Brownian motors in the micro-scale domain: Enhancement of efficiency by noise

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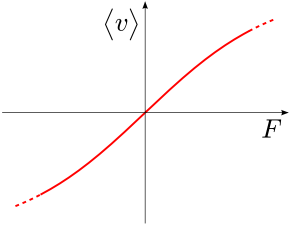
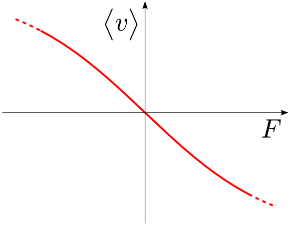
Le Chatèlier-Braun principle

$$\langle v \rangle = \mu F.$$



Typical response



Absolute negative mobility

Driven Brownian motor

Minimal model exhibiting ANM expressed in the dimensionless variables

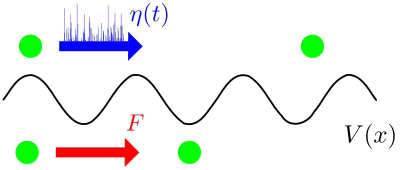
$$\ddot{x} + \gamma \dot{x} = -V'(x) + a\cos(\omega t) + F + \sqrt{2\gamma D_G}\,\xi(t).$$

We replace the constant static load F with the random force $\eta(t)$

$$F \to \eta(t), \quad \text{assuming} \quad \langle \eta(t) \rangle = F.$$

Fundamental question

Can noise $\eta(t)$ induce more effective transport than the constant force F?



Nonequilibrium noise

Generalized white Poissonian noise

$$\eta(t) = \sum_{i=1}^{n(t)} z_i \delta(t - t_i),$$

where $n(t)$ is Poissonian counting process

$$Pr\{n(t) = k\} = \frac{(\lambda t)^k}{k!} e^{-\lambda t}.$$

The process $\eta(t)$ presents white noise of finite mean and a covariance given by

$$\langle \eta(t) \rangle = \lambda \langle z_i \rangle, \quad \langle \eta(t)\eta(s) \rangle - \langle \eta(t) \rangle \langle \eta(s) \rangle = 2D_P \delta(t - s).$$

Its intensity reads

$$D_P = \frac{\lambda \langle z_i^2 \rangle}{2}.$$

Distribution of the amplitudes $\{z_i\}$ of the δ-kicks

$$\rho(z) = \zeta^{-1}\theta(z)e^{-z/\zeta}, \quad \langle z_i^k \rangle = k!\zeta^k, \quad k = 1, 2, \dots$$

The mean value and the intensity of white Poissonian shot noise

$$\langle \eta(t) \rangle = \lambda\zeta = \sqrt{D_P \lambda} \geq 0, \quad D_P = \lambda\zeta^2.$$

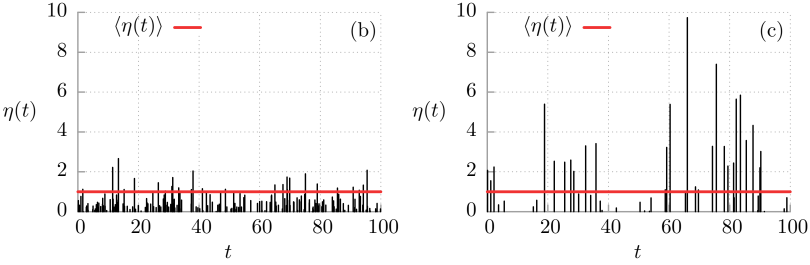


Figure : In (b): $\lambda = 2$, $D_P = 0.5$; in (c): $\lambda = 0.5$, $D_P = 2$.

Quantities of interest

- Average velocity

$$\lim_{t\to\infty} \langle \dot{x}(t) \rangle = \langle v \rangle + v_\omega(t) + v_{2\omega}(t) + \ldots$$

$$\langle v \rangle = \lim_{t\to\infty} \frac{\omega}{2\pi} \int_t^{t+2\pi/\omega} ds \prec v(s) \succ .$$

- Velocity fluctuations

$$\sigma_v^2 = \langle v^2 \rangle - \langle v \rangle^2, \quad v(t) \in [\langle v \rangle - \sigma_v, \langle v \rangle + \sigma_v].$$

- Stokes efficiency, $P_{out} = \gamma \langle v \rangle^2$, $P_{in} = \gamma[\langle v \rangle^2 + \sigma_v^2 - D_G]$

$$\varepsilon_S = \frac{P_{out}}{P_{in}} = \frac{\langle v \rangle^2}{\langle v \rangle^2 + \sigma_v^2 - D_G}.$$

Average velocity

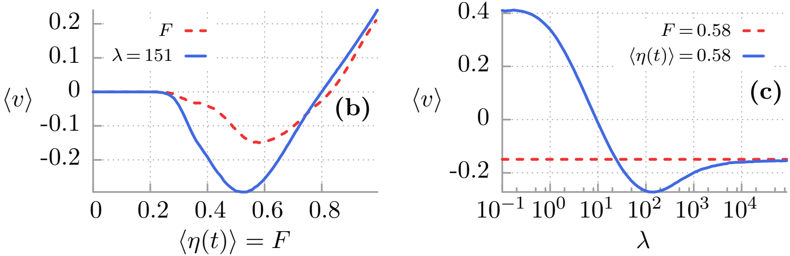


Figure : Anomalous transport regime. Parameters are $a = 8.95$, $\omega = 3.77$, $\gamma = 1.546$, $D_G = 0.001$, $\lambda = 151$.

$$\sigma_v^2 = \langle v^2 \rangle - \langle v \rangle^2.$$

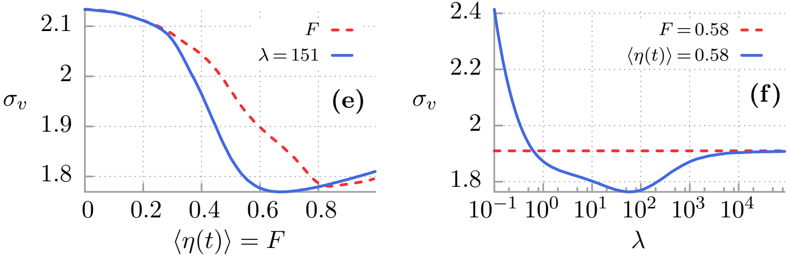
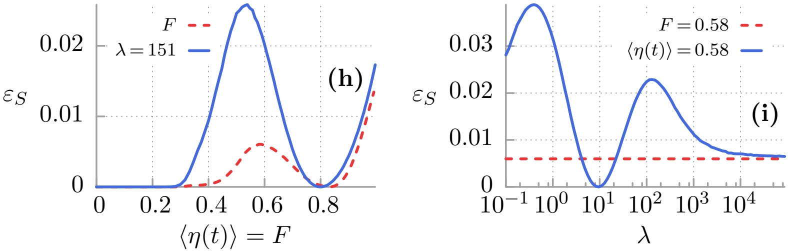


Figure : Anomalous transport regime. Parameters are $a = 8.95$, $\omega = 3.77$, $\gamma = 1.546$, $D_G = 0.001$, $\lambda = 151$.

Stokes efficiency

$$\varepsilon_S = \frac{\langle v \rangle^2}{\langle v \rangle^2 + \sigma_v^2 - D_G}.$$



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Take home message

Fundamental question

Can noise $\eta(t)$ induce more effective transport than the constant force F?

Answer

Yes, the Brownian motor can move much faster, its velocity fluctuations are much smaller and the motor efficiency increases several times in both normal and absolute negative mobility regimes.

New operating principle: consider replacing the constant force by nonequilibrium noise!

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