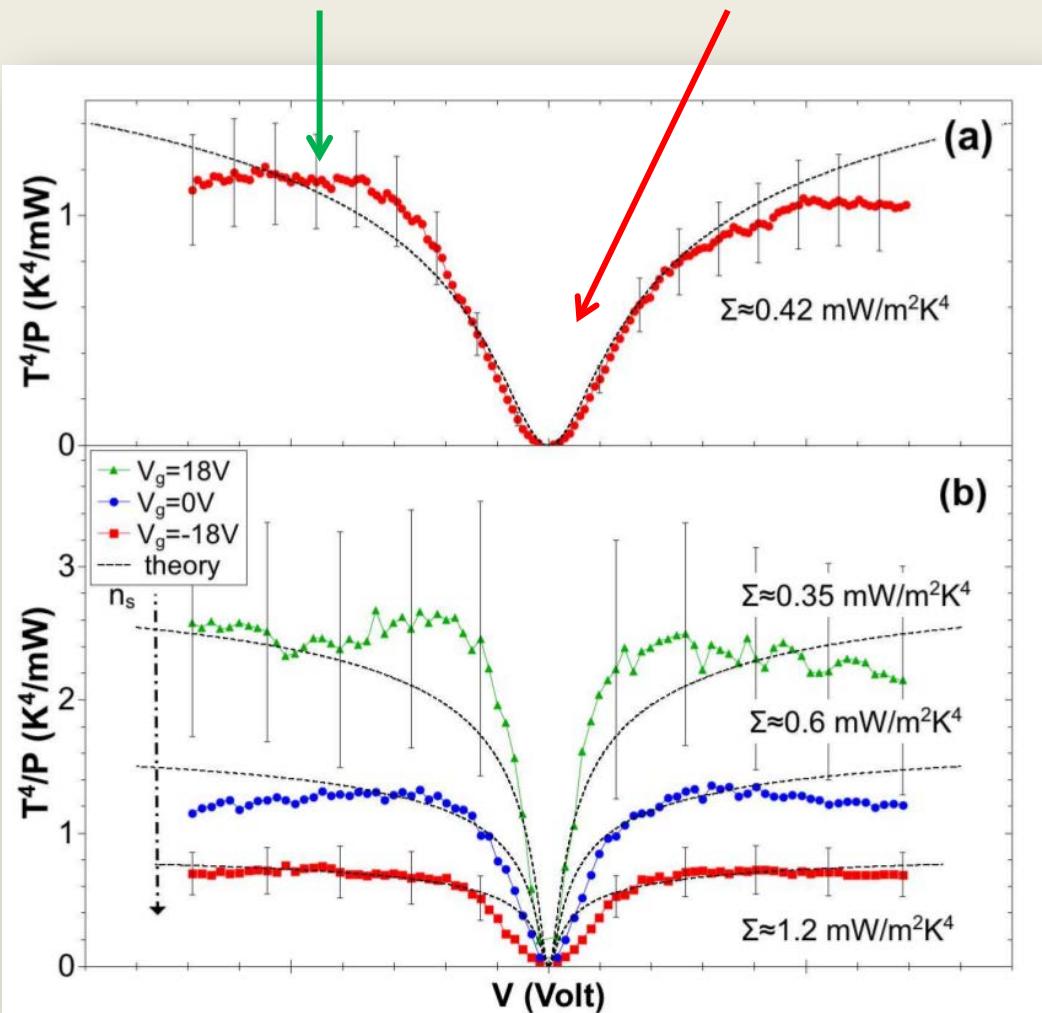
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$$L_o = \pi^2 k_B^2 / 3e^2 \text{ (Lorenz number)}$$

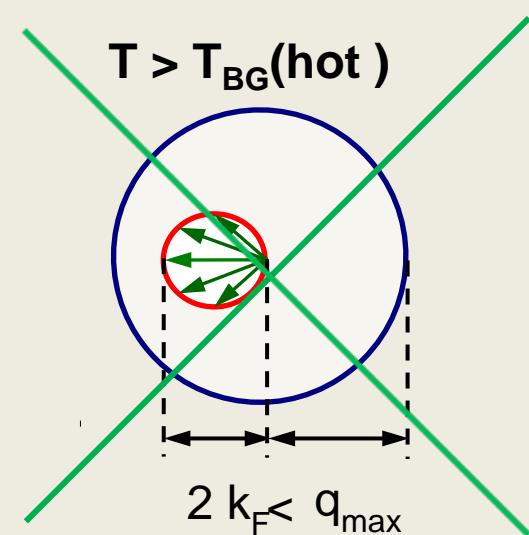
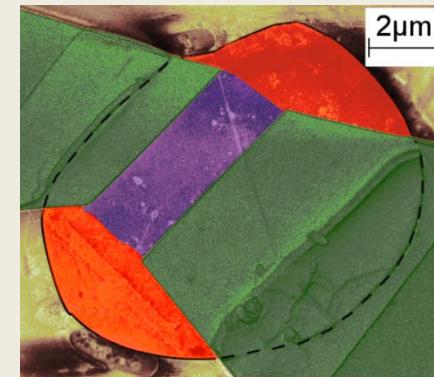
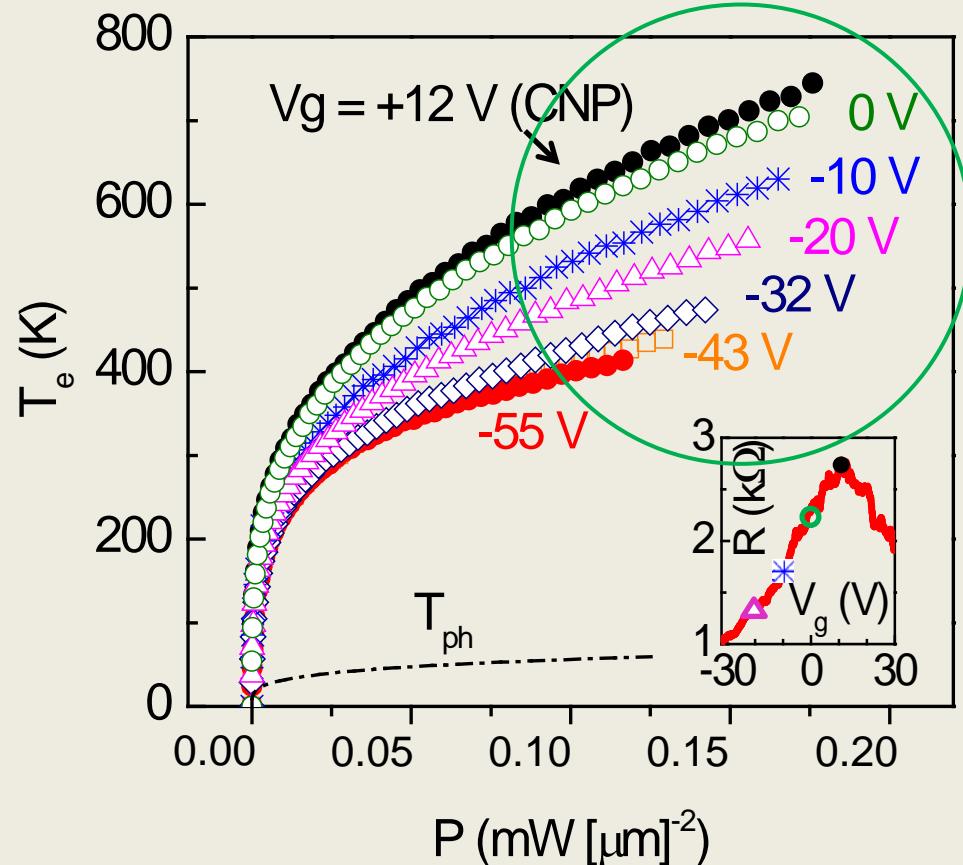


**T<sup>4</sup>-BG - hot electrons**



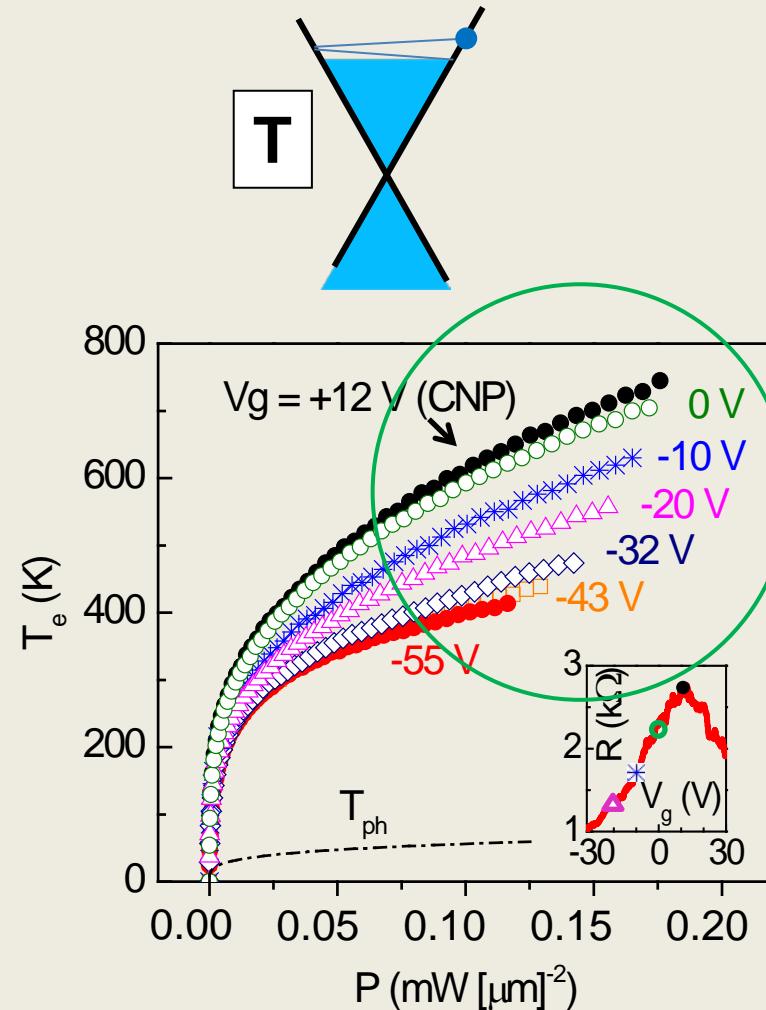
A. Betz et al. / Phys. Rev. Lett. 109 (2012) 056805

## Hot-phonon regime (low doping)

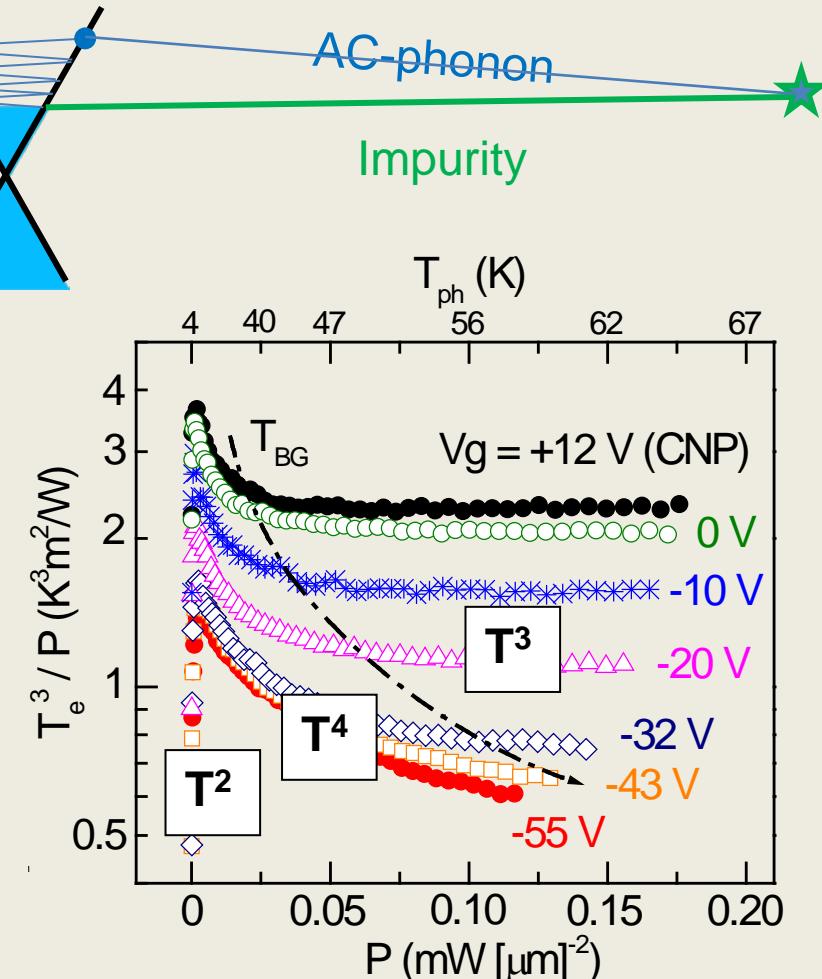


# The « supercollision » regime

Ordinary electron-phonon collision



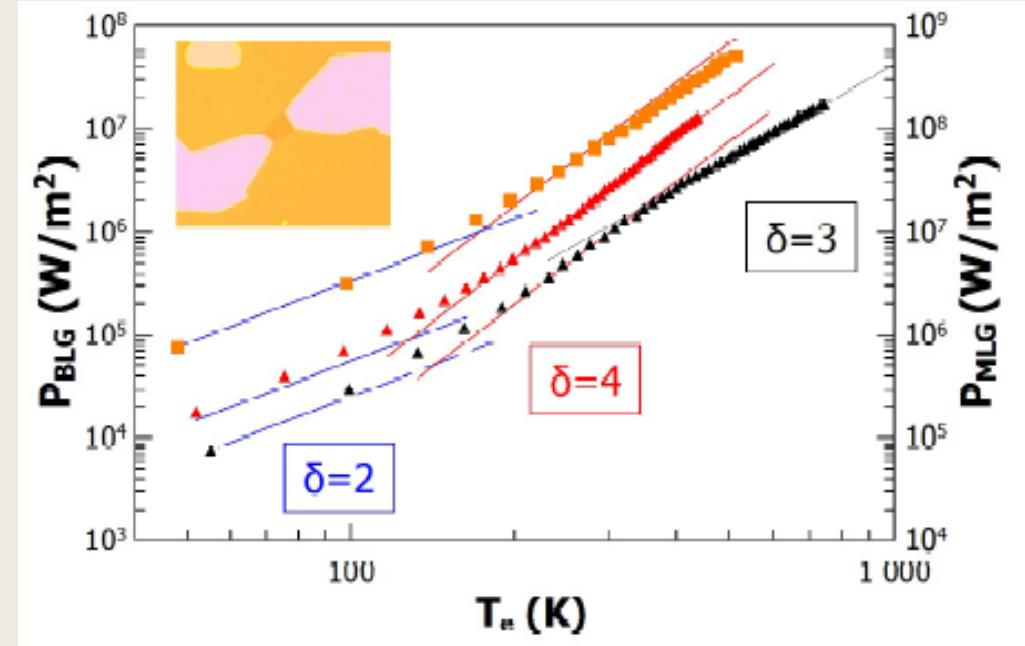
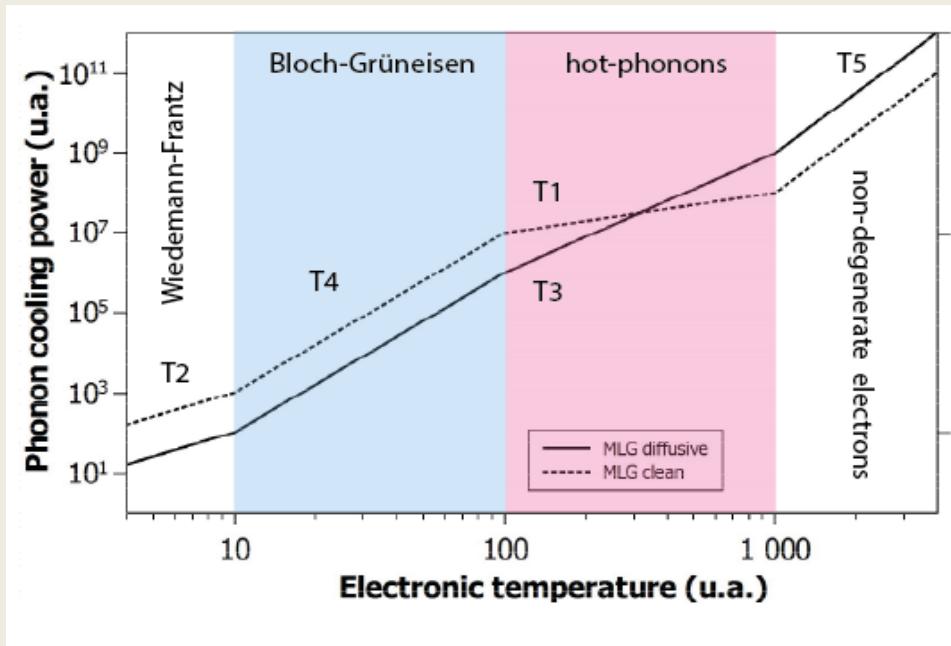
3-body electron-phonon impurity



Exp. : Betz et al. / Nat. Phys. 9 (2012)

Th. : Song-Levitov / PRL (2013)

# Acoustic-phonon cooling (summary)

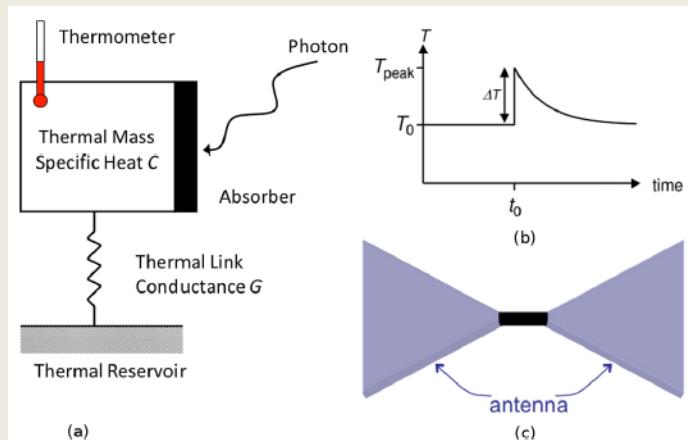


C. Voisin and B. Plaçais / special issue “hot carriers in graphene”, J. Phys.: Cond. Matter 27 (April 2015)

- $T^4$  A. Betz et al., Phys. Rev. Lett. 109 (2012) 056805
- $T^4$  K.C. Fong and K.C. Schwab, PRX 2, (2012) 031006
- $T^3$  J.C.W. Song et al., Phys. Rev. Lett. 109 (2012) 10660
- $T^3$  A. Betz et al., Nat. Phys. 9 (2012) 109
- $T^3$  M.W. Graham et al., Nat. Phys. 9 (2012) 109; Nano Letters 13, (2012) 5497
- $T^3$  M.W. Graham et al., Nat. Phys. 9 (2012) 109; Nano Letters 13, (2012) 5497
- $T^3$  .../...
- $T^3, T^5$  A. Laitinen et al. / Nano Lett.. 14 (2012) 3009.

# Acoustic-phonon cooling (applications)

Hot electron Bolometers for single photon detection :  
tiny electronic heat capacity + weak electron-phonon relaxation



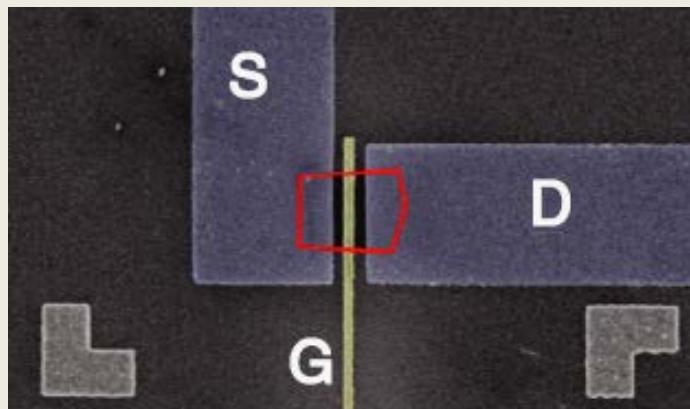
$$\varepsilon\delta(t) = LW \sum T_e^4 - \frac{L_o L^2}{2R} \frac{\partial^2 T_e^2}{\partial x^2} + \gamma LW \frac{\partial T_e^2}{\partial t}$$

e.g. : Yale group :

- McKitterick et al., JAP 113, 044512 (2013)
- McKitterick et al., JLTP 176, 291 (2014)
- B. Karasik et al., JLTP 176, 249 (2014)
- McKitterick et al., JPCM 27 164203 (2015)
- E. Pallecchi et al., JPAP 47, 094004 (2014)

$$\begin{aligned} P &= \sum_{e-ph} T^4 \\ &= 100 \frac{aW}{\mu m^2} (T < 1K) \\ &= 1 \text{photon}/100ps \end{aligned}$$

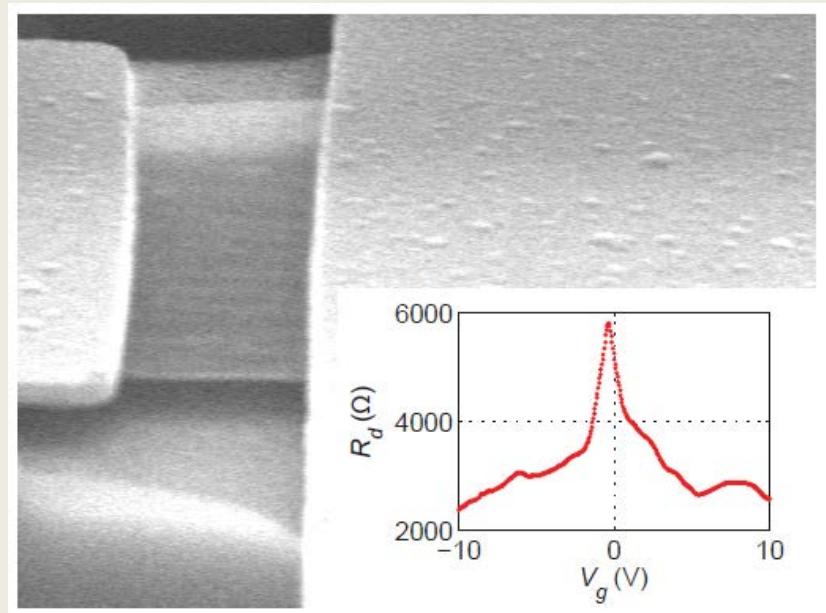
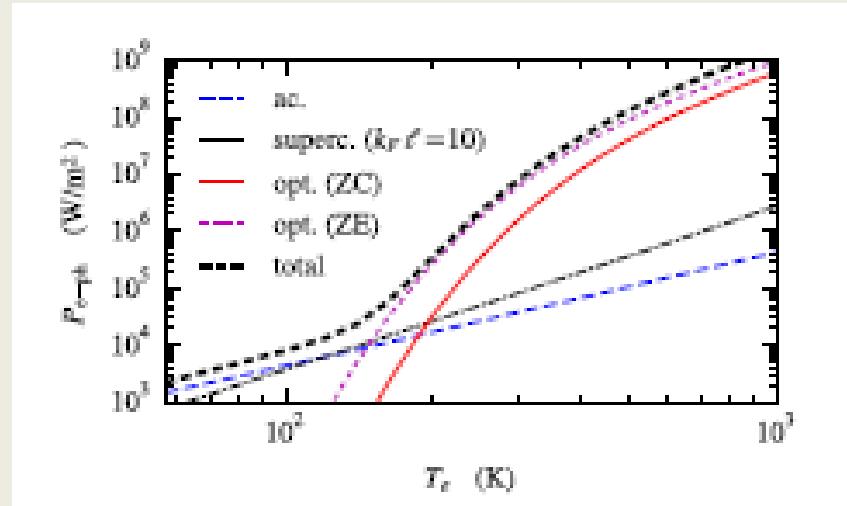
Electronics : Hot-electrons limit the resolution of RF charge detectors



e.g. : LPA graphene group :

- E. Pallecchi et al., JPAP 47, 094004 (2014)

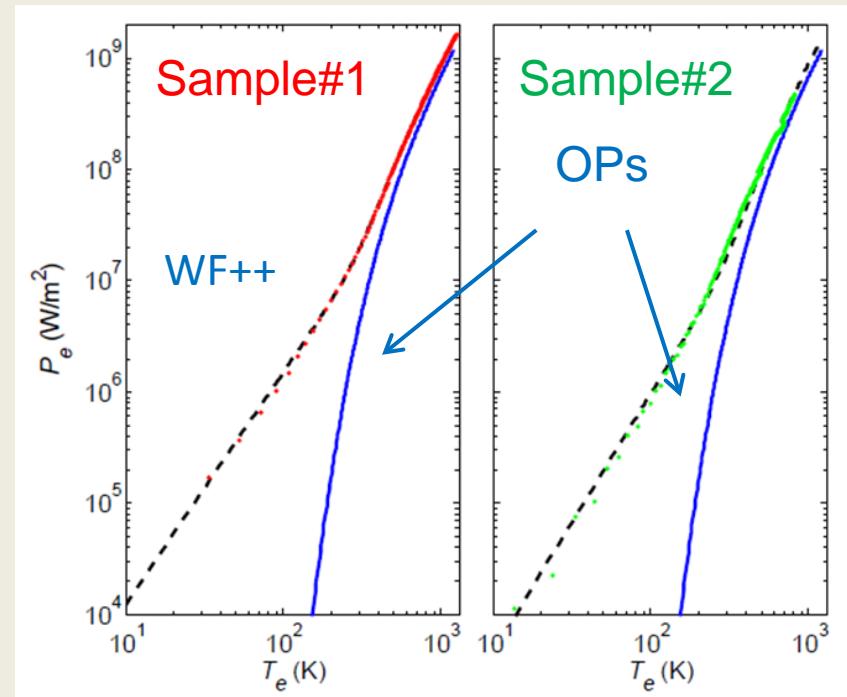
# Hot electrons reveal optical-phonons



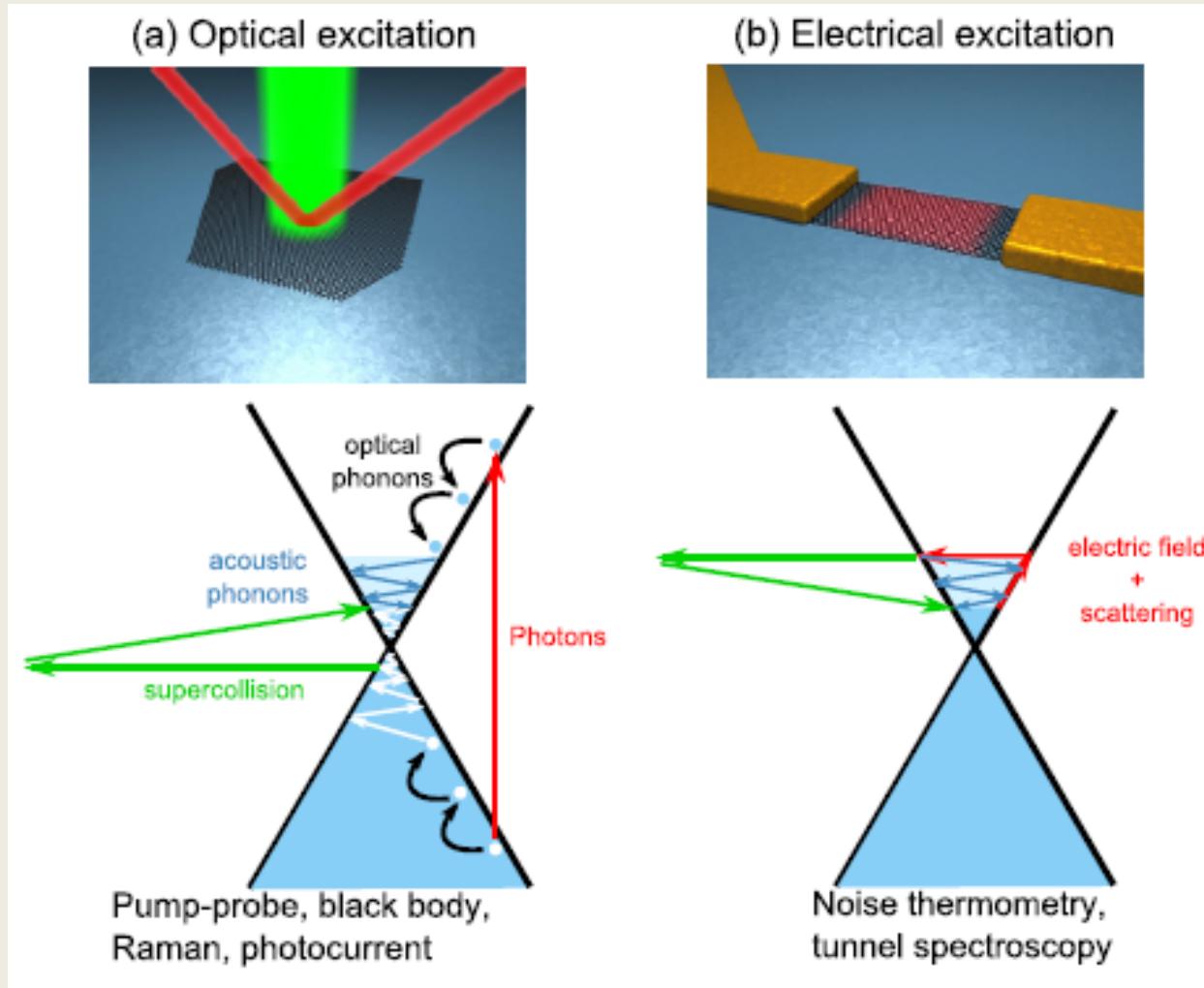
A. Laitinen et al., Phys. Rev. B 91, 121414(R) (2015)

OP-phonon energy  $\approx 2000$  K

Use suspended BLG :  
 => rid of substrate phonons  
 => AC-phonon is suppressed  
 => But a large WF++



# Hot electrons and substrate-phonons



UPoN1: Benchmarking optical and Joule heating

UPoN2: Investigate interactions with substrate polar phonons (SPPs)

Brunel, Berthou et al., J. Phys. : Condens. Matter 27, 164208 (2015)

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## Low-frequency 1/f noise in graphene devices

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Low-frequency noise with a spectral density that depends inversely on frequency has been observed in a wide variety of systems including current fluctuations in resistors, intensity fluctuations in music and signals in human cognition. In electronics, the phenomenon, which is known as 1/f noise, flicker noise or excess noise, hampers the operation of numerous devices and circuits, and can be a significant impediment to the development of practical applications from new materials. Graphene offers unique opportunities for studying 1/f noise because of its two-dimensional structure and widely tunable two-dimensional carrier concentration. The creation of practical graphene-based devices will also depend on our ability to understand and control the low-frequency noise in this material system. Here, the characteristic features of 1/f noise in graphene and few-layer graphene are reviewed, and the implications of such noise for the development of graphene-based electronics including high-frequency devices and sensors are examined.

UPoN3 : new clues on 1/f noise and Hooge's law using tunable graphene ?

See next talk by M. Macucci

# Hot-plasmon noise in graphene

UPoN4 : investigate interplay between electrons and plasmons in 2D  
See pm-talk by L. Varani

□ Graphene as tunable 2D semi-metal

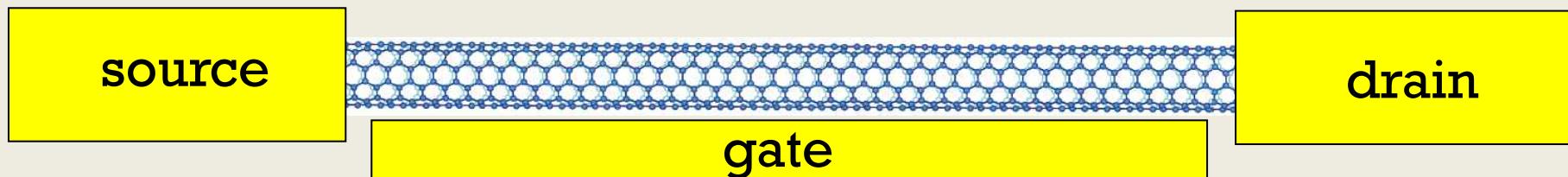
- a) Quantum shot noise in graphene (a brief review)
- b) Noise thermometry of hot electrons : electron-phonon in 2D
- c) Applications : HEBs, LNAs, Photo-detectors,

□ Carbon Nanotubes as single mode nano-conductors (a review)

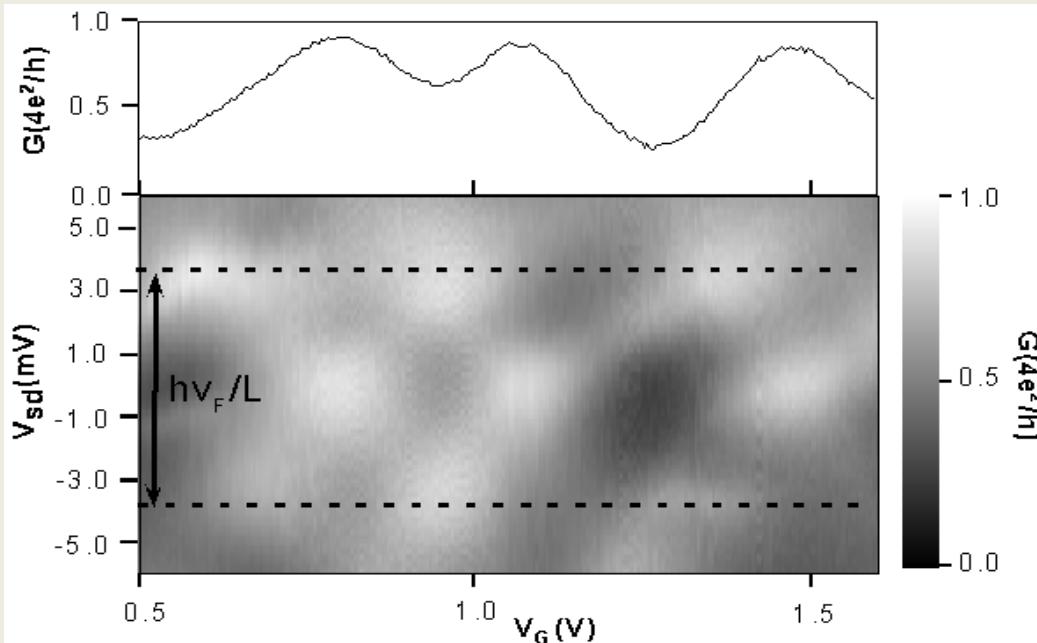
- a) Quantum shot noise in carbon nanotube devices
- b) Thermal noise in CNT wires and CNT-FETs: the noise conductance

# Fabry-Pérot CNT devices

good contacts + ballistic carbon nanotube  $\Rightarrow$  Fabry-Pérot electronic cavity



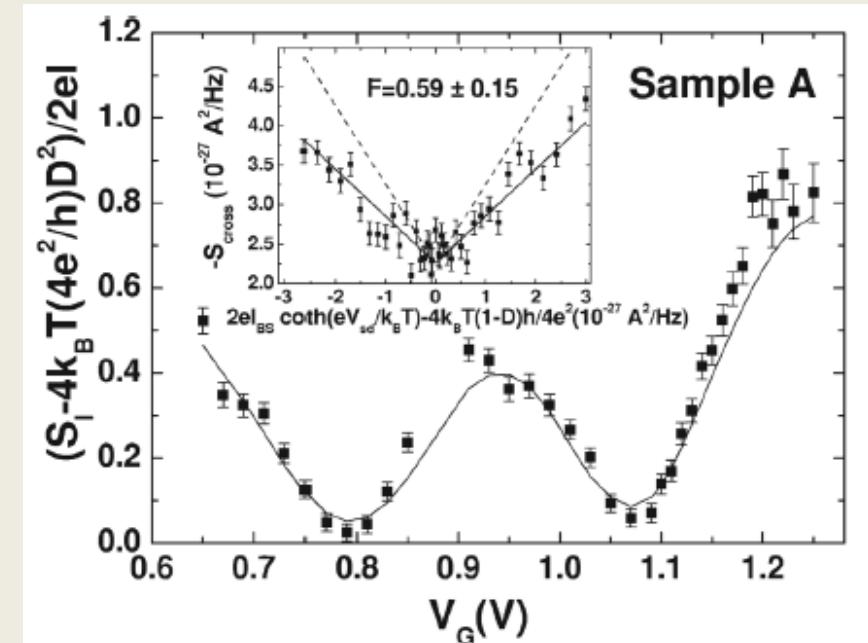
Checkerboard conductance pattern



=>

QSN suppression :

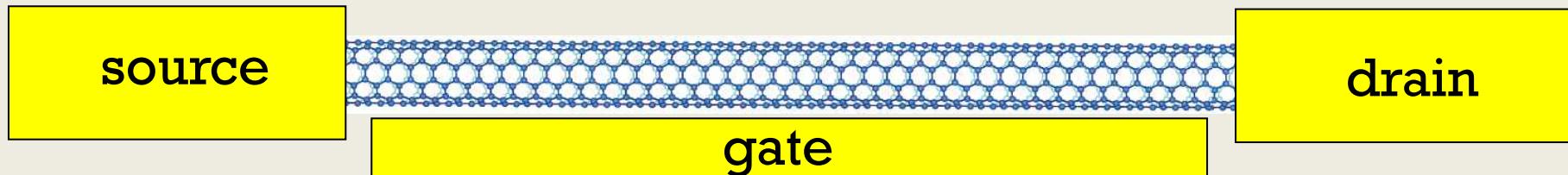
$$S_I = 2eI(1 - T)$$



L. Hermann et al., Phys. Rev. Lett. 99 (2007) 156804

# Kondo CNT-devices

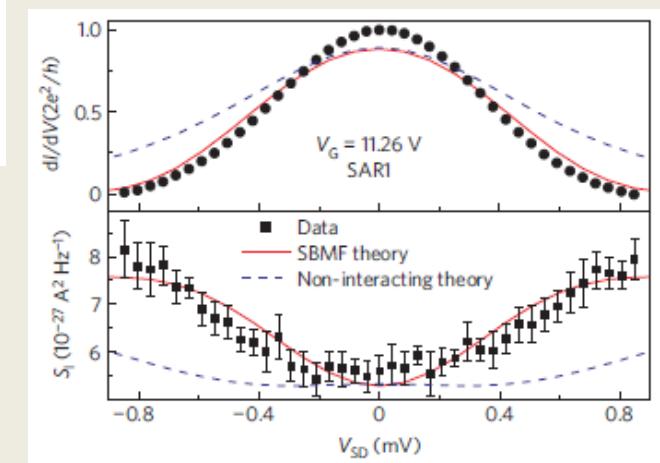
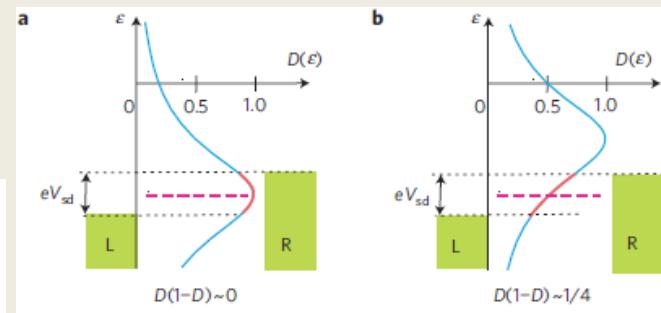
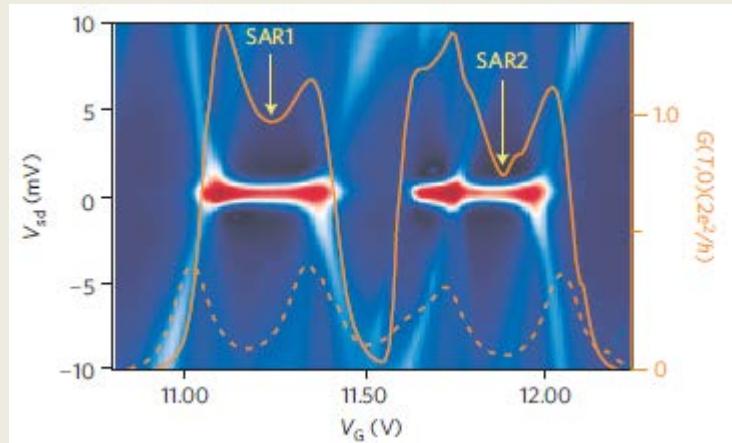
Medium contacts + interactions + odd e-number => Kondo effect



Kondo ridge

=>

QSN suppression

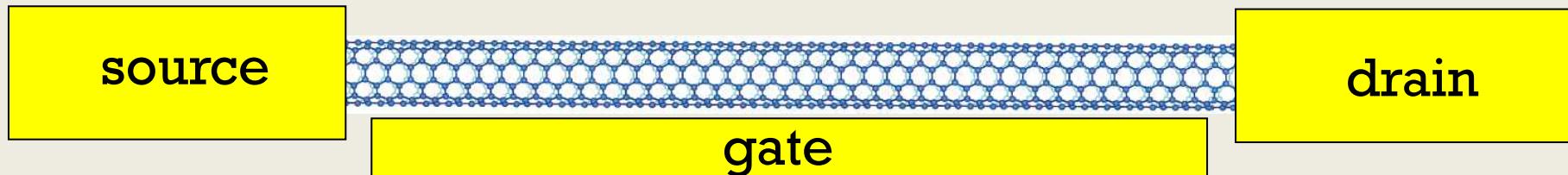


T. Delattre et al., Nat. Phys. 99 (2007) 156804

# Coulomb-blockade devices

Poor contacts + interactions = quantum dot

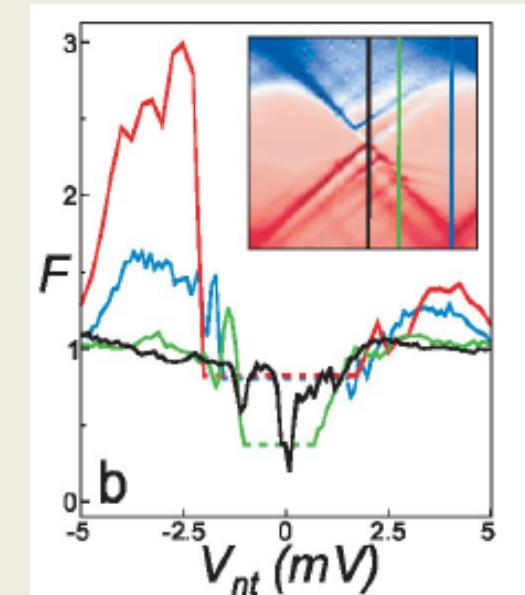
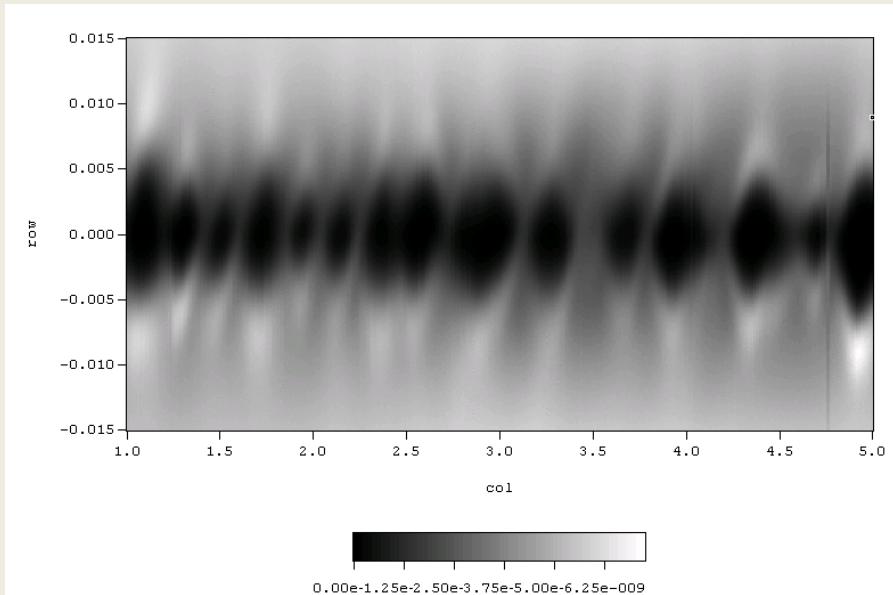
=> Coulomb blockade



Coulomb Blockade + inelastic cotunneling

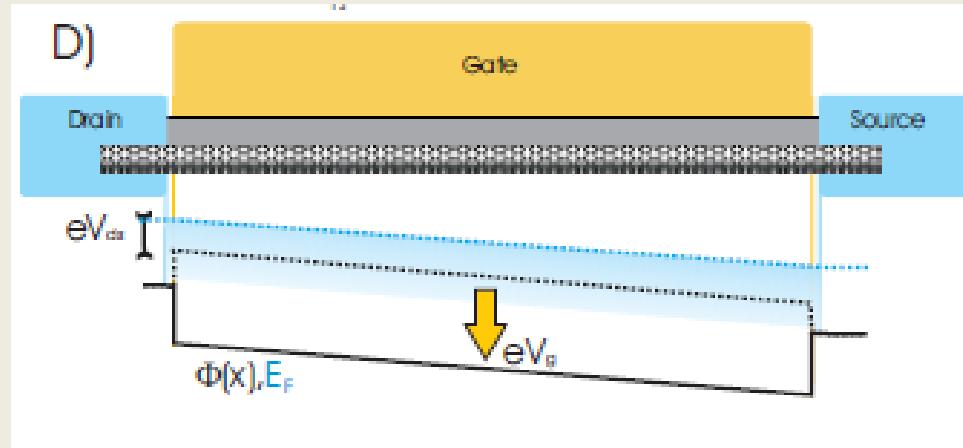
=>

superpoissonian noise



E. Onac et al., Phys. Rev. Lett. 96, 026803 (2006)

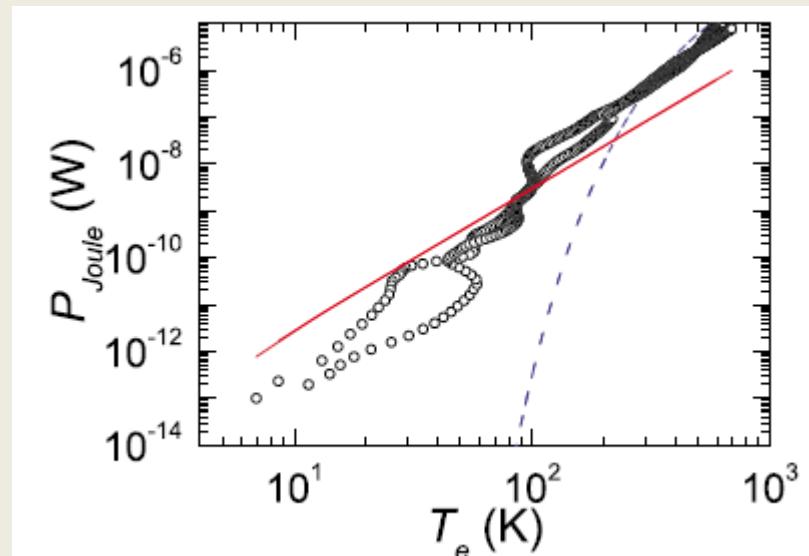
# Hot electrons in 1D carbon nanotubes



$$P_{ph} = \sum (T_e^{d+2} - T_{ph}^{d+2})$$

Graphene :  $P_{ph} = \sum (T_e^4 - T_{ph}^4)$

Carbon nanotube :  $P_{ph} = \sum (T_e^3 - T_{ph}^3)$



F. Wu et al., Appl. Phys. Lett. 97, 262115 (2010)

Thermal noise in field effect transistors : JA. van der Ziel, Proc. IRE 50, 1808 (1962)

### Two-terminal conductors

$$S_I(\omega) = 4k_B T_e \times G_{diff}(\omega)$$

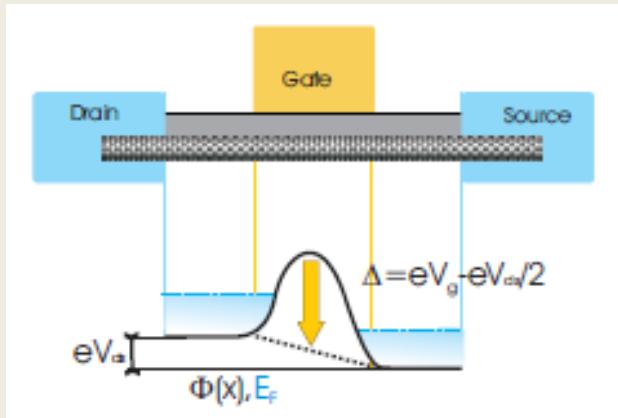
### Three-terminal conductors

$$S_I(\omega) = 4k_B T_e \times G_{noise}$$

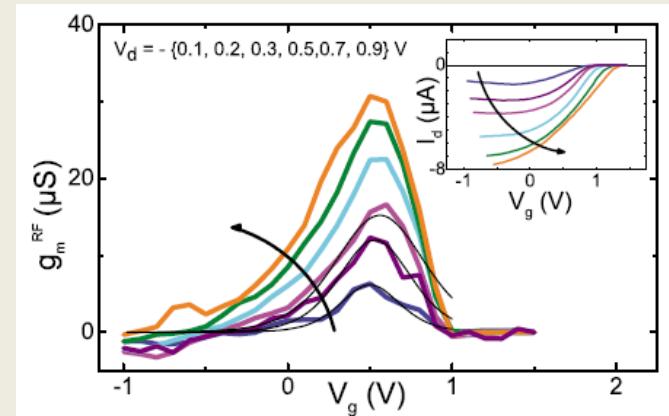
See also : talk on noise temperature fluctuations and the  
Noise Thermal Impedance by E. Pinsolle and B. Reulet

# The noise conductance of nano-FETs

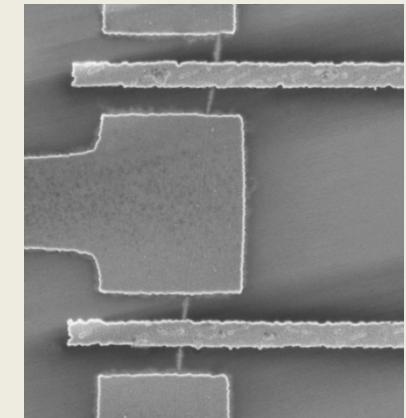
CNT-nano-FET



transconductance



GHz-CNT-FET



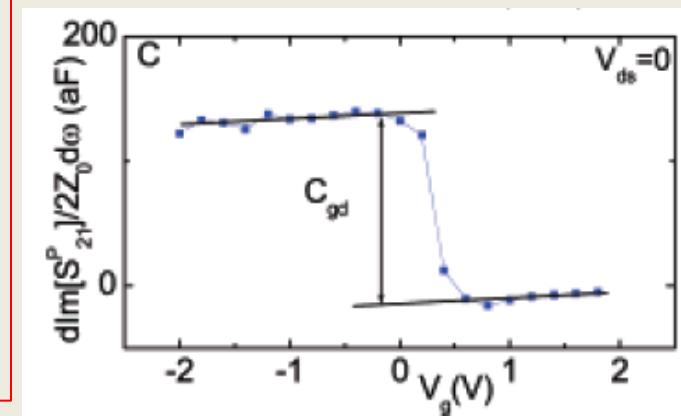
$$G_{noise} = G_{ds} + G_m \frac{C_Q}{2C_{gs}}$$

$$G_{ds} = \frac{4e^2}{h} \times f_d(\Delta)$$

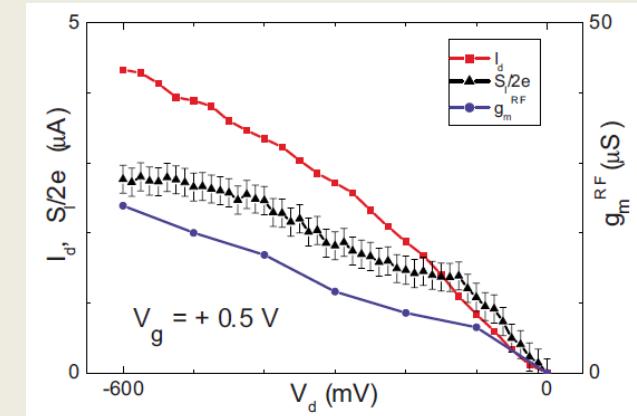
$$G_m = \frac{4e^2}{h} \times [f_s(\Delta) - f_d(\Delta)] \times \frac{C_{gs}}{C_Q}$$

$$G_{noise} = \frac{4e^2}{h} \times [f_s(\Delta) + f_d(\Delta)]$$

Quantum capacitance



current noise



J. Chaste et al., Nano Lett. 8, 525 (2008); J. Chaste et al., Appl. Phys. Lett. 96, 192103 (2010)

Thank you for your attention