The role of the coupling in the energy transfer between two stochastic systems coupled to different thermal baths

- Two electric circuits

- Two Brownian particles

On the heat flux and entropy produced by thermal fluctuations

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JSTAT P12014 (2013)

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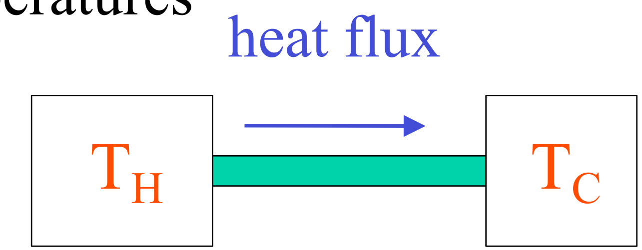
On the heat flux between two reservoirs at different temperture

A) In the stationary case for the heat flux between two reservoirs at different temperatures

$$\ln \frac{P(Q_\tau)}{P(-Q_\tau)} = \left(\frac{1}{T_C} - \frac{1}{T_H}\right) \frac{Q_\tau}{k_B}$$



Theory : no experiments

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The Nyquist problem

JULY, 1928 PHYSICAL REVIEW VOLUME 32

THERMAL AGITATION OF ELECTRIC CHARGE IN CONDUCTORS*

By H. Nyquist

Abstract

The electromotive force due to thermal agitation in conductors is calculated by means of principles in thermodynamics and statistical mechanics. The results obtained agree with results obtained experimentally.

Dr. J. B. Johnson[1] has reported the discovery and measurement of an electromotive force in conductors which is related in a simple manner to the temperature of the conductor and which is attributed by him to the thermal agitation of the carriers of electricity in the conductors. The work to be resported in the present paper was undertaken after Johnson's results were available to the writer and consists of a theoretical deduction of the electromotive force in question from thermodynamics and statistical mechanics.[2]

Consider two conductors each of resistance R and of the same uniform temperature T connected in the manner indicated in Fig. 1. The electromotive force due to thermal agitation in conductor I causes a current to be set up in the circuit whose value is obtained by dividing the electromotive force by $2R$. This current causes a heating or absorption of power in conductor II, the absorbed power being equal to the product of R and the square of the current. In other words power is transferred from conductor I to conductor II. In precisely the same manner it can be deduced that power is transferred from conductor II to conductor I. Now since the two conductors are at the same temperature it follows directly from the second law of thermodynamics that the power flowing in one direction is exactly equal to that flowing in the other direction. It will be noted that no assumption has been made as

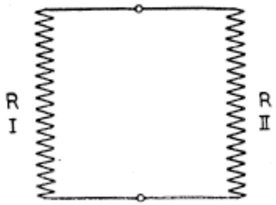
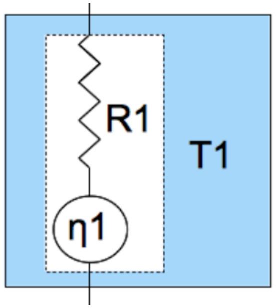


Fig. 1.

Power spectral density of the electric noise

$$|\tilde{\eta}|^2 = 4k_B R T$$

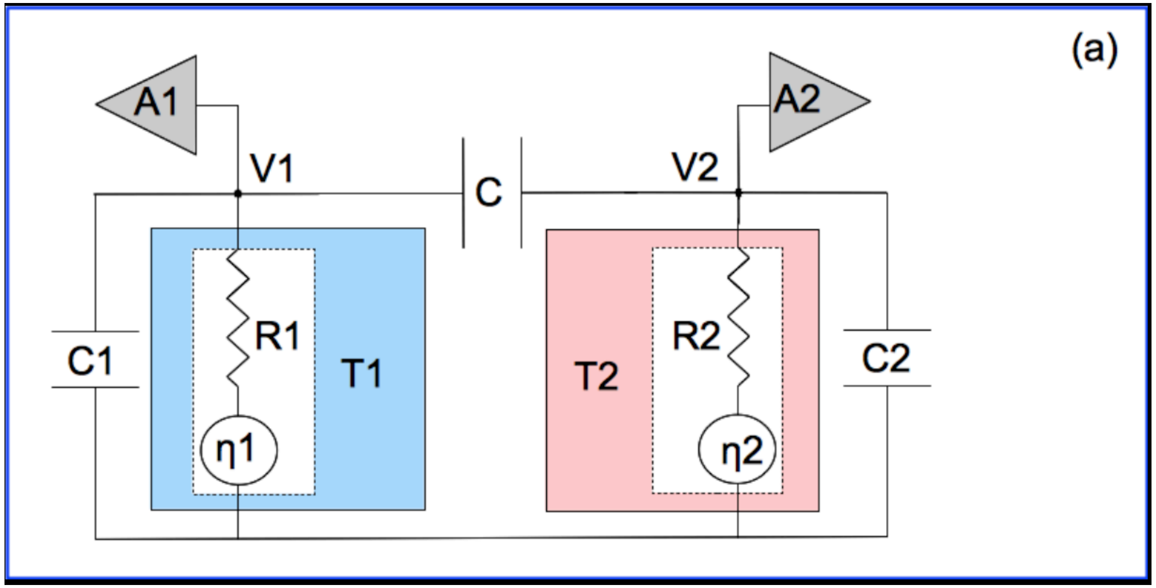


In 1928 well before Fluctuation Dissipation Theorem (FDT), this was the second example, after the Einstein relation for Brownian motion, relating the dissipation of a system to the amplitude of the thermal noise.

What are the consequences of removing the Nyquist equilibrium conditions ?

What are the statistical properties of the energy exchanged between the two conductors kept at different temperature ?

We analyse these questions in an electric circuit within the framework of FT.



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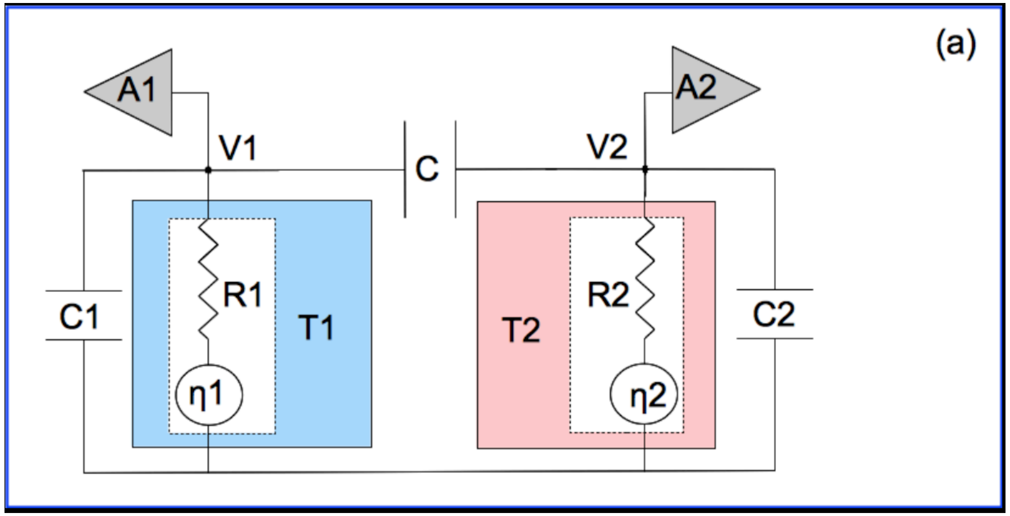

How the variance of V_1 and V_2 are modified because of the heat flux ?

What is the role of correlation between V_1 and V_2 ?

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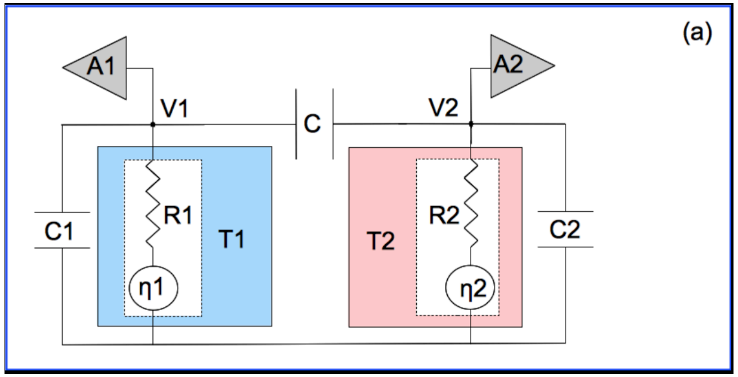
Electric Circuit



T_1 is changed with a nitrogen vapor circulation

T_2 =296K is kept fixed

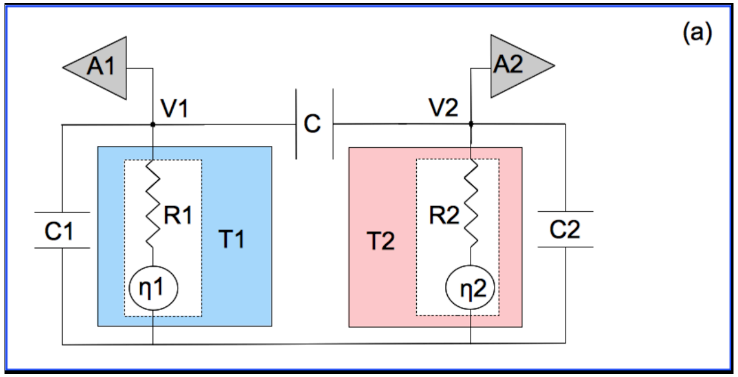
C is the coupling capacitance = 100pF and 1000pF

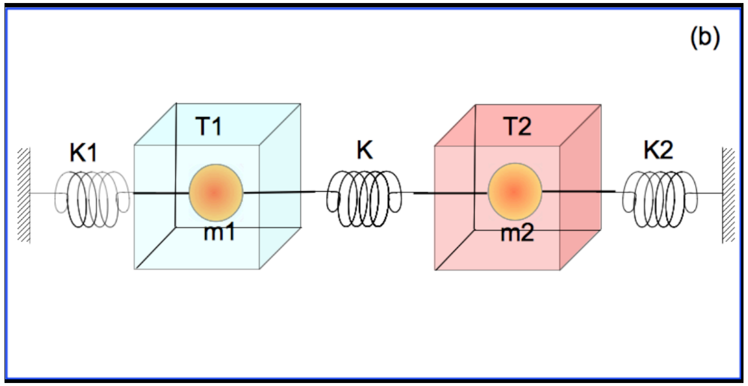
C1 and C2 are the cable and amplifier capacitances $\simeq 500pF$

$R_1 = R_2 = 10M\Omega$

$$\tau_o \simeq 0.01s$$

Electric Circuit and the mechanical equivalent





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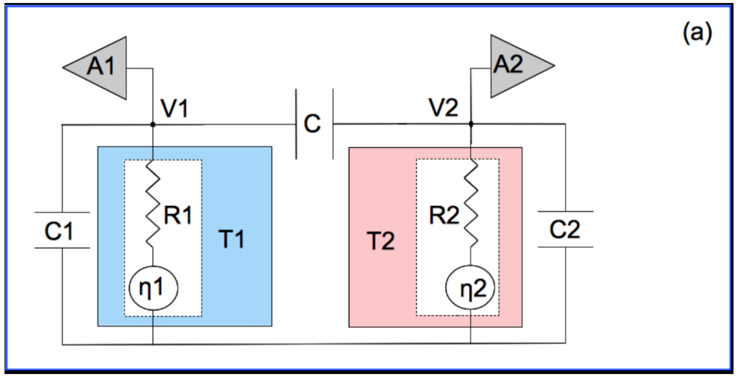
Electric Circuit



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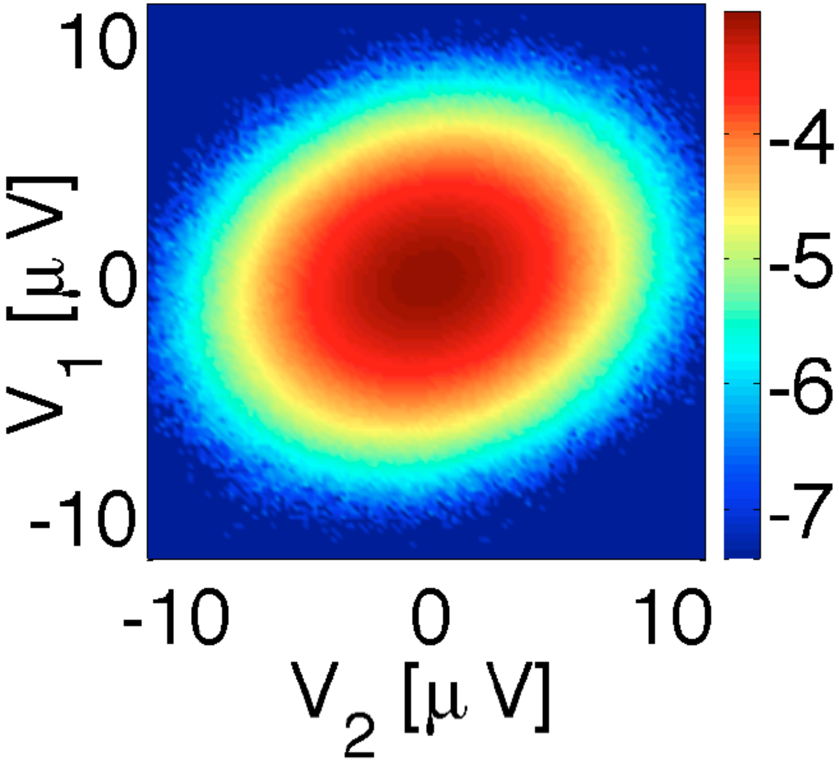
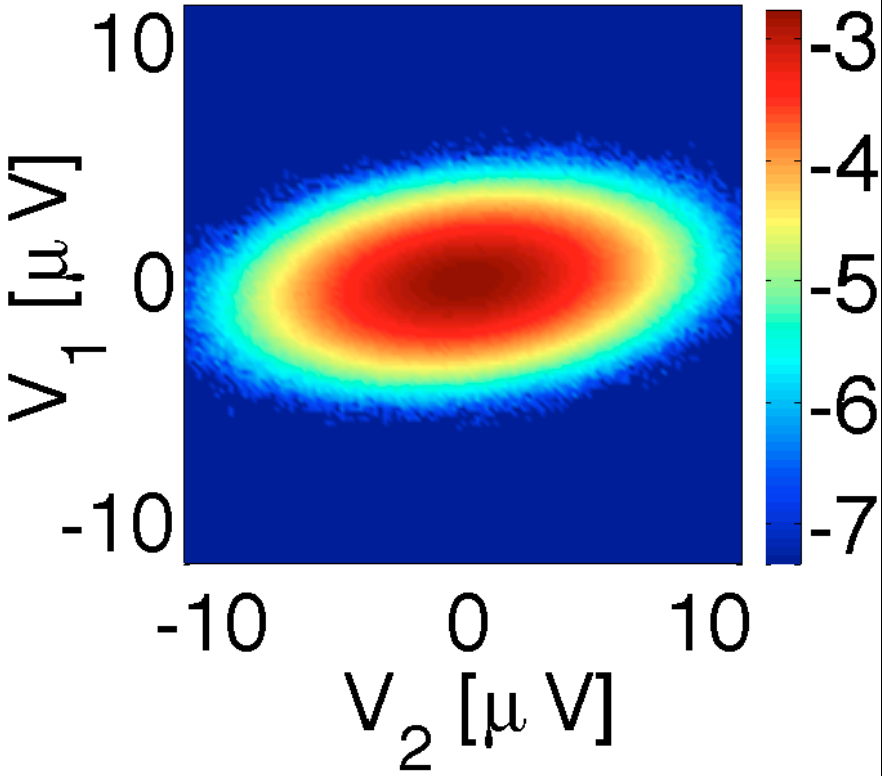
Joint probability of V_1 and V_2

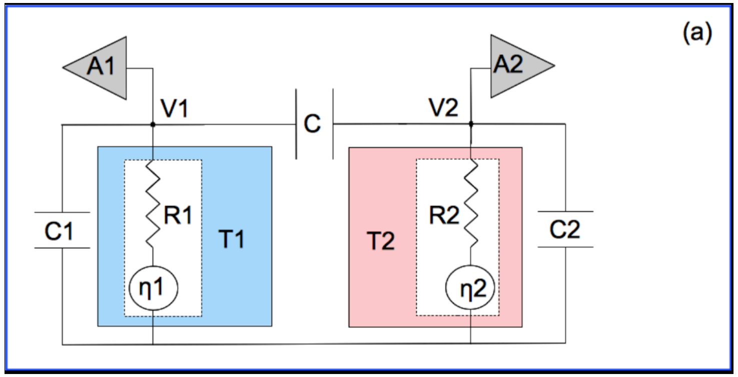
$\log_{10} P(V_1, V_2)$
at $T_1 = T_2 = 296k$



$\log_{10} P(V_1, V_2)$
at $T_1 = 88K$ and $T_2 = 296K$



Electric Circuit and the dissipated energy



i_m current flowing in the resistance m

i_{C_m} current flowing in the capacitance C_m

i_C current flowing in the capacitance C

$$\dot{Q}_m = V_m \, i_m$$

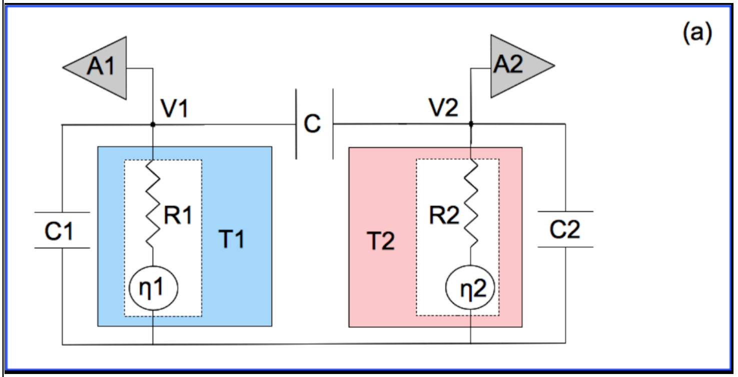
Power dissipated in the resistance m=1,2

$$i_m = i_C - i_{C_m} \qquad i_{C_m} = C_m \frac{dV_m}{dt} \qquad i_C = C \frac{d(V_2 - V_1)}{dt}$$

$$\dot{Q}_m = V_m i_m = \frac{V_m}{R_m}(V_m - \eta_m) = V_m \left[(C_m + C)\dot{V}_m - C\dot{V}_{m'} \right]$$

$m' = 2$ if $m = 1$, and $m' = 1$ if $m = 2$

Electric Circuit and the dissipated energy



Power dissipated in the resistance m=1,2

$$\dot{Q}_m = V_m i_m = V_m[(C_m + C)\dot{V}_m - C\dot{V}_{m'}]$$

$m' = 2$ if $m = 1$, and $m' = 1$ if $m = 2$

Integrating on a time τ

$$Q_{m,\tau} = W_{m,\tau} - \Delta U_{m,\tau}$$

$$Q_{m,\tau} = \int_t^{t+\tau} i_m \, V_m \, dt$$

heat flowed in the time τ from reservoir m' to reservoir m

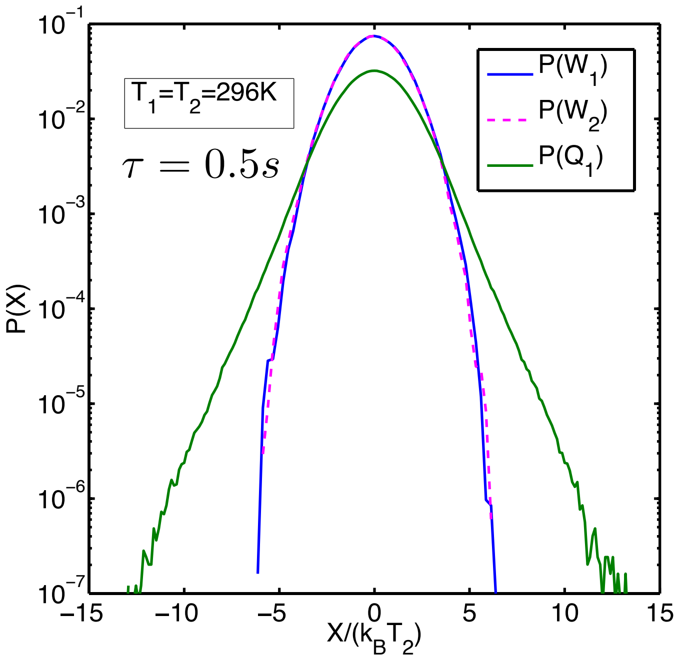
$$W_{m,\tau} = \int_t^{t+\tau} C V_m \frac{dV_{m'}}{dt} dt$$

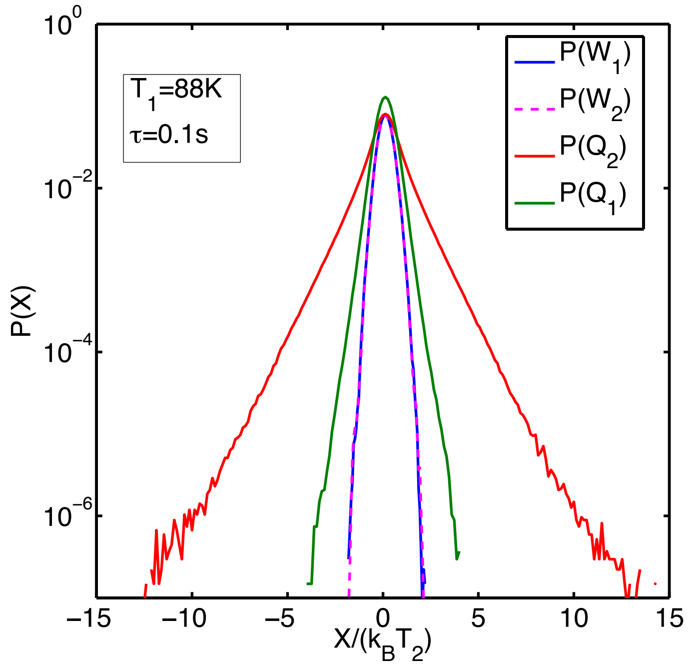
work performed by the circuit m on m' in the time τ

$$\Delta U_{m,\tau} = \frac{(C_m + C)}{2}(V_m(t+\tau)^2 - V_m(t)^2)$$

Potential energy change of the circuit m in the time τ.

Statistic of the work and heat



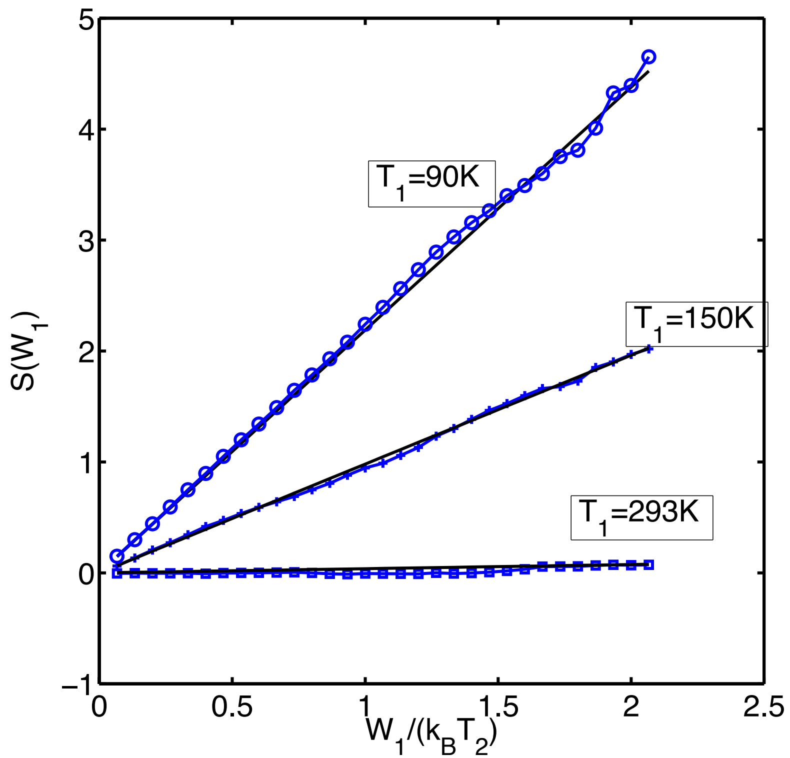


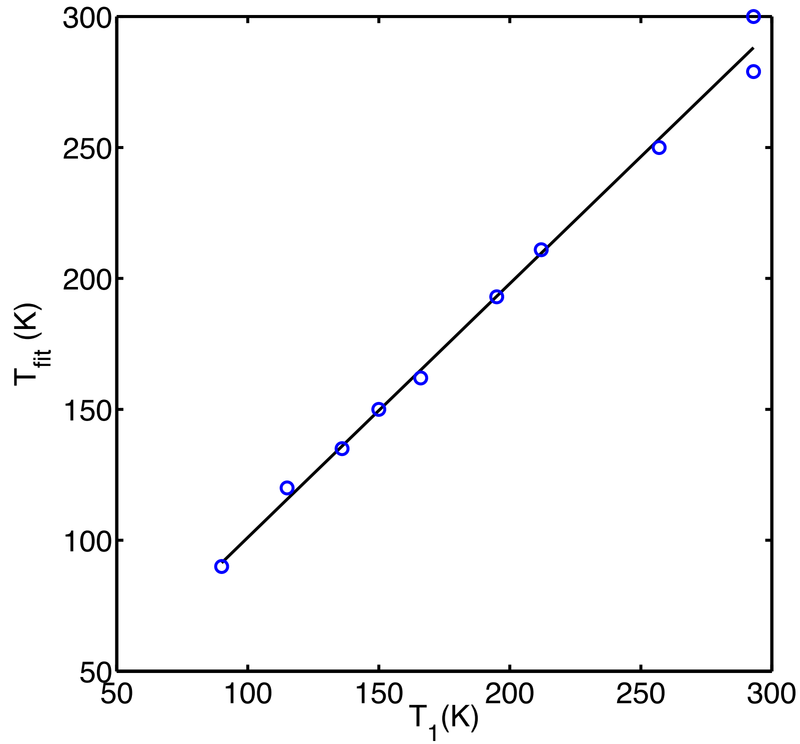
FT for W_τ et Q_τ
for $\tau \to \infty$

$$S(X_{m,\tau}) = \log \frac{P(X_{m,\tau})}{P(-X_{m,\tau})} = \Delta\beta \frac{X_{m,\tau}}{k_B T_2}$$

with $\Delta\beta = (T_2/T_1 - 1)$

On the heat flux and entropy produced by thermal fluctuations



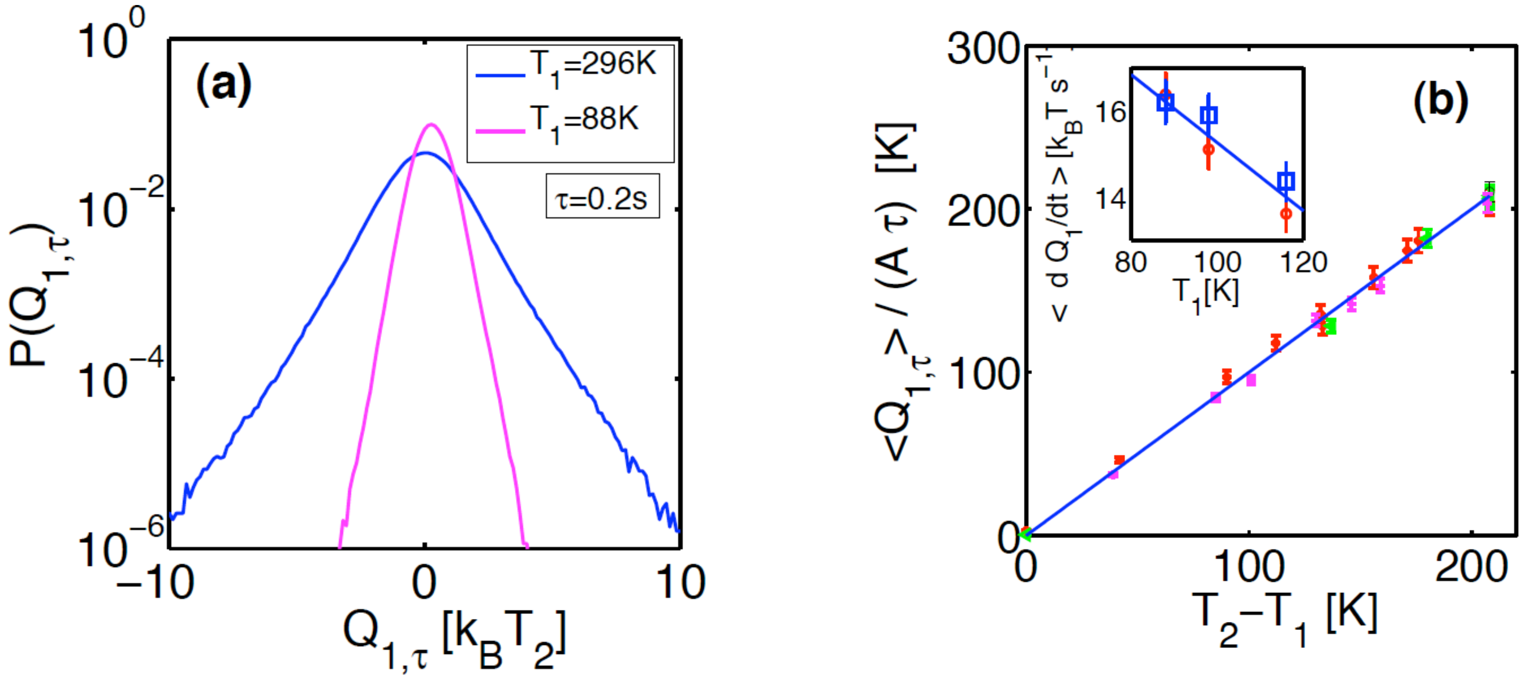


$$S(X_{m,\tau}) = \log \frac{P(X_{m,\tau})}{P(-X_{m,\tau})} = \Delta\beta \frac{X_{m,\tau}}{k_B T_2}$$

$$\text{with } \Delta\beta = (T_2/T_1 - 1)$$

$$T_{fit} = T_2/(\Delta\beta + 1)$$

The heat flux as a function of T_2-T_1



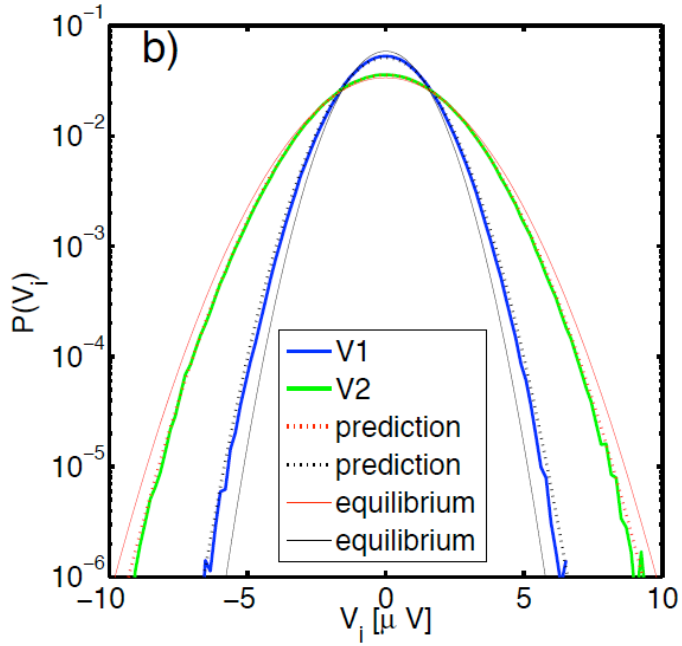
$$\left\langle \dot{Q}_1 \right\rangle = A\,(T_2 - T_1) = \frac{C^2 \Delta T}{XY}$$

$$X = C_2\,C_1 + C\,(C_1 + C_2)$$

$$Y = [(C_1 + C)R_1 + (C_2 + C)R_2] \text{ and } A = C^2/(X\,Y)$$

How the equilbrium variance of V_1 and V_2 is modified

σ_m^2 is
the variance of V_m



$$\sigma_m^2(T_m, T_{m'}) = \sigma_{m,eq}^2(T_m) + < \dot{Q}_m > R_m$$

$$\sigma_{m,\mathrm{eq}}^2(T_m) = k_B T_m (C + C'_m)/X$$

which is an extension to two temperatures of the Harada-Sasa relation

On the entropy produced by thermal fluctuations

$$\Delta S_{r,\tau} = Q_{1,\tau}/T_1 + Q_{2,\tau}/T_2$$

related to the heat exchanged with the reservoirs

Following Seifert, (PRL 95, 040602, 2005)
who developed this concept for a single heat bath,
we introduce a trajectory entropy for the evolving system

$$S_s(t) = -k_B \log P(V_1(t), V_2(t))$$

and the entropy production on the time τ

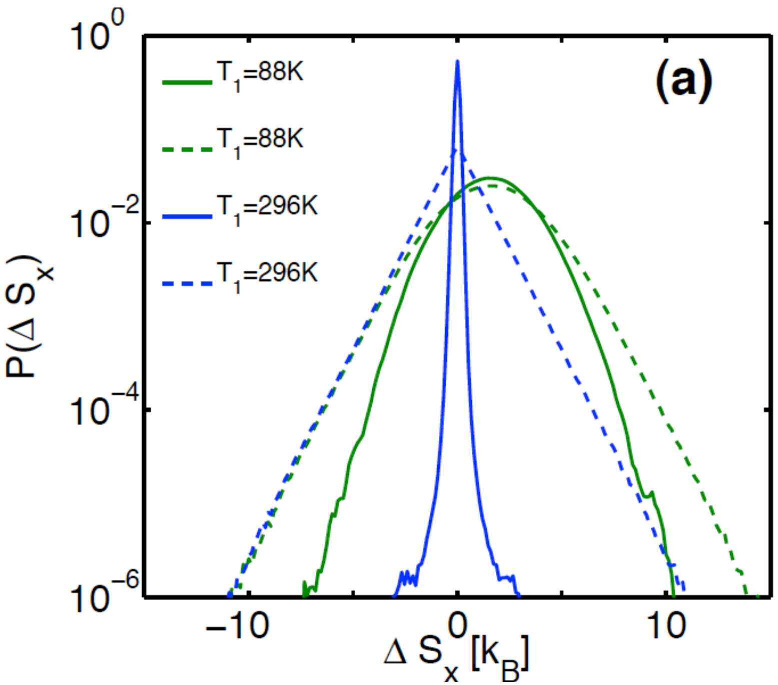
$$\Delta S_{s,\tau} = -k_B \log \left[\frac{P(V_1(t+\tau), V_2(t+\tau))}{P(V_1(t), V_2(t))} \right].$$

The total entropy is :

$$\Delta S_{tot,\tau} = \Delta S_{r,\tau} + \Delta S_{s,\tau}$$

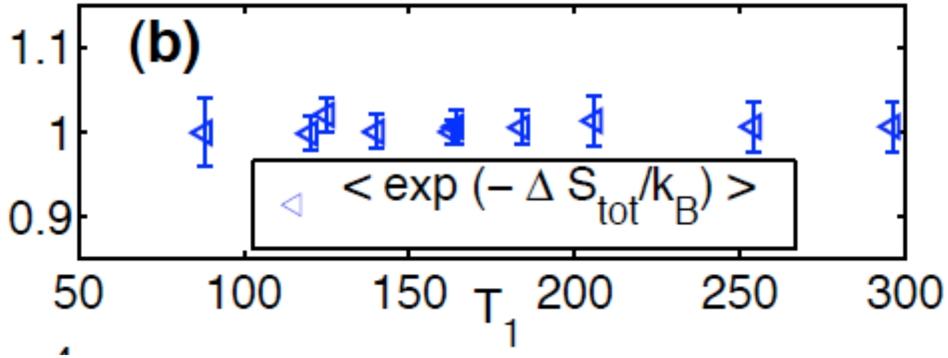
Statistical properties of the total entropy

$$\Delta S_{tot,\tau} = \Delta S_{r,\tau} + \Delta S_{s,\tau}$$



$\Delta S_{tot,\tau}$

$\Delta S_{r,\tau}$



independently of ΔT and of τ, the following equality always holds

$$\langle \exp(-\Delta S_{tot}/k_B) \rangle = 1$$

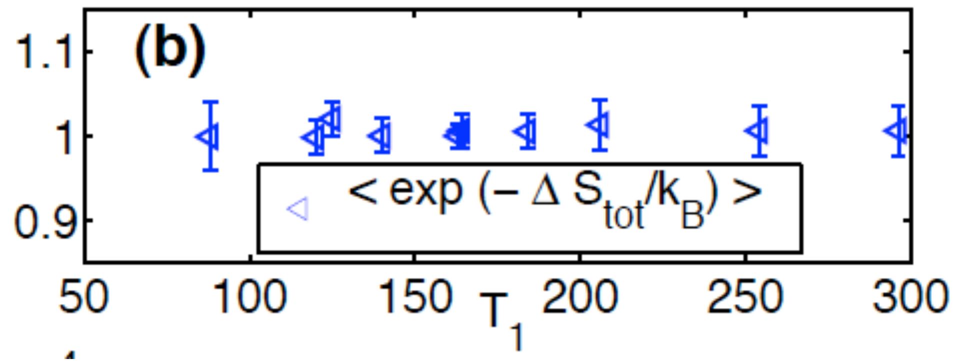
Statistical properties of the total entropy

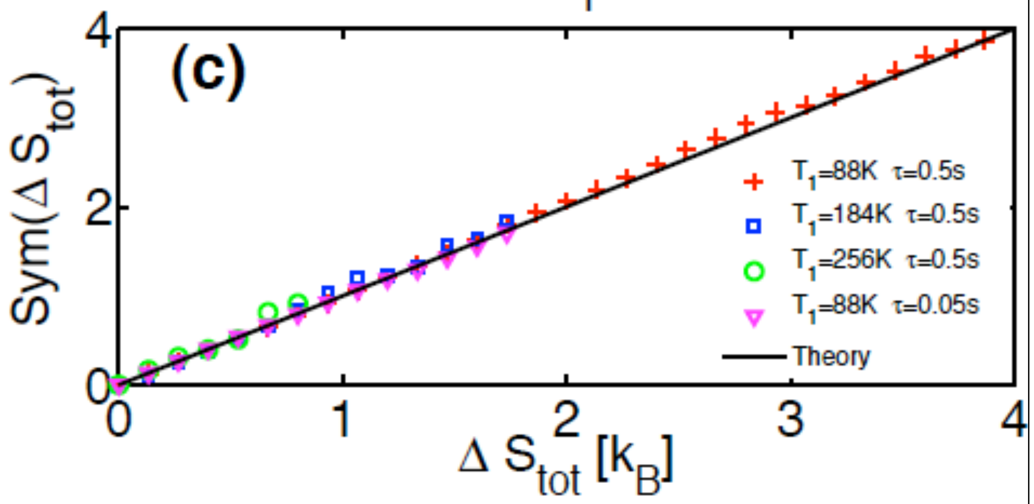
$$\langle \exp(-\Delta S_{tot}/k_B) \rangle = 1$$

implies that $P(\Delta S_{tot})$ satisfies a FT

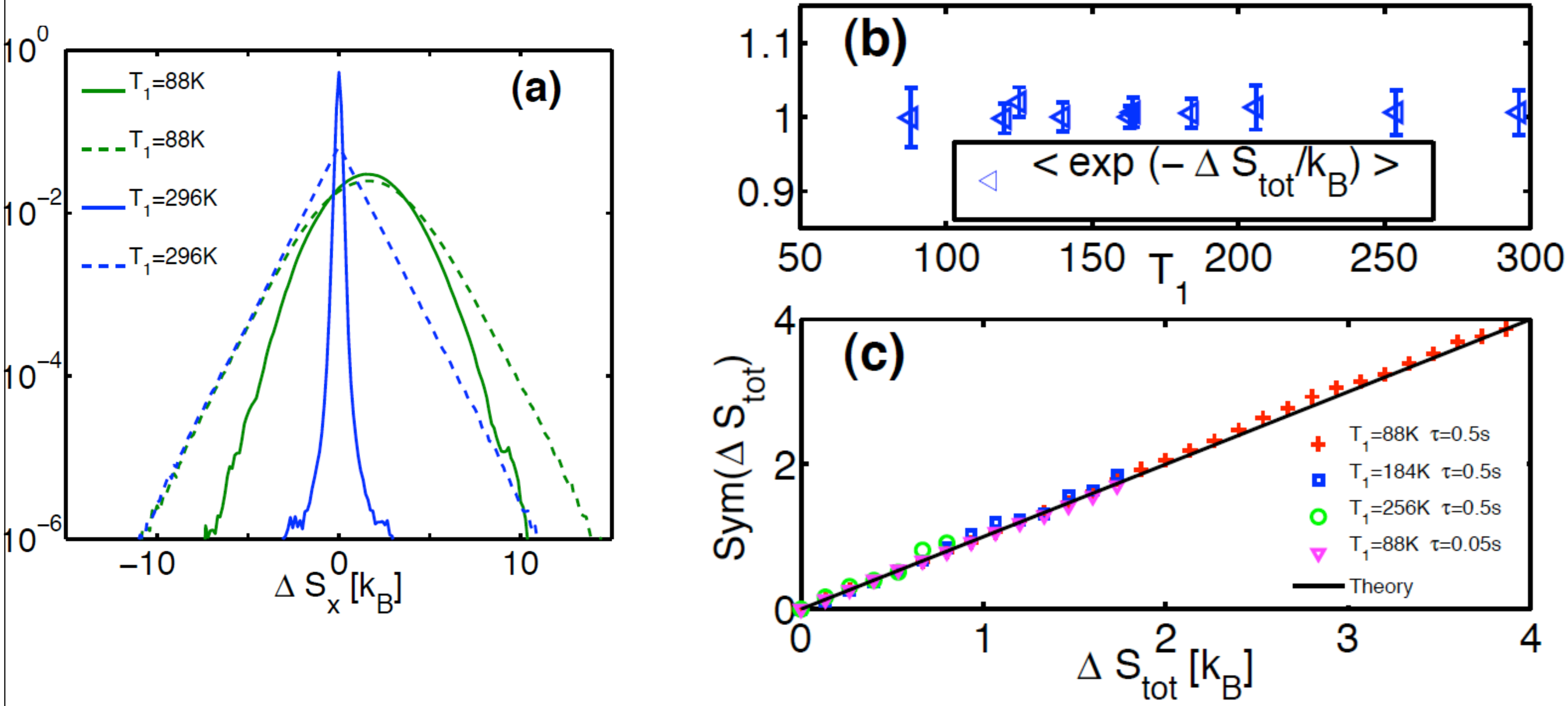
$$\log\left[\frac{P(\Delta S_{tot})}{P(-\Delta S_{tot})}\right] = \frac{\Delta S_{tot}}{k_B}$$

$$\forall \tau, \Delta T$$





On the heat flux and entropy produced by thermal fluctuations



Summary of the experimental and theoretical results
''On the heat flux and entropy produced by thermal fluctuations''

- The mean heat flux $< \dot{Q} > \propto (T_2 - T_1)$

- The pdf of $W_m / < W_m >$ satisfies an asymptotic FT whose prefactor is the entropy production rate $< W_m > (1/T_m - 1/T_{m'})$.

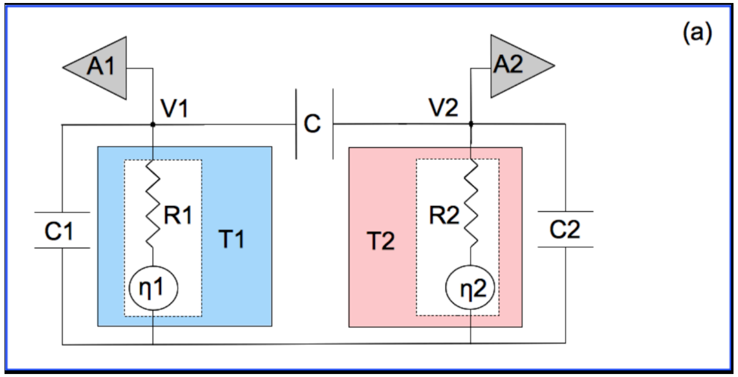
- The out of equilibrium variance :
  $\sigma^2_m(T_m, T_{m'}) = \sigma^2_{m,eq}(T_m) + < \dot{Q}_m > R_m$
  (Extension of Harada-Sasa relation)

- The total entropy ΔS_{tot} satisfies a conservation law which implies the second law and imposes the existence of a FT which is not asymptotic in time.

- ΔS_{tot} is rigorously zero in equilibrium, both in average and fluctuations

- The electrical-mechanical analogy makes these results very general and useful

On the heat flux and entropy produced by thermal fluctuations
Theory



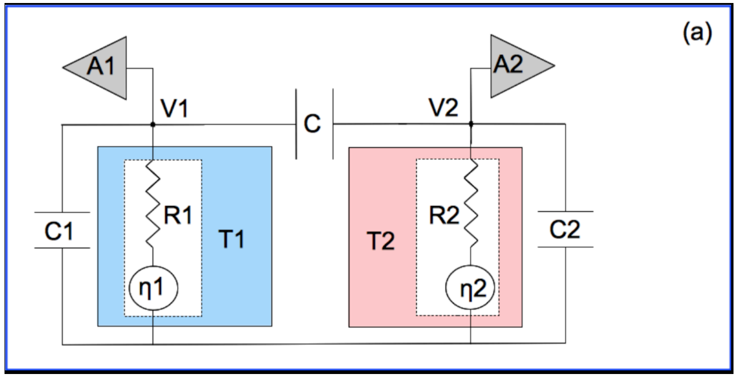
q_m is the charge flowed in the resistance R_m

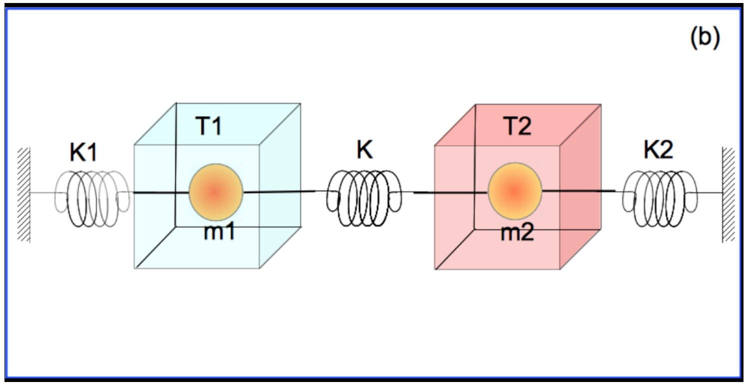
$$
\begin{aligned}
q_1 &= (V_1 - V_2)\,C + V_1\,C_1 \\
q_2 &= (V_1 - V_2)\,C - V_2\,C_2
\end{aligned}
$$

$$
\begin{aligned}
R_1 \dot{q}_1 &= -q_1 \frac{C_2}{X} + (q_2 - q_1)\frac{C}{X} + \eta_1 \\
R_2 \dot{q}_2 &= -q_2 \frac{C_1}{X} + (q_1 - q_2)\frac{C}{X} + \eta_2
\end{aligned}
$$

$$
\langle \eta_i(t)\eta_j(t')\rangle = 2\delta_{ij} k_B T_i R_j \delta(t - t')
$$

Electric Circuit and the mechanical equivalent





$$R_1\dot{q}_1 = -q_1\frac{C_2}{X} + (q_2 - q_1)\frac{C}{X} + \eta_1$$

$$R_2\dot{q}_2 = -q_2\frac{C_1}{X} + (q_1 - q_2)\frac{C}{X} + \eta_2$$

$$\langle \eta_i(t)\eta_j(t')\rangle = 2\delta_{ij}k_B T_i R_j \delta(t-t')$$

$$X = C_2\, C_1 + C\,(C_1 + C_2)$$

q_m the displacement of the particle m

i_m its velocity

$K_m = 1/C_m$ the stiffness of the spring m

$K = 1/C$ the stiffness of the coupling spring

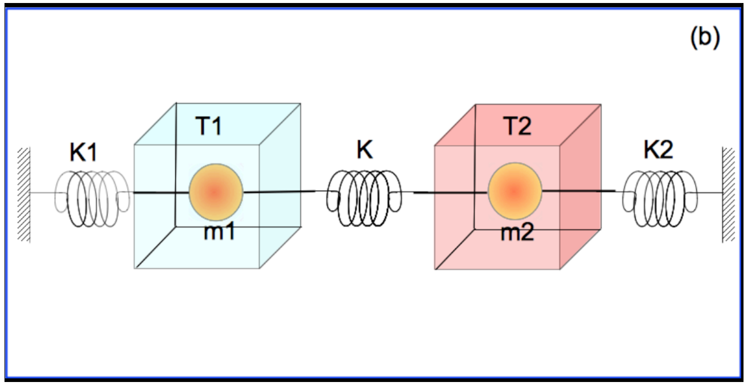
R_m the viscosity.

On the heat flux bewteen two particles at two different temperature

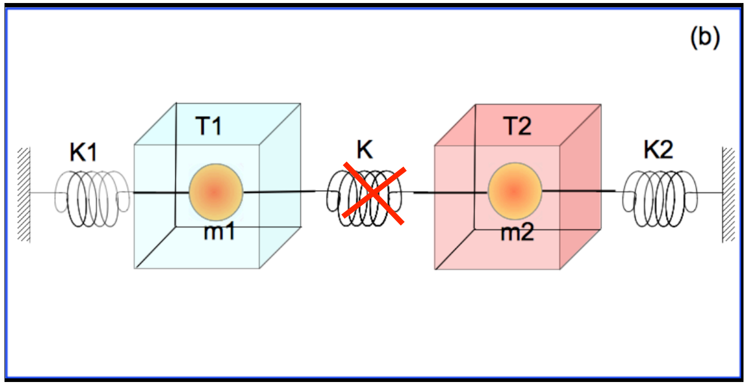
A. Bérut, A. Petrosyan and S. Ciliberto,

Laboratoire de Physique, C.N.R.S. UMR5672,
Ecole Normale Supérieure, France

Energy flow between two hydrodynamically coupled particles kept at different effective temperatures
A. Bérut, A. Petrosyan and S. Ciliberto, EPL, 107 (2014) 60004

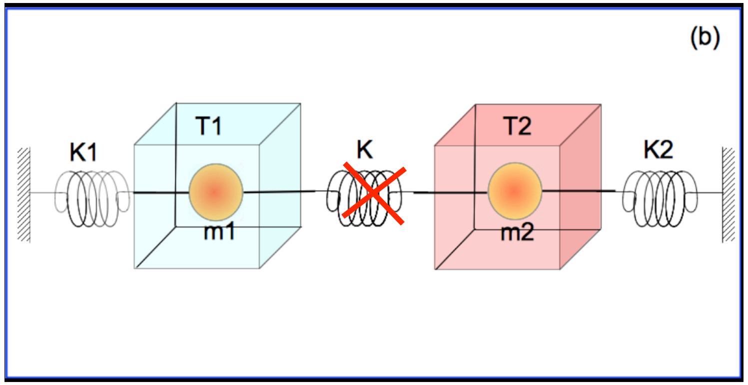


Two Brownian particles trapped by two laser beams.



Two Brownian particles trapped by two laser beams.

Difficulty of having an harmonic coupling between the particles.
The main source of coupling is hydodynamic (viscous)



Two Brownian particles trapped by two laser beams.

Difficulty of having an harmonic coupling between the particles.
The main source of coupling is hydodynamic (viscous)

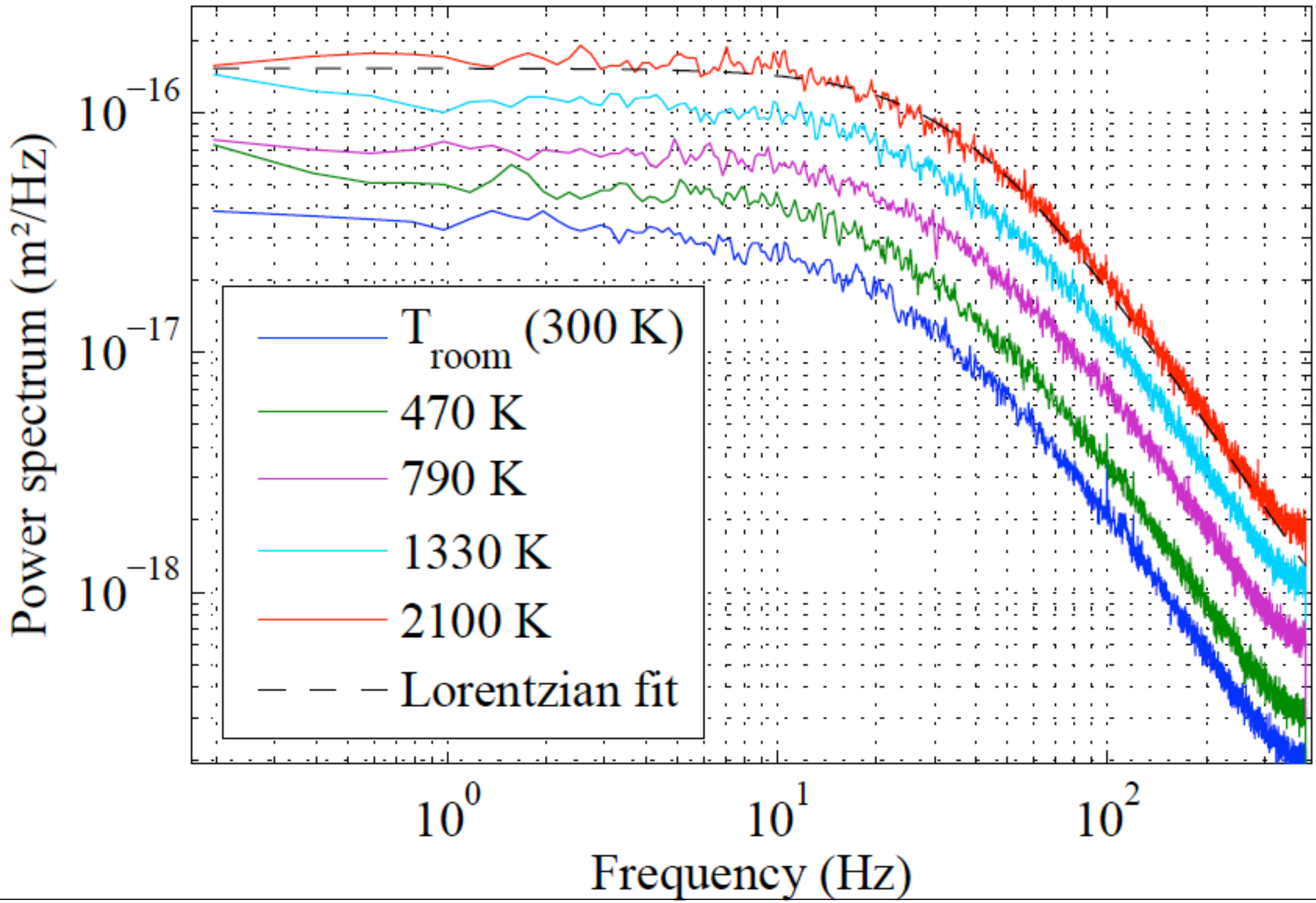
Difficulty of having two close Brownian particles at two different temperatures

The temperature gradient is done by forcing the motion of one particle with an external random force

ENS DE LYON

Experimental results

Spectra of excited particle



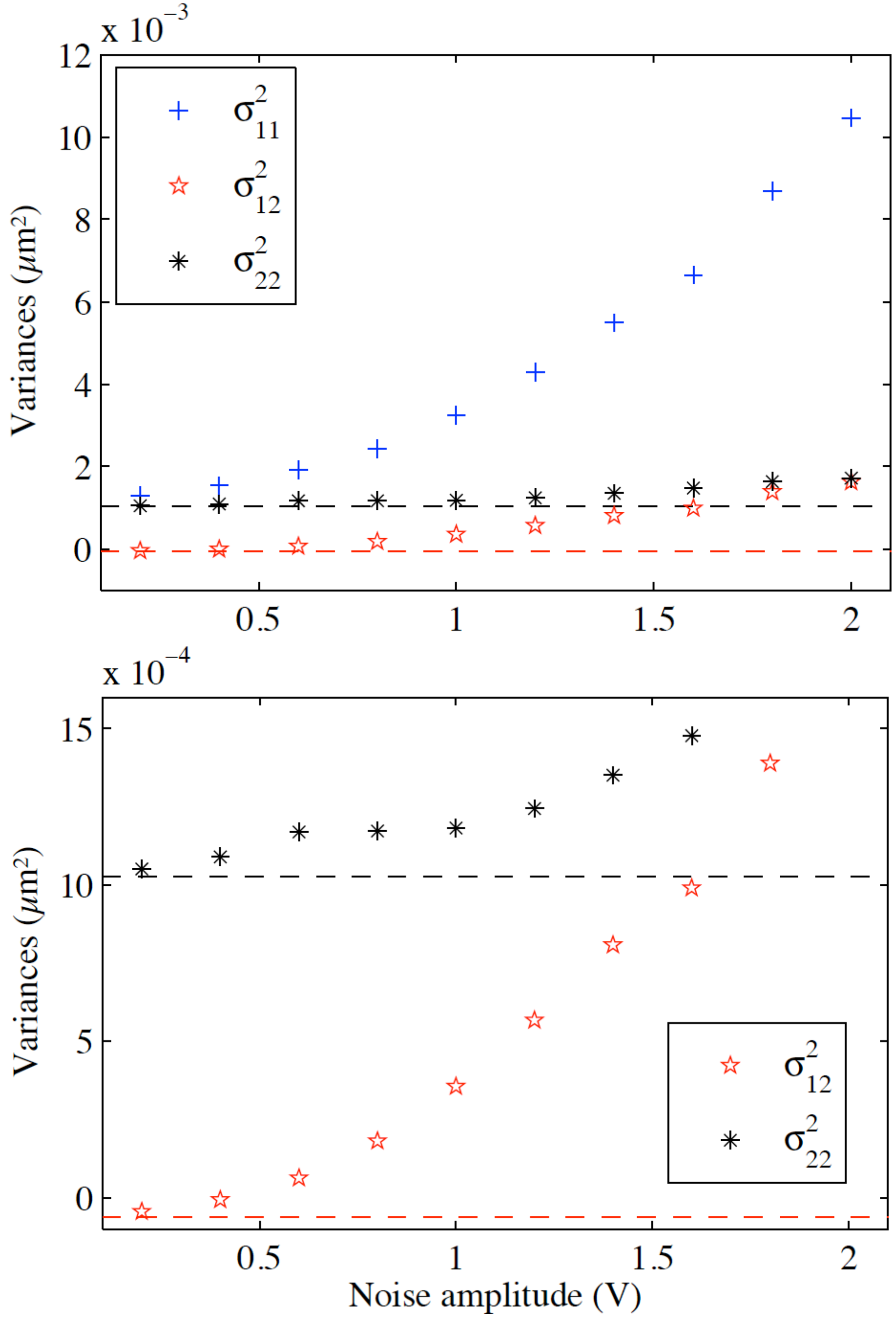
$$\sigma_{11}^2 = \langle x_1 x_1 \rangle$$

$$\sigma_{12}^2 = \langle x_1 x_2 \rangle$$

$$\sigma_{22}^2 = \langle x_2 x_2 \rangle$$

Variances and cross variances as a function of the random driving voltage (force) at

$$d = 3.2\,\mu m$$



ENS DE LYON

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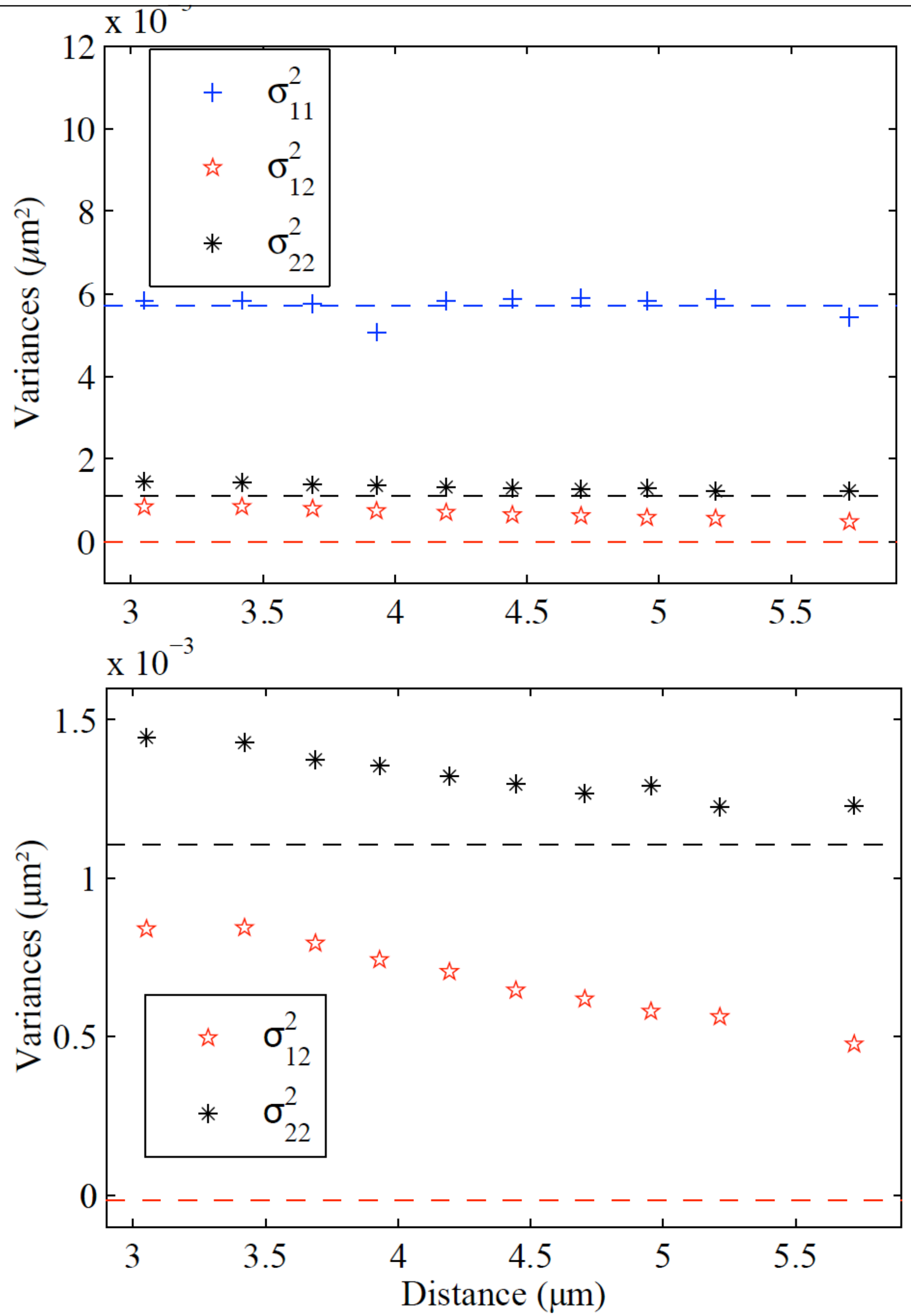


Variances and cross variances as a function of the distance betwen the beads for a fixed driving of 1.5V

From a suitable hydrodynamic model one can compute the variances

$$\sigma_{11}^2 = \langle x_1 x_1 \rangle = \frac{k_B(T+\Delta T)}{k_1} - \frac{k_2}{k_1}\frac{\epsilon^2 k_B \Delta T}{k_1+k_2}$$
$$\sigma_{12}^2 = \langle x_1 x_2 \rangle = \frac{\epsilon k_B \Delta T}{k_1+k_2}$$
$$\sigma_{22}^2 = \langle x_2 x_2 \rangle = \frac{k_B T}{k_2} + \frac{\epsilon^2 k_B \Delta T}{k_1+k_2}$$

where :

ϵ is the coupling coefficient of the particle.
It has to depend on the distance but not
on the random driving amplitude

ΔT is the temperature difference induced by the random driving.

k_1 and k_2 are the stiffness of the optical traps.

From a suitable hydrodynamic model one can compute the variances

$$\sigma_{11}^2 = \langle x_1 x_1 \rangle = \frac{k_{\mathrm{B}}(T+\Delta T)}{k_1} - \frac{k_2}{k_1}\frac{\epsilon^2 k_{\mathrm{B}}\Delta T}{k_1+k_2}$$

$$\sigma_{12}^2 = \langle x_1 x_2 \rangle = \frac{\epsilon k_{\mathrm{B}}\Delta T}{k_1+k_2}$$

$$\sigma_{22}^2 = \langle x_2 x_2 \rangle = \frac{k_{\mathrm{B}} T}{k_2} + \frac{\epsilon^2 k_{\mathrm{B}}\Delta T}{k_1+k_2}$$

where :

ϵ, T and ΔT are the unknown

ϵ is the coupling coefficient of the particle.
It has to depend on the distance but not on the random driving amplitude

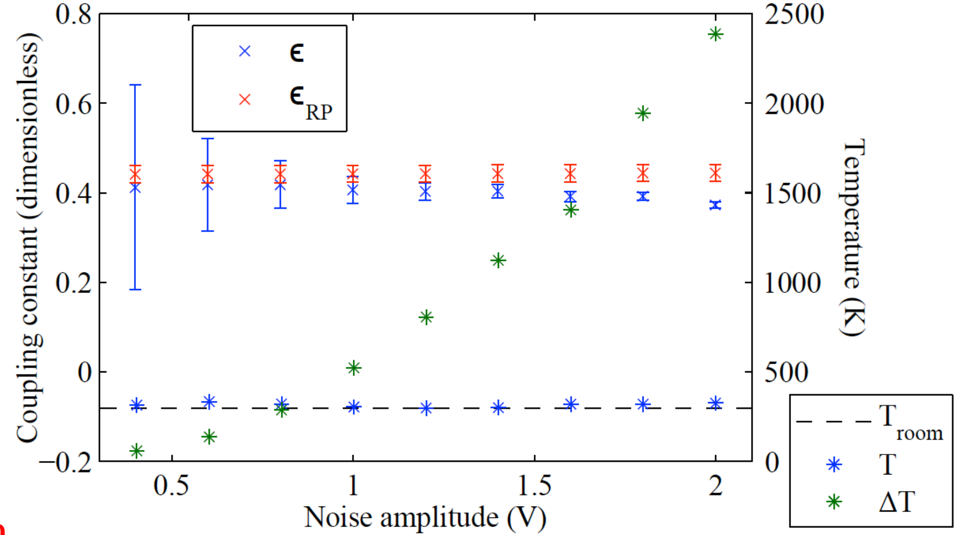
ΔT is the temperature difference induced by the random driving.

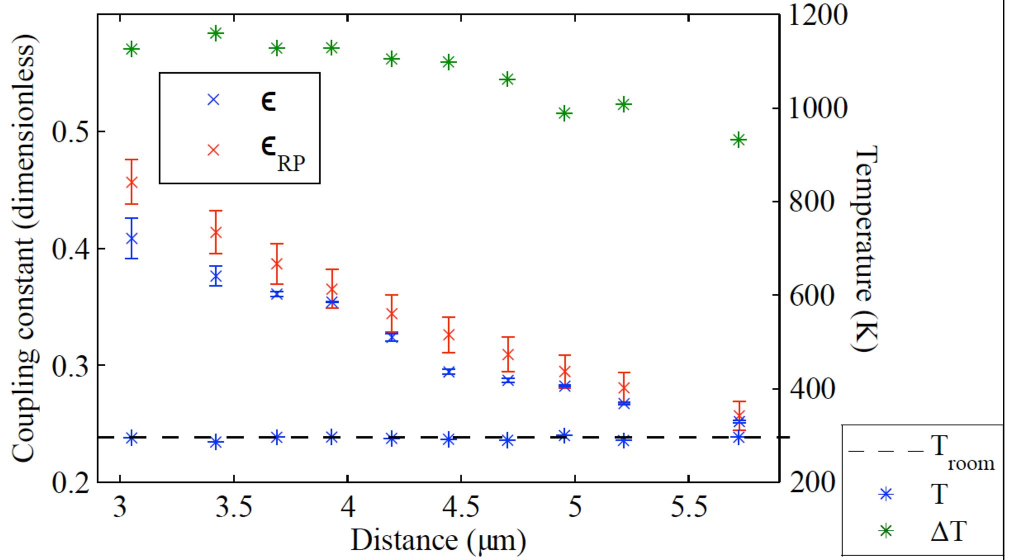
k_1 and k_2 are the stiffness of the optical traps.

Values of the parameters from the experiment

$$\sigma_{11}^2 = \langle x_1 x_1 \rangle = \frac{k_B(T+\Delta T)}{k_1} - \frac{k_2}{k_1}\frac{\epsilon^2 k_B \Delta T}{k_1+k_2}$$
$$\sigma_{12}^2 = \langle x_1 x_2 \rangle = \frac{\epsilon k_B \Delta T}{k_1+k_2}$$
$$\sigma_{22}^2 = \langle x_2 x_2 \rangle = \frac{k_B T}{k_2} + \frac{\epsilon^2 k_B \Delta T}{k_1+k_2}$$

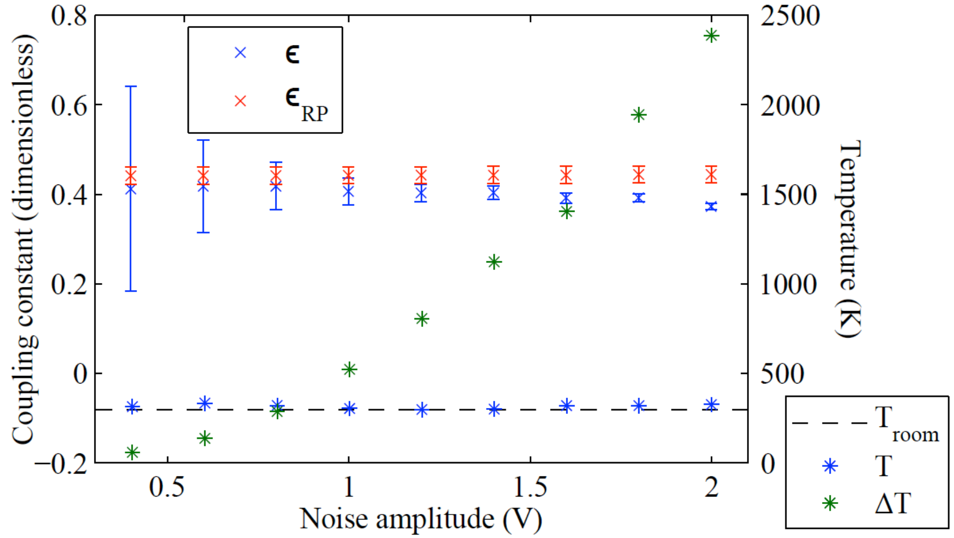
ϵ, T and ΔT are the unknown



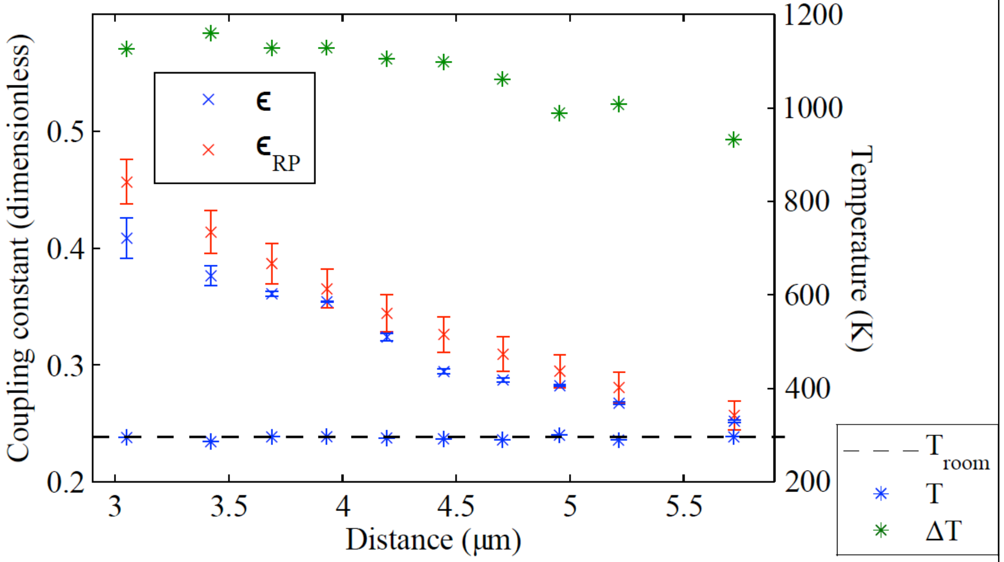


Values of the parameters from the experiment

$$\sigma_{11}^2 = \langle x_1 x_1 \rangle = \frac{k_B(T+\Delta T)}{k_1} - \frac{k_2}{k_1}\frac{\epsilon^2 k_B \Delta T}{k_1+k_2}$$
$$\sigma_{12}^2 = \langle x_1 x_2 \rangle = \frac{\epsilon k_B \Delta T}{k_1+k_2}$$
$$\sigma_{22}^2 = \langle x_2 x_2 \rangle = \frac{k_B T}{k_2} + \frac{\epsilon^2 k_B \Delta T}{k_1+k_2}$$



Can we interpret the term proportional to ΔT as the heat flux between the two particles ?



The standard hydrodynamic model

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \mathcal{H} \times \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}$$

two coupled Langevin equations

coupling Rotne-Prager diffusion tensor

$$\mathcal{H} = \begin{pmatrix} 1/\gamma & \epsilon/\gamma \\ \epsilon/\gamma & 1/\gamma \end{pmatrix}$$

$$\epsilon = \frac{3R}{2d} - \left(\frac{R}{d}\right)^3$$

and forces

$$F_i = -k_i \times x_i + f_i$$

in equilibrium

$$\langle f_i(t) \rangle = 0$$

$$\langle f_i(t) f_j(t') \rangle = 2k_{\mathrm{B}}T\,(\mathcal{H}^{-1})_{ij}\,\delta(t-t')$$

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Out of Equilbrium : forcing on bead 1 $f^* = k_1 x_0(t)$

f^* is a delta correlated noise

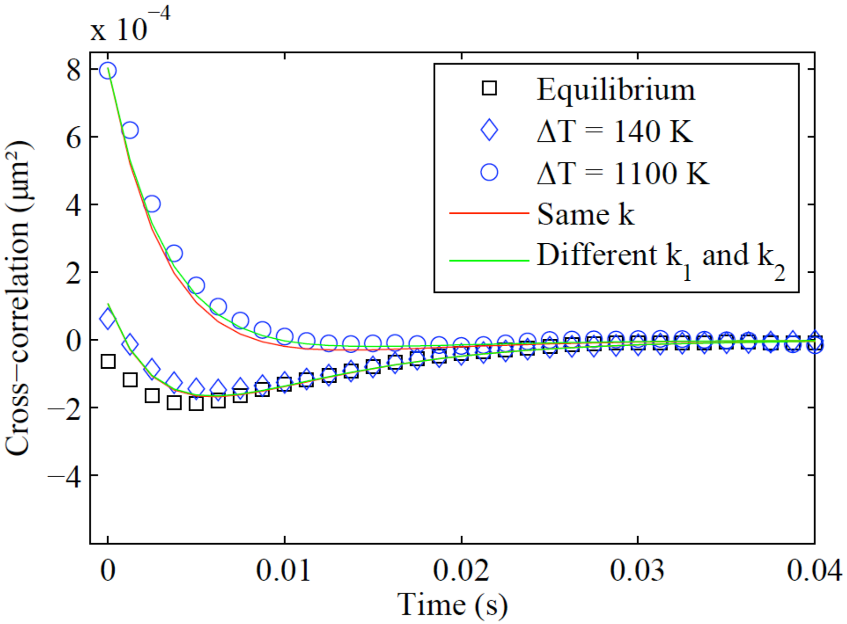
Bead 1 has an effective temperature $T^* = T + \Delta T$

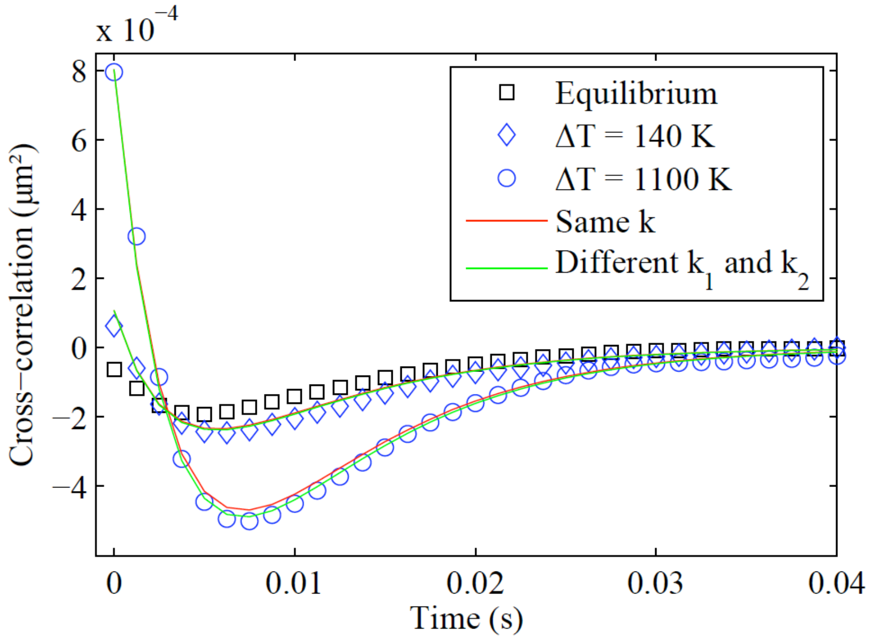
The standard hydrodynamic model

It follows that the system of equations is:

$$\begin{cases} \gamma \dot{x}_1 = -k_1 x_1 + \epsilon(-k_2 x_2 + f_2) + f_1 + f^* \\ \gamma \dot{x}_2 = -k_2 x_2 + \epsilon(-k_1 x_1 + f_1 + f^*) + f_2 \end{cases}$$

The correlation functions





$$< x_1(t)x_2(0) >$$

$$< x_1(0)x_2(t) >$$

The standard hydrodynamic model

It follows that the system of equations is:

$$\begin{cases} \gamma \dot{x}_1 = -k_1 x_1 + \epsilon(-k_2 x_2 + f_2) + f_1 + f^* \\ \gamma \dot{x}_2 = -k_2 x_2 + \epsilon(-k_1 x_1 + f_1 + f^*) + f_2 \end{cases}$$

comparison with the electric case

$$\begin{aligned} R_1 \dot{q}_1 &= -q_1 \frac{C_2}{X} + (q_2 - q_1)\frac{C}{X} + \eta_1 \\ R_2 \dot{q}_2 &= -q_2 \frac{C_1}{X} + (q_1 - q_2)\frac{C}{X} + \eta_2 \end{aligned}$$

The standard hydrodynamic model

It follows that the system of equations is:

$$\begin{cases} \gamma \dot{x}_1 = -k_1 x_1 + \epsilon(-k_2 x_2 + f_2) + f_1 + f^* \\ \gamma \dot{x}_2 = -k_2 x_2 + \epsilon(-k_1 x_1 + f_1 + f^*) + f_2 \end{cases}$$

heat exchanged by the bead i in the time τ

$$Q_i(\tau) = \int_0^\tau (\gamma \dot{x}_i - \gamma \xi_i)\, \dot{x}_i \, \mathrm{dt}$$

$$\begin{aligned} \xi_1 &= \frac{1}{\gamma}(f_1 + \epsilon f_2 + f^*) \\ \xi_2 &= \frac{1}{\gamma}(f_2 + \epsilon f_1 + \epsilon f^*) \end{aligned}$$

$$Q_i(\tau) = k_i q_{ii} + \epsilon k_j q_{ij}$$

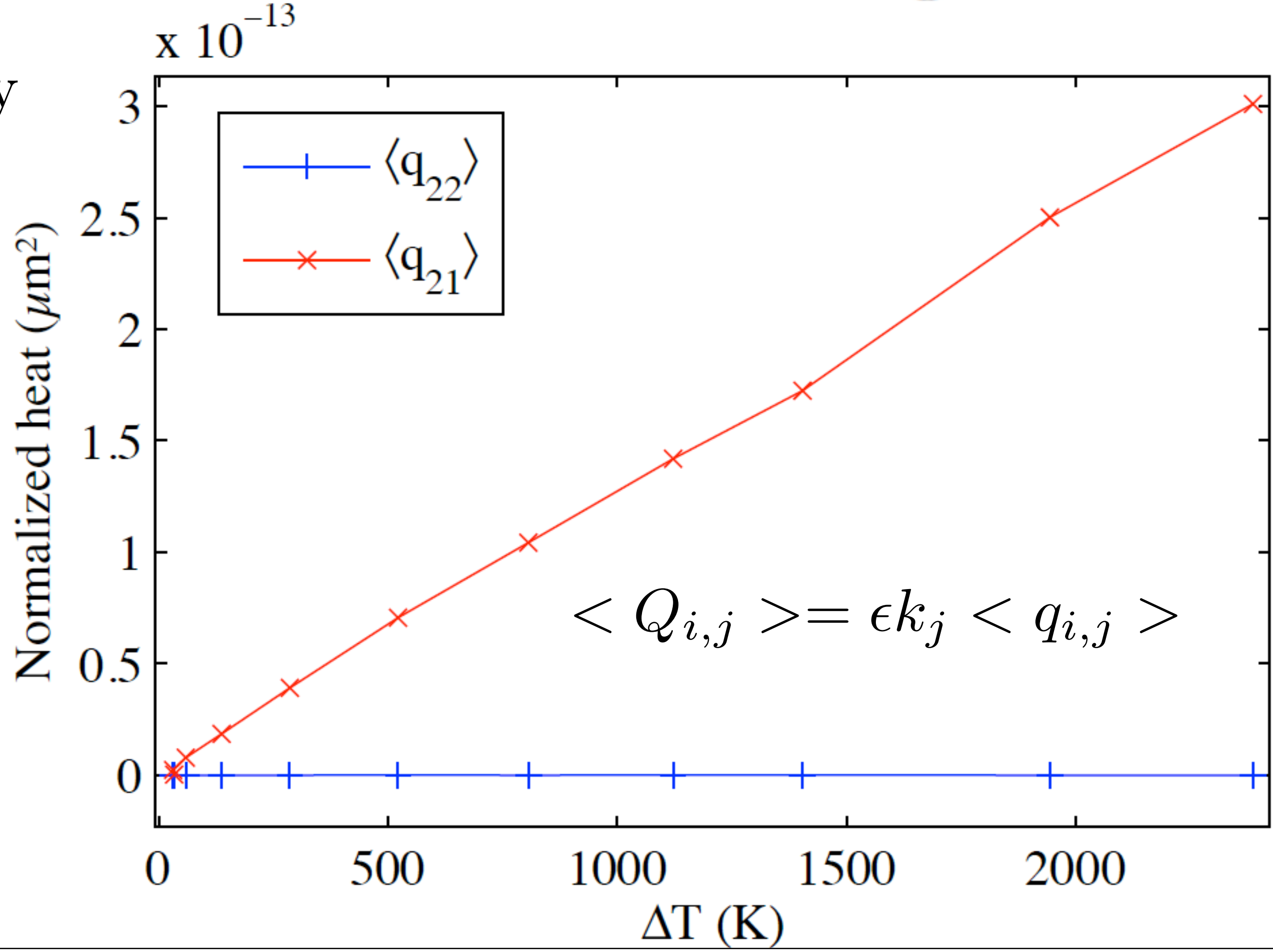
$$\begin{aligned} q_{ii} &= -\int_0^\tau x_i \dot{x}_i \, \mathrm{dt} \\ q_{ij} &= -\int_0^\tau x_j \dot{x}_i \, \mathrm{dt} \end{aligned}$$

The heat flux

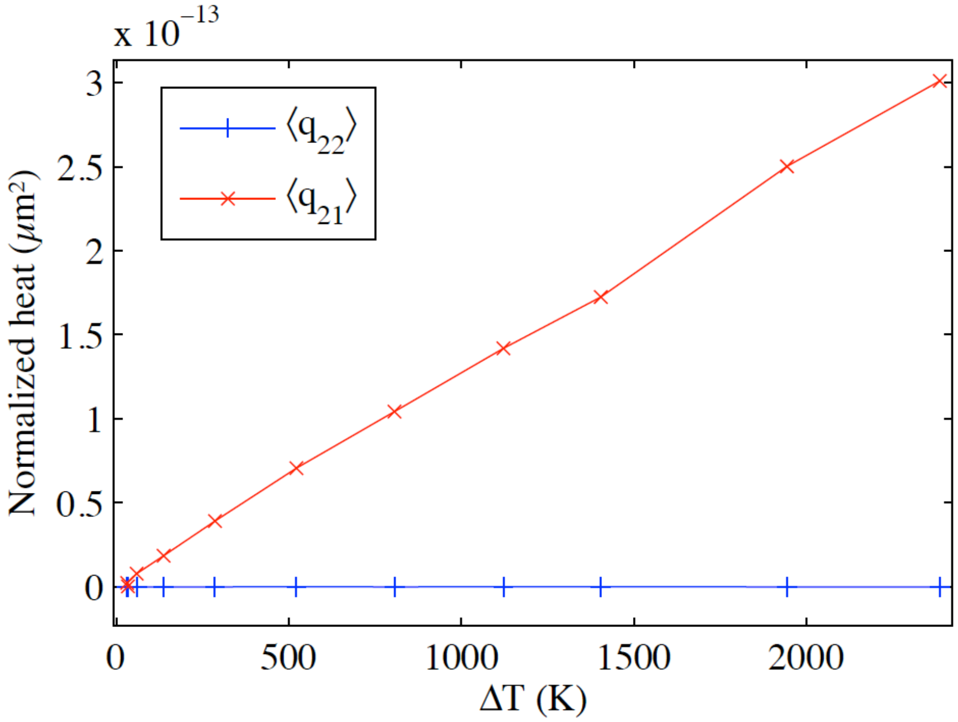
$$Q_i(\tau) = k_i q_{ii} + \epsilon k_j q_{ij}$$

$$q_{ii} = -\int_0^\tau x_i \dot{x}_i \, \mathrm{d}t$$
$$q_{ij} = -\int_0^\tau x_j \dot{x}_i \, \mathrm{d}t$$

potential energy
$< q_{ii} >= 0$



The heat flux



$$< Q_{i,j} >= \epsilon k_j < q_{i,j} >$$

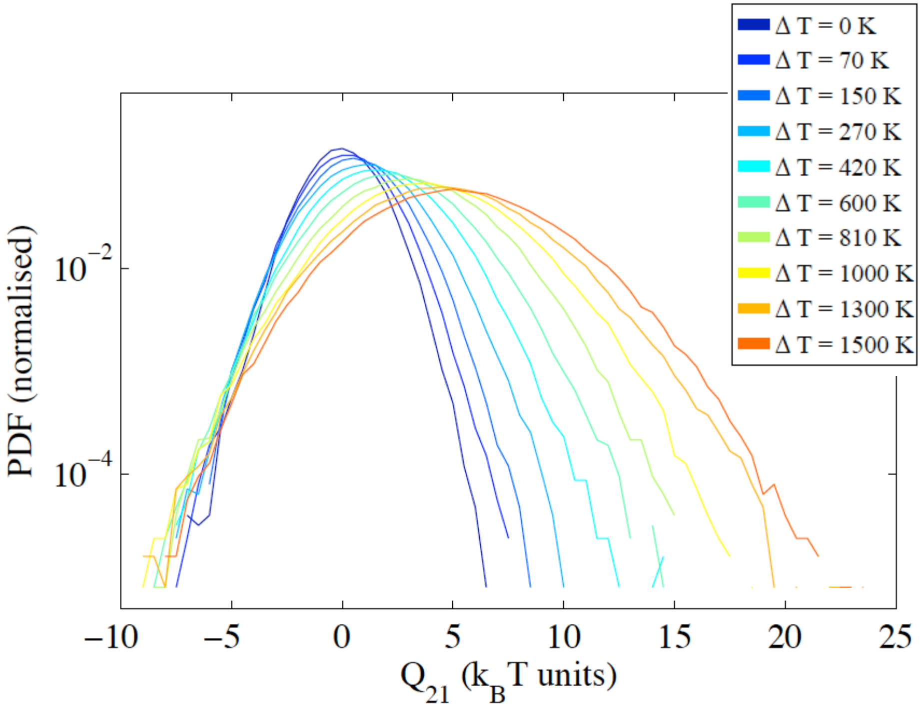
As for the electric case one obtains that

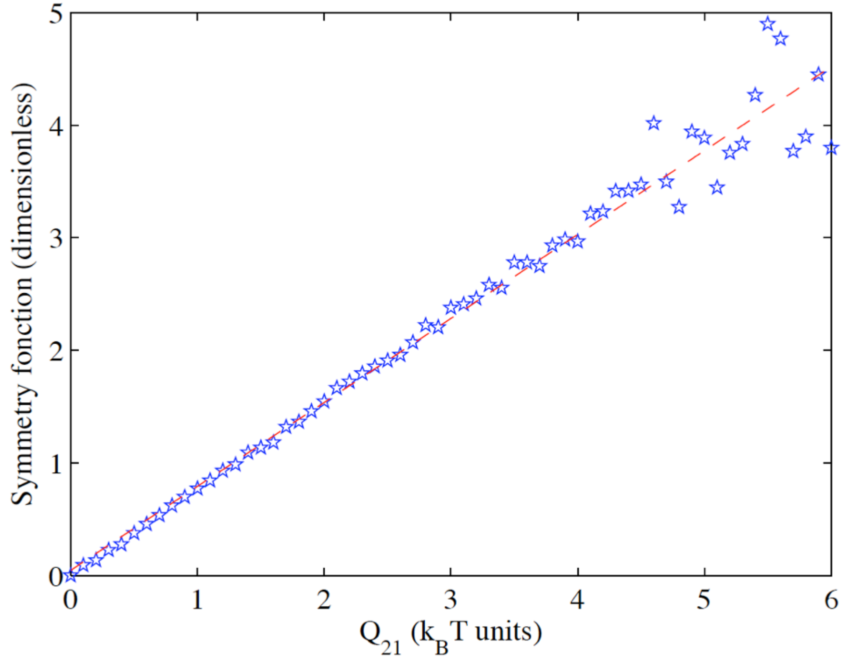
$$\sigma_i^2 - \sigma_{i,equilibrium}^2 \propto < Q_i >$$

but $< Q_{2,1} >= -\frac{k_1}{k_2} < Q_{1,2} >$ and

$$< Q_{2,1} > + < Q_{1,2} > \neq 0$$

The Fluctuation Theorem and the effective Temperature

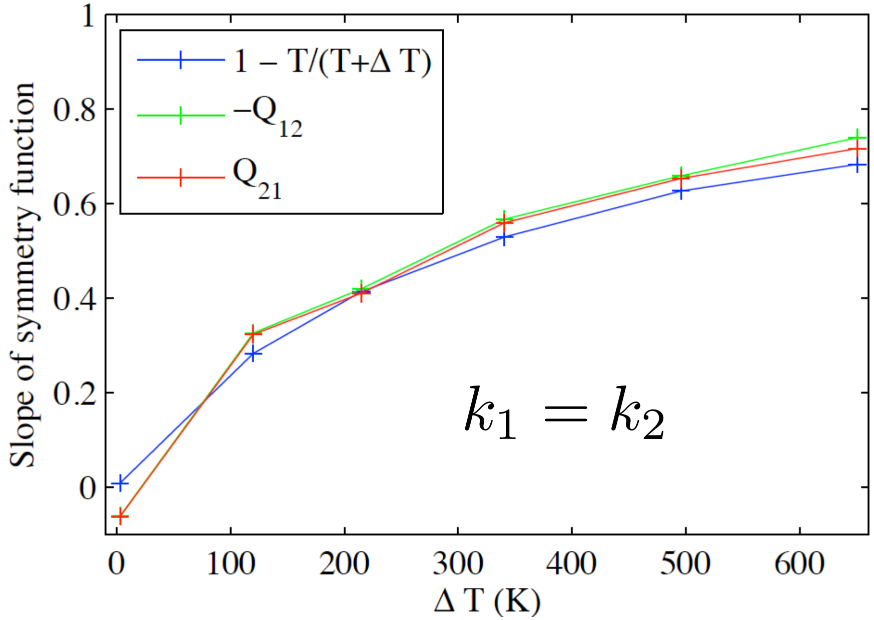


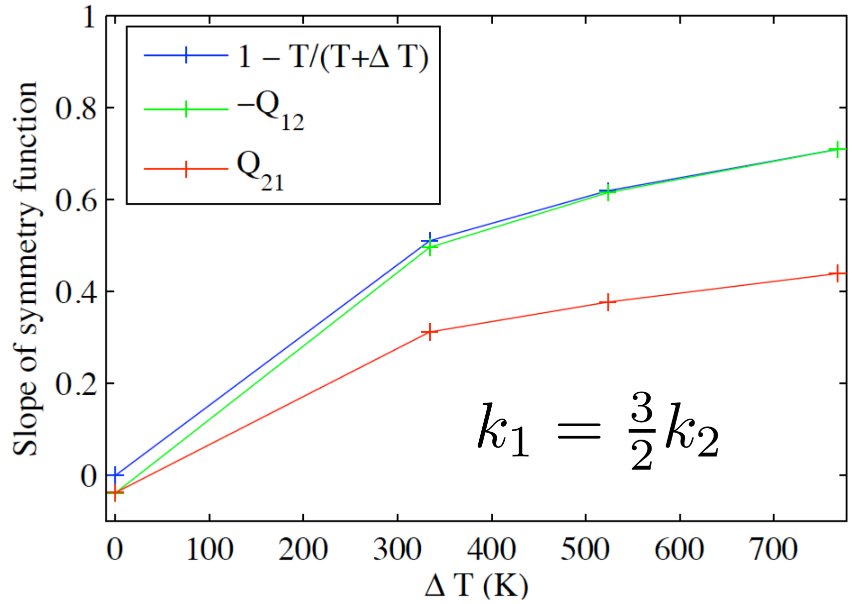


$$S(Q_{2,1}) = \log \frac{P(Q_{2,1})}{P(-Q_{2,1})} = \Delta\beta_{2,1} \frac{Q_{2,1}}{k_B T_2} \qquad \text{with } \Delta\beta_{2,1} = \frac{k_2}{k_1}(1 - T_2/T_1)$$

$$S(Q_{1,2}) = \log \frac{P(Q_{1,2})}{P(-Q_{1,2})} = \Delta\beta_{1,2} \frac{Q_{1,2}}{k_B T_2} \qquad \text{with } \Delta\beta_{1,2} = (1 - T_2/T_1)$$

Dependence of $\Delta\beta$ on ΔT





FT is satisfied both for $Q_{2,1}$ and $Q_{1,2}$ but with different $\Delta\beta$

Conclusions on particle interactions

- The differrence between out-equilibrium and equilibrium variance is proportional to the heat flux
- A hydrodynamic models precisely described the experimental data
- The FT seems to correctly estimate the effective temperature within experimental errors.
- The definition of heat is doubtful !