Stochastic facilitation in the brain?

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Unsolved Problems of Neural Noise

- To what extent may the oscillatory behavior of individual neurons, or linked populations of neurons, best be characterized as noise-driven quasi-cycles or as noisy limit cycles (or both)?

- What role(s) does neural noise play in the synchronization of, and information transmission between, neural populations located far from one another in the brain?

- To what extent do noise-driven quasi-patterns arise in the brain and affect its overall functioning?
Neural oscillations

Measure electrical activity of brain in any of several ways to get fluctuating broadband signal with roughly 1/f poser spectrum with bumps at some particular frequencies

Greenwood, McDonnell, Ward, 2015, Neural Computation
Noisy limit cycles or quasi-cycles?

- A common model of interaction of Excitatory and Inhibitory neurons in cortex of brain
- $S_{EE}, S_{II}, S_{EI}, S_{IE}$: Synaptic efficacies between neurons
Noisy limit cycles or quasi-cycles?


  \[ \tau_E dV_E(t) = \left[ -V_E(t) + g \left[ a_E (S_{EE} V_E(t) - S_{EI} V_I(t) - \theta_E + P_E(t)) \right] \right] dt + \sigma_E dW_E(t) \]

  \[ \tau_I dV_I(t) = \left[ -V_I(t) + g \left[ a_I (S_{II} V_I(t) - S_{IE} V_E(t) - \theta_I) \right] \right] dt + \sigma_I dW_I(t) \]

  \[ \tau_E, \tau_I: \text{time constants} \]

- \( V_E \): firing rate/LFP of excitatory neurons;
  \( P_E(t) \): input current

- \( V_I \): firing rate/LFP of inhibitory neurons

- \( S_{EE}, S_{II}, S_{EI}, S_{IE} \): synaptic efficacies between neurons

- \( g \): threshold function, usually \( g(x) = 1/(1+e^x) \), i.e. logistic

- \( W_E(t), W_I(t) \): standard Brownian motions (NOISE!)

- \( \Theta_E, \Theta_I, \sigma_E, \sigma_I, a_E, a_I \): constants
Noisy limit cycles

\[
\tau_E dV_E(t) = \left[ -V_E(t) + g \left[ a_E (S_{EE} V_E(t) - S_{EI} V_I(t) - \theta_E + P_E(t)) \right] \right] dt + \sigma_E dW_E(t)
\]

\[
\tau_I dV_I(t) = \left[ -V_I(t) + g \left[ a_I (S_{II} V_I(t) - S_{IE} V_E(t) - \theta_I) \right] \right] dt + \sigma_I dW_I(t)
\]

- \( g(x) = \frac{1}{1+e^x} \)

- \( \sigma_E = \sigma_I = 0 \): deterministic limit cycles around unstable fixed point

- \( \sigma_E = \sigma_I > 0 \): noisy limit cycles

Wallace et al., 2011
Quasi-cycles

\[ \tau_E dV_E(t) = \left[ -V_E(t) + g \left[ a_E (S_{EE} V_E(t) - S_{EI} V_I(t) - \theta_E + P_E(t)) \right] \right] dt + \sigma_E dW_E(t) \]

\[ \tau_I dV_I(t) = \left[ -V_I(t) + g \left[ a_I (S_{II} V_I(t) - S_{IE} V_E(t) - \theta_I) \right] \right] dt + \sigma_I dW_I(t) \]

- \( g(x) = x; \ a_E = a_I = 1 \)
- \( P_E(t) = 0; \ \Theta_E = \Theta_I = 0 \)
- \( \sigma_E = \sigma_I = 0 \): damps to stable fixed point
- \( \sigma_E, \sigma_I > 0 \): quasi-cycles – i.e. noise-driven oscillations at frequency determined by \( S_{EE}, S_{II}, S_{EI}, S_{IE} \) and \( \tau_E, \tau_I \)
- Inputs to neurons are typically Poisson-like, i.e. noise-driven

Wallace et al., 2011
Both models can generate narrow band oscillations.

Both models generate bursting, but with different mechanisms.

Both models generate a physiological range of local field potential oscillation frequencies.

We need new methods to distinguish the mechanism in data from brain recordings.

Greenwood, McDonnell, Ward, 2015, *Neural Computation*
Noisy synchronization?

- Noise can enhance synchronization between oscillators.
- Both noisy limit cycles and quasi-cycles display synchronization between distant populations.
- Synchronization is supposed to benefit neural information transfer.
- Does noise benefit information transfer via synchronization?

Greenwood, McDonnell, Ward, 2015, unpublished

Daffertshoffer & vanWijk, 2011, Frontiers
Noisy multiplexing?

- Population neural codes are noisy – sparse neural firing
- Noisy oscillations can be modulated by a signal and decoded at another area to recover the signal
- Does the noise play a computational role? Via quasi-cycles?

Akam & Kullman, 2014, Nat Rev Neurosci
Spatial quasi-patterns?

- Noise-driven systems can exhibit both quasi-cycles and quasi-patterns.
- Quasi-patterns are analogous to quasi-cycles but are spatial and characterized by a wave number instead of a frequency.
- Demonstrated in predator-prey and epidemiological models.
- Quasi-patterns of quasi-cycles in brains? Do they affect information processing and information transmission?

Butler & Goldenfeld, 2011, PRE

McKane et al, 2014, Bull Math Biol
Cortical waves

- Binocular-rivalry-induced cortical traveling waves: waves of dominance move across the visual cortex as perception of dominant stimulus changes.

- EEG standing alpha (8-12 Hz) waves: areas with similar colour are roughly in phase; dark and light are 180 deg out of phase.


Mexican Hat coupling on a 100 x 100 lattice of stochastic E-I processes

- Quasi-cycles at \( \approx 70 \) Hz at each lattice location
- Local Mexican hat coupling only; run for 10,000 iterations
- Quasi-Turing patterns at \( \approx 5-6 \) cycles in amplitudes (100/19=5.26)
- Little mathematical understanding

Greenwood & Ward, 2015, unpublished
Summary

- Noisy limit cycles or quasi-cycles?
- Noise-driven synchronization and information transfer?
- Spatial quasi-patterns mixed with noisy or noise-driven oscillations?